The Wigner Function in Signal Processing of Nanostructure

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ABSTRACT
The Wigner function is a powerful tool in signal processing, especially for analyzing time-frequency characteristics of signals. It is particularly useful when the signals are non-stationary or have complex frequency contents. The Wigner function provides a coherent time-frequency representation of signals, allowing us to visualize the evolution of spectral content over time. This makes it a valuable tool in various applications, such as speech recognition, image processing, and communication systems.

INTRODUCTION
The Wigner Function, denoted as $W(x,t)$, is a bilinear transform that provides a time-frequency representation of a signal. It is defined as:

$$W(x,t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} df \, e^{2\pi i f \tau} \left[ \hat{f}(x') \hat{f}^*(x - x') - \frac{1}{2} \frac{d}{dx} \hat{f}(x') \right] \delta(x' - \tau - x - f \tau)$$

where $\hat{f}(x')$ is the Fourier transform of the signal, $f$ is frequency, and $\tau$ is time. The Wigner function is useful in analyzing signals whose frequency content changes with time, such as those encountered in high-speed or high-frequency applications.

FUNDAMENTALS OF THE WIGNER FUNCTION
The Wigner function allows us to visualize the time-frequency content of a signal. It is especially useful for signals that are not easily analyzed by other means, such as those with time-varying frequency content.

Multi plots or data zooming

CALCULATION OF THE CURRENT DENSITY FROM THE MAGNETIC FIELD AT A DISTANCE Z USING THE WIGNER FUNCTION

Introduction:

The Wigner-Ville distribution [2-5] is a powerful tool for analyzing signals whose frequency content changes with time. It is especially useful for analyzing signals with high-frequency components, such as those encountered in high-speed or high-frequency applications.

The simulated magnetic field $B_y$ was calculated from a current density $J_x$ with four parallel traces, each of $20\, \mu m$ width, whose intensity and profile plots are shown in Figs. 10 and 11, respectively. The magnetic field $B_y$ at $z=10\, \mu m$ was calculated using the Wigner function of the simulation's magnetic field at $z=10\, \mu m$. The inverse Wigner transform was calculated using a random approximation method.

The plots clearly show the coarse structure as we increase the increment in frequency. From the above discussion we can conclude that, the resolution in the frequency for ‘$N$’ number of points in the signal with ‘$n$’ increment in the calculation of FFT, will be same as for ‘$2N$’ points and ‘$2n$’ increments. But in the latter case, the signal is better extracted from the background noise.

In the above plot we can see that the frequency $800\, \text{Hz}$ is clearer as we increased the number of points from 1024 to 4096. The disappearance of the lines corresponding to the other frequencies can be explained on the basis of the folding up of lines because of increased resolution.

CONCLUSIONS
The Wigner function is the tool through which we can obtain the accurate frequency and temporal information, even with very high-velocity signals. In our research we have applied the Wigner transforms for the calculation of the current densities from simulated magnetic fields and at 2-10 microns (high velocity cases). The inverse Wigner transform was calculated using a random approximation method.

REFERENCES

SPIE Optics East, Boston, MA (October 2006)