Probabilistic Behavior of Floods of Record in the United States

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Abstract: Literature on the probability distribution function (PDF) of annual maximum flood discharges is extensive, yet the literature on the PDF of the flood of record (FOR) is sparse. This is partially due to two facts: (1) the PDF for record events is more complex than the PDF for annual maxima; and (2) data sets for observed FORs are much smaller than for the annual maximum flood series from which they derive. We show that, if annual floods arise from a generalized extreme values (GEV) distribution, then the FOR also arises from another GEV distribution, which we term GEV_{max}. We also derive the moments and L-moments for the PDF of GEV_{max}. Using record flood observations at over 1,500 basins in the United States, we compared theoretical and empirical properties of observed values of the FOR. We found that, at both regional and national scales, the FOR values are on average more extreme than would be expected if they occurred randomly, and that they tend to form spatial clusters.

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Introduction

A record, in the context of this study, is defined as the largest recorded value in a time series. For example, a flood of record (FOR) is the largest recorded flood in a time series of annual floods measured at a particular site. In addition to the oft quoted observation that "in this world, nothing is certain but death and taxes" (attributed to Benjamin Franklin in a letter written in 1789), there is at least one other thing of which we can be certain: a record, no matter how large or long standing, will eventually be broken (Glick 1978).

The literature available on estimation of the magnitude and frequency of floods is enormous. In contrast, relatively little has been written on the behavior of the FOR. This is due to at least two factors: (1) the probability theory used to describe record events such as the FOR is much more complex than the theory which describes annual maximum floods; and (2) the sample size of an observed FOR is small for the simple fact that, while the number of observed annual floods at a site is equal to the number of years of data, there can be only one observed FOR for a given site.

Estimation of properties of the FOR in a region has been of fundamental interest in hydrology for well over a century. Envelope curves, which are derived from plots of the FOR versus their drainage area, have had widespread application in hydrology. Cudworth (1989) and England (2005) describe how envelope curves are used (1) in traditional flood hydrology studies; (2) in studies which compare design flood peaks for new and existing dams; and (3) as a way to judge the adequacy of estimates of the probable maximum flood. However, with the exception of a recent study by Castellarin et al. (2005), previous envelope curve applications have not provided a probabilistic interpretation of the envelope curve or the FOR. The impetus behind this study is primarily to derive a probability distribution function (PDF) of the FOR to better enable future probabilistic interpretations of regional envelope curves. After deriving a PDF of the FOR, we summarize the characteristics of observed FORs for large basins across the United States.

Previous Work

Conventional flood frequency analysis involves estimating the probability of extreme events based on historical stream gauge data for a site or a region. A major limitation to applying this method to evaluating the frequency of the FOR is the length of the historical data set; typical annual flood time series from gauged sites range from 50 to 100 years in length, and yet a flood near the upper bound of flood experience for a region may occur over a much greater time interval. Jarrett (1990), Jarrett and Tomlinson (2000), and others have developed methods for extending existing flood data with paleoflood estimates. Conover and Benson (1963), Carrigan (1971), and Wahl (1982) extended flood frequency analysis to FORs by scaling the FOR at each site within a region so that they could be assumed to have the same underlying PDF and therefore be grouped together. Malamud et al. (1996) applied power-law (fractal) extreme-value statistics and partial-duration series to flood flow frequency analysis and found they were better able to match maximum flood discharges estimated from paleoflood data for the Mississippi and Colorado rivers using power-law scaling than the log-Pearson type 3 PDF.

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Several recent studies have attempted to broadly characterize the behavior of the FOR. Vogel et al. (2001) introduced the probability distribution and moments for the expected number of record events in an n-year period. They found that the average number of record flood events in the United States behaved as one would expect serially independent floods to behave, as long as the effect of spatial correlation was accounted for. Benestad (2004) performed a similar analysis to the one by Vogel et al. (2001) but for global temperatures; however, their analysis did not correct for spatial correlation of the records. An intriguing study by Yongquan (1993) evaluates the relationship between solar activity and maximum floods (discharge greater than 10,000 cubic meters per second) for 141 rivers throughout the world. He found that the majority of flood maxima occurred within the period of three years prior to the sunspot minima and one to two years before and after the sunspot maxima. More recently, Castellarin et al. (2005) introduced an estimator of the exceedance probability associated with a regional envelope curve (REC), which accounts for the impact of intersite cross-correlation of floods. Monte-Carlo experiments were performed to assess the performance of their estimator, and generalized regional envelope curves were presented for assessing the impact of regional intersite cross-correlation on the likelihood of exceeding an observed REC. Troutman and Karlinger (2003) introduce an approach for determining the probability that at least one site in a region will experience a T-year flood in any given year. Although Troutman and Karlinger (2003) did not focus attention on the FOR, the foundation of their analysis is the determination of the joint distribution of annual maximum floods for all sites in a region, which is very closely related to the problem addressed by Castellarin et al. (2005) and this study.

Estimating the Exceedance Probability of the Flood of Record

The literature on the theoretical behavior of records is growing, so much so that a book (Arnold et al. 1998) has recently been published on the subject. In it, PDFs for records arising from exponential, Weibull, Pareto, and power-function distributions are presented, as well as methods of parameter estimation and statistical inference. Ang and Tang (1990) present exact distributions for maxima and minima generated from exponentially and uniformly distributed series of finite length, n. They and Lambert and Li (1994) provide asymptotic forms, following the Type I, Type II, and Type III extremal functions categorized by Gumbel (1958), for approximating the distribution of maxima and minima for large n taken from normal, Rayleigh, exponential, and extreme value PDFs. We are not aware of anyone who has derived similar expressions for the maxima generated from a generalized extreme value (GEV) distribution. Vogel and Wilson (1996) summarize investigations around the world and show that a consensus is emerging that the distribution of annual maximum floods is perhaps best approximated by a generalized extreme value (GEV) PDF. They also document that the GEV PDF provides a good fit to annual flood series at 1,490 sites across the continental United States. For this reason, we derive the PDF, moments, and L-moments for maxima (i.e., FOR) from a series of finite length n generated by a GEV distribution.

Derivation of CDF, Quantile Function, Moments, and L-moments of the Flood of Record

Let X be a random variable with a known cumulative density function (CDF) $F_X(x)$. The maximum value from the independent series X_i, $i=1,\ldots,n$ is

$$Y_{\max} = \max(X_1, X_2, \ldots, X_n) \tag{1}$$

For any value of y, $Y_{\max}$ is less than or equal to y if and only if all the Xs are less than or equal to y. The CDF for $Y_{\max}$ is then

$$\begin{aligned} F_{\max}(y) &= P(Y_{\max} \le y) \\ &= P(X_1 \le y, X_2 \le y, \ldots, X_n \le y) \\ &= P(X_1 \le y)P(X_2 \le y)\ldots P(X_n \le y) \\ &= F_1(y)F_2(y)\ldots F_n(y) \\ &= [F_X(y)]^n \end{aligned} \tag{2}$$

Therefore, the CDF of the maximum value from an independent sample of length n is a function of both the CDF of the parent distribution, $F_X(y)$, and n. This is convenient, because the maximum value of a sample of independent random variables can be estimated from the parameters of the distribution of the original random variable, X.

If X arises from a generalized extreme value (GEV) distribution, then its CDF is

$$F_X(x) = \exp\left\{-\exp\left(\frac{1}{\kappa}\cdot \ln\left[1 - \kappa \cdot \frac{(x-\xi)}{\alpha}\right]\right)\right\} \quad \text{for } \kappa \neq 0 \tag{3}$$

where ξ=location parameter; α=scale parameter; and κ=shape parameter (Jenkinson 1955). The range of x in Eq. (3) is $-\infty < x < \xi + (a/\kappa)$ for $\kappa > 0$ and $\xi + (a/\kappa) \le x < \infty$ for $\kappa < 0$, so that the annual flood series (as well as the FOR, as is shown later) will have an upper bound when $\kappa > 0$, as suggested by Enzel et al. (1993). As the shape parameter κ approaches zero, the GEV distribution approaches a Gumbel (extreme value type I) distribution. Further details on the GEV distribution, such as its product moments, L-moments, parameter estimators, and goodness-of-fit tests, can be found in Hosking and Wallis (1997), Stedinger et al. (1993), and Chowdhury et al. (1991).

Combining Eqs. (2) and (3) leads to the CDF for the maxima $Y_{\max}$ generated from GEV samples of length n:

$$F_{\max}(y) = \left[\exp\left\{-\exp\left(\frac{1}{\kappa}\cdot \ln\left[1 - \kappa \cdot \frac{(y-\xi)}{\alpha}\right]\right)\right\}\right]^n \tag{4}$$

The inverse or quantile function for $Y_{\max}$ is

$$y(p) = \xi + \frac{\alpha}{\kappa}\left[1 - \left(\frac{-\ln(p)}{n}\right)^{\kappa}\right] \tag{5}$$

where $p = F_{\max}(y)$. Eq. (5) is similar in form to the quantile function for the original GEV variate X:

$$x(p) = \xi + \frac{\alpha}{\kappa}[1 - (-\ln p)^{\kappa}] \tag{6}$$

Note that if $\kappa \to 0$, then $n^{\kappa} \to 1$; therefore, if the distribution of floods follows a Gumbel distribution, the distribution of the FOR will also be Gumbel. This is consistent with the findings of Ang and Tang (1990), Lambert and Li (1994), and others.

In this study, we are interested in the exceedance probability of the FOR, so the quantile function was expressed as a function of the exceedance probability for $Y_{\max}$, $p_n=1-p$, by substituting p_n into Eq. (5). The quantile function can be used to generate the rth ordinary moment about the origin, μ_r, using

$$\mu_r = \int_0^1 [y(p)]^r dp \tag{7}$$

Combining Eqs. (6) and (7), the mean, $\mu_{\max}$, and variance, $\sigma^2_{\max}$, of $Y_{\max}$ are

$$\mu_{\max} = \mu_1 = \int_0^1 [y(p)]dp = \xi + \frac{\alpha}{\kappa}\left[1 - \frac{\Gamma(1+\kappa)}{n^\kappa}\right] \tag{8}$$

$$\sigma^2_{\max} = \mu_2 - (\mu_1)^2 = \int_0^1 [y(p)]^2 dp - \mu^2_{\max}$$

$$= \left(\frac{\alpha}{\kappa \cdot n^\kappa}\right)^2 \{\Gamma(1+2\kappa) - [\Gamma(1+\kappa)]^2\} \tag{9}$$

Similar to the quantile function, the first two moments of $Y_{\max}$ differ in form from those of the GEV only by the additional term, n^κ. Hosking and Wallis (1997), Hosking (1990), and Stedinger et al. (1993) recommended the use of L-moments for fitting the GEV distribution. L-moments are similar in concept to ordinary moments except that L-moments are computed from linear combinations of order statistics rather than from the square or the cube of residuals (deviations from the mean). The first two L-moments for $Y_{\max}$ are

$$\lambda_{1\max} = \mu_{\max} = \xi + \frac{\alpha}{\kappa}\left[1 - \frac{\Gamma(1+\kappa)}{n^\kappa}\right] \tag{10}$$

$$\lambda_{2\max} = \frac{\alpha}{\kappa}\frac{\Gamma(1+\kappa)}{n^\kappa}(1-2^{-\kappa}) \tag{11}$$

Again, the expressions of L-moments for the $Y_{\max}$ differ from those for X only by the term n^κ. Interestingly, expressions for the L-moment ratios of $Y_{\max}$, L-skewness (τ_3) and L-kurtosis (τ_4), are the same as for the original GEV variate X, as presented in Hosking and Wallis (1997). The equivalence of the relationship between τ_3 and τ_4 for both X and $Y_{\max}$ was confirmed using a Monte Carlo experiment [see Douglas (2002) for details]. Thus, if X follows a GEV distribution, then the FOR generated from X is also GEV and the tail behavior of the FOR is identical to the tail behavior of X. It is only their means and coefficients of variation that differ. Hence, to describe the distribution of the FOR, all one needs to do is to estimate its mean and variance, because estimates of τ_3 and τ_4 can be estimated from the original observations of X.

Probabilistic Behavior of Observed Floods of Record

Regional Analysis

We investigated the regional behavior of floods of record (FORs) across the United States by first grouping them into nine superregions [as defined in Douglas et al. (2000) and shown in Fig. 1] and then estimating the exceedance probabilities associated with observed FORs at each site using Eq. (4). Analyses in this study were performed using data contained in the Hydro-Climatic Data Network (HCDN), a data set compiled by Slack et al. (1993), which is comprised of average streamflow values recorded on a daily, monthly, and annual basis at 1,571 gauging stations across the continental United States. Only stations with records suitable at a daily timescale (Time_Scale equal to D) were used in this study, which reduced the total number of usable stations to 1,474. We selected stations that had 41 years of continuous annual peak streamflows (1948–1988) so that the analysis would not be confounded by missing data and varying time series lengths. The 41-year length maximized the number of site-years of continuous data used from the overall HCDN. For completeness, we also analyzed all sites within each region regardless of record length or data continuity.

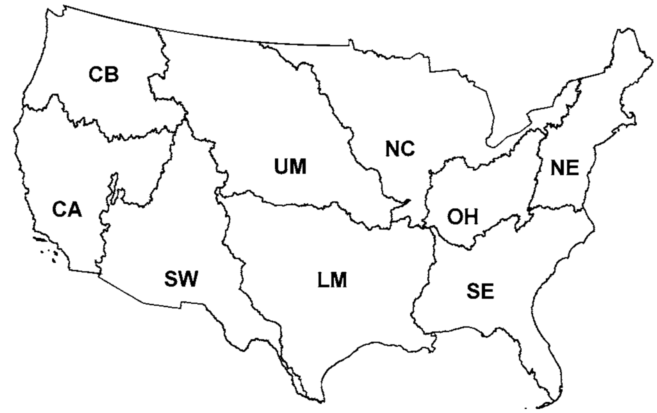

Fig. 1. Regional delineations [superregions as defined in Douglas et al. (2000)] used in this analysis

We estimated the GEV parameters, α, κ, and ξ, associated with each annual flood series x, using the L-moment estimators presented in Stedinger et al. (1993) and Hosking and Wallis (1997)

$$\hat{\kappa} = 7.8590c + 2.9554c^2 \quad \text{where} \quad c = \frac{2\lambda_2}{\lambda_3 + 3\lambda_2} - \frac{\ln(2)}{\ln(3)}$$

$$\hat{\alpha} = \frac{\kappa\lambda_2}{\Gamma(1+\kappa)(1-2^{-\kappa})}$$

$$\hat{\xi} = \lambda_1 + \frac{\alpha}{\kappa}[\Gamma(1+\kappa) - 1] \tag{12}$$

The maximum flood, y, for each annual flood series was selected and its exceedance probability was estimated as $p_n=1-F_{\max}(y)$ using Eq. (4). Figs. 2(a and b) are box plots of the regional exceedance probabilities p_n, computed in this manner for the sites with continuous 41-year time series and for all sites, respectively. If the FOR are independent across sites, p_n will be uniformly distributed within each region, with a median of 0.50 and upper and lower quartiles at 0.25 and 0.75, respectively. In general, p-values arising from any set of independent experiments, such as a set of hypothesis tests, are distributed uniformly on the interval (0,1) (Casella and Berger 1990). For further discussion of this result, see Loucks et al. (1981; p. 109) and Douglas (2002). Figs. 2(a and b) demonstrate that the p_n values are not uniformly distributed, as expected, and that in most regions the median p_n is lower than the expected value of 0.50. The fact that the regional median values are substantially lower than the expected regional median of 0.5 [Fig. 2(a)] in CB (Pacific Northwest), LM (Lower Midwest), and NE (Northeast) indicates that

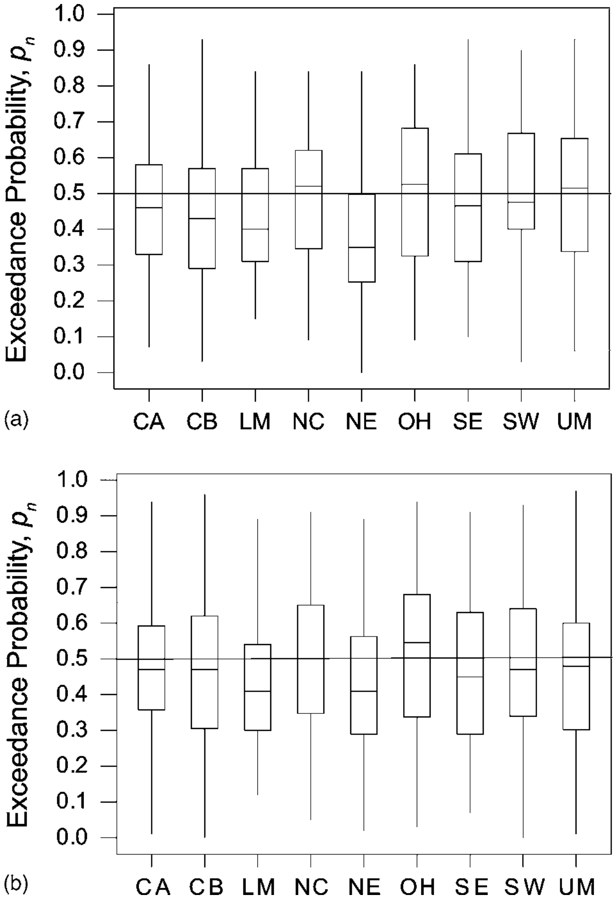

Fig. 2. Distribution of exceedance probabilities for observed floods of record, p_n: (a) for sites with continuous time series between 1948 and 1988 within each superregion; (b) for all sites within each superregion

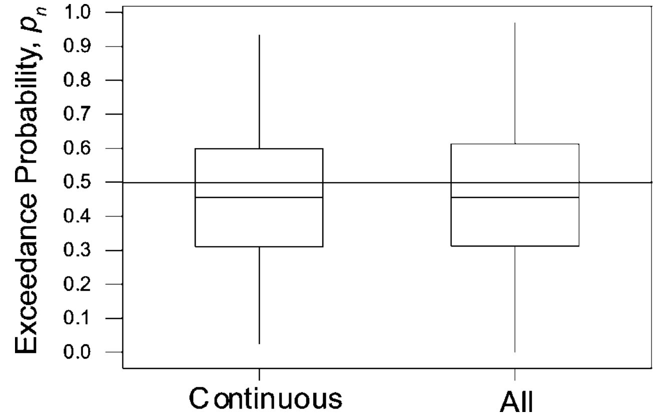

Fig. 3. Distribution of all p_n values across the United States: "continuous"=combination of all sites with continuous time series between 1948 and 1988; "all"=combination of all HCDN sites regardless of time series length

the FOR in these regions have been more extreme than would be expected if they were strictly random (independent) events.

Analysis Over the Entire Continental United States

To investigate the influence of the small number of sites in each region in the analysis, the regional p_n values were combined and evaluated as a single group (entire United States). Time series lengths at all HCDN sites range from 9 to 115 years, with a median of 44 years. Box plots (Fig. 3) and histograms [Figs. 4(a and b)] show the distribution of all p_n values calculated at HCDN sites in the United States. The box plots (Fig. 3) again illustrate that the distribution of exceedance probabilities is not uniform and that the median values are slightly below the expected median of 0.5 for a uniform distribution. The histograms [Figs. 4(a and b)] document that the mode of these distributions is significantly lower than 0.5.

Our analysis of the distribution of p_n values at regional and national scales suggests that, on average, FORs tend to be more extreme than would be expected if they occurred randomly (independently) in space. This pattern persisted even when randomly selected subsets of the p_n data were plotted. Interestingly, the nonuniform distribution of p_n shows a much lower than expected number of FORs in the tails of the distribution (very high or very low exceedance probabilities). Such a result could be an indica-

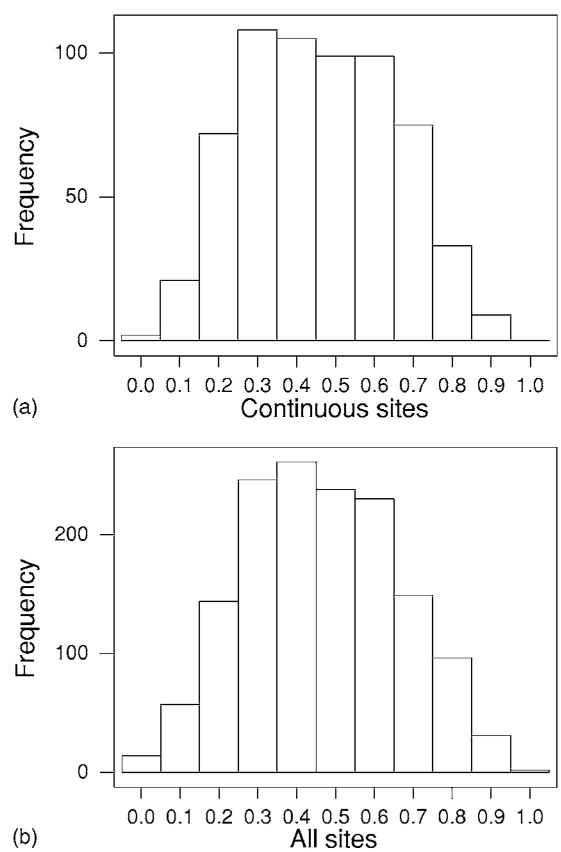

Fig. 4. Histogram of p_n values: (a) for combined HCDN sites with continuous 41-year time series; (b) for all HCDN sites in the United States

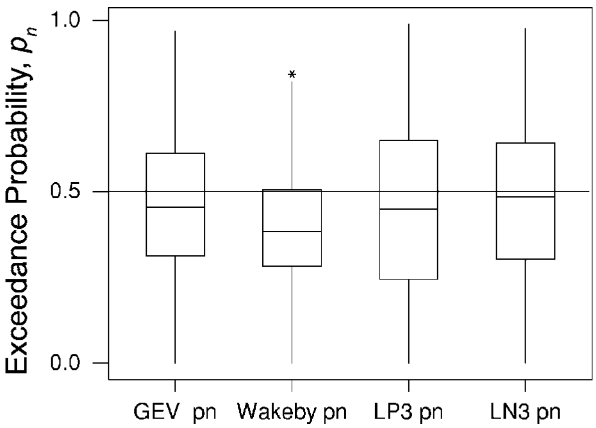

Fig. 5. Comparison of p_n values computed using generalized extreme value (GEV), Wakeby, Log-Pearson type III (LP3), and three-parameter lognormal (LN3) distributions

tion of an invalid distributional assumption associated with the original annual peak streamflow series. To test this, we fit the Wakeby (WAK), Log-Pearson type III (LP3), and three-parameter lognormal (LN3) distributions to all HCDN annual flood series using L-moments and computed p_n values using the relationship

$$p_n = 1 - (1 - p_e)^n \tag{13}$$

where n=length of the annual flood series; and p_e=exceedance probability of the FOR at each site computed from the distribution fit to each annual flood series. Fig. 5 shows box plots of p_n computed from the four different distributions. Although the shape of the distribution of p_n values does appear to depend on the distribution of the flood series from which they are generated, all are nonuniform, with median values less than the expected median value of 0.5. The histograms (not shown) for these distributions confirmed the lack of high and low p_n values in three of the four distributions (GEV, Wakeby, and LN3). The LP3 results in a more uniform looking distribution than the others, suggesting that the observed nonuniformity in p_n values may, in part, be due to our assumption of a GEV annual flood distribution, but the fact that, in all cases, the p_n distributions are thicker in the middle and thinner at the tails indicates that other factors may be contributing to this behavior as well. To better understand the apparent nonrandom behavior of FORs in the United States, we next turned our attention to the spatial and temporal behavior of the FOR.

Spatial and Temporal Distribution of Floods of Record across the United States

It does not seem unreasonable to hypothesize that a storm of sufficient magnitude to generate the FOR at one site would be large enough in areal extent to generate the FOR at nearby sites as well. If true, then one would expect to see FORs at adjacent sites within the same year and with similar exceedance probability values, given the assumption of regional homogeneity. We suspected that the nonuniformity of the p_n distribution was due, at least in part, to the fact that some of the FORs were related in both space and time. Because there is only one FOR at each site, we could not perform traditional multivariate analysis, such as estimation of the spatial correlation coefficient or principal components analysis, which would require having more than one FOR at each site. Instead, to gain a qualitative understanding of spatial and temporal relationships between the FORs, we plotted the station locations and estimated p_n values and the year in which each FOR occurred on separate maps of the United States. Figs. 6 and 7 show values of p_n and the year in which the FOR occurred, respectively, generated from the 623 sites with continuous annual maximum flood series between 1948 and 1988. In Fig. 6, p_n values between 0.35 and 0.65 are plotted with open circles; more extreme p_n values (<0.35) are plotted in gray and black triangles and less extreme (>0.65) are plotted in squares.

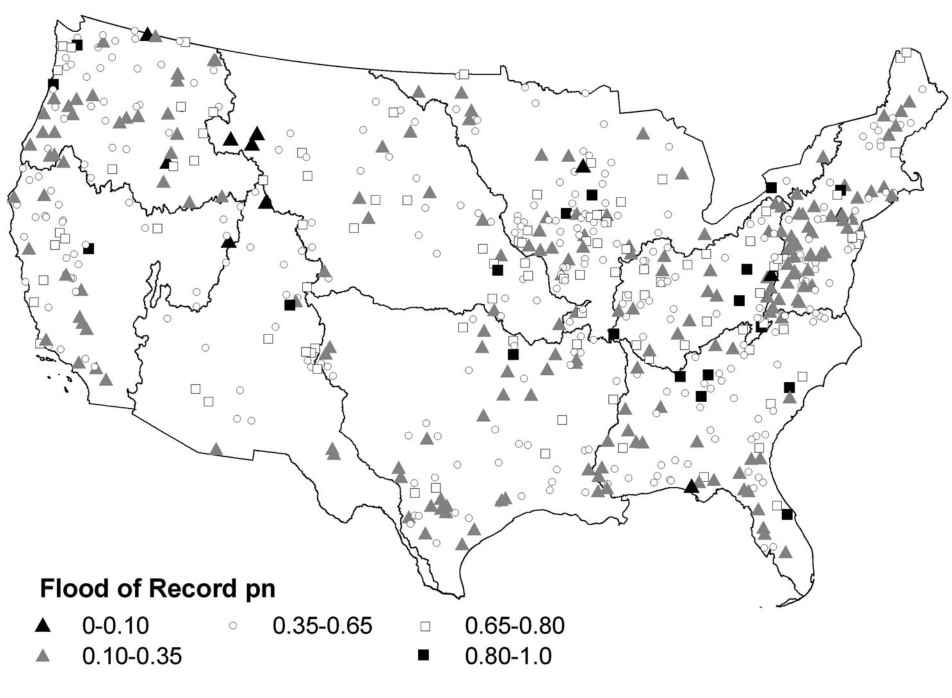

Fig. 6. Spatial distribution of p_n values for sites with continuous 41-year time series

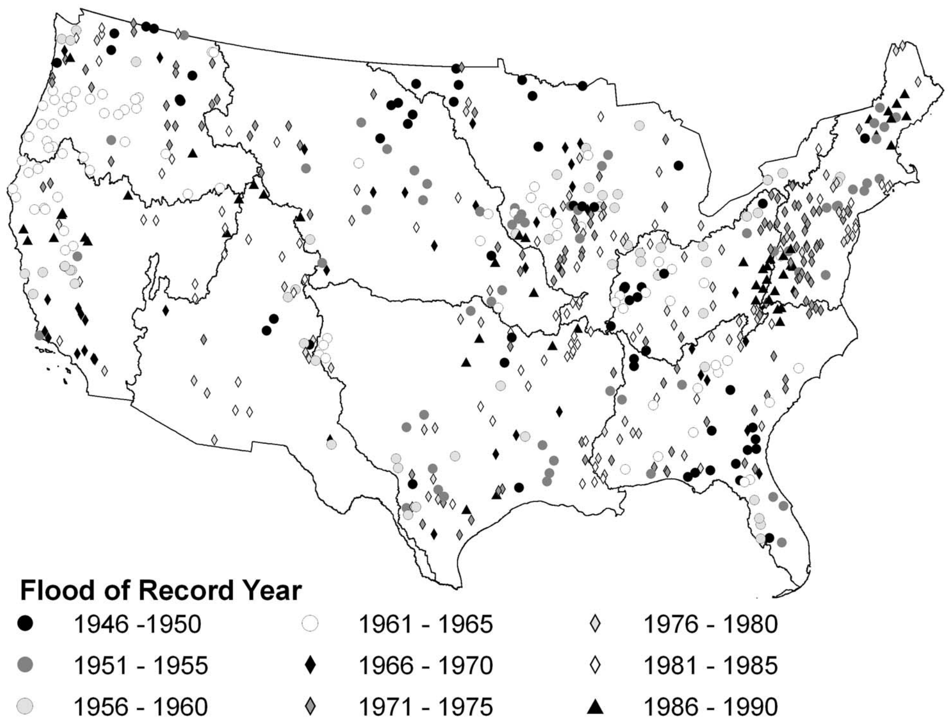

Fig. 7. Spatial distribution for FOR year for sites with continuous 41-year time series (1948–1988)

If FORs were randomly generated and spatially independent, then one would expect to see values interspersed and evenly distributed across the maps. This type of pattern is apparent in Fig. 6 in the central portions of the country, and more so for less extreme events (squares). However, the more extreme p_n values (gray triangles) exhibit both spatial and temporal clustering, especially in the northeast, along the Gulf coast, and in the Pacific Northwest. For instance, there is a cluster of sites with more extreme p_n values that extends through central Pennsylvania and into western New York State. All the FORs at these sites occurred in 1972 and were probably a result of Hurricane Agnes. Hurricane Agnes reached landfall in North Carolina and appears to have generated FORs as it moved northward through the mid-Atlantic states, as indicated by the line of gray diamonds (1971-1975) in Fig. 7 extending from North Carolina into western New York. The storm ceased its northerly progression over western New York and reversed direction, traveling southward over central Pennsylvania again before finally dissipating. This is likely the reason why the sites in central Pennsylvania and western New York possess more extreme p_n values than those in Maryland, Virginia, and North Carolina (see Fig. 6). This cluster of more extreme FORs is located entirely in the NE (northeast) region. Also within NE is a cluster of extreme p_n values over Connecticut and southeastern New York that are associated with FORs generated in 1955, probably by Hurricane Diane. These two clusters of sites with more extreme p_n values, along with another over eastern West Virginia and western Virginia and Maryland at which the FORs occurred in 1986 and 1987, appear to explain the much lower than expected average p_n for this region.

Conclusions

In this study, we summarized the literature on the FOR and derived a probability distribution with which to evaluate its exceedance probability, p_n, when annual maximum floods follow a GEV distribution. We showed that, when annual floods are GEV, the FOR is also GEV with the same shape parameter. To describe the distribution of the FOR (from the PDF of the GEV annual floods), one need only compute its mean and variance. Using the theory introduced here, we computed the exceedance probability p_n for observed floods of record in the United States and found that they are not uniformly distributed as would be expected if they occurred independently across space. Instead, we found that in many regions, and in the United States as a whole, floods of record are on average more extreme than would be expected if they occurred randomly. We found instances where more extreme floods of record ($p_n < 0.35$) are spatially clustered and can be temporally related to storm events that appear to have produced floods of record over large areas. This may explain the nonuniform distribution of the values of p_n and the fact that the regional median and average p_n values were less than the expected value of 0.5.

Acknowledgment

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