The Wigner Function in Signal Processing of Nanostructure

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ABSTRACT

To demonstrate the potential of the Wigner function in the processing of signals with very low SNR such as those generated by nanostructures, we present the results from two preliminary studies: visualization of the frequency content of a simulated signal of SNR as low as 0.2, and the recovery of the current density from the simulated magnetic field at a separation of z=10μm.

INTRODUCTION

Since its introduction in 1932 by Wigner[1], the Wigner function has been applied to the field of signal processing, where it is commonly known as the Wigner-Ville distribution[2-5]. The mathematical definition for the Wigner function of a signal x(t)

$$W(f,t) = \int_{-\infty}^{\infty} d\tau\, e^{-j2\pi f\tau}\, x(t+\tau)\, x(t-\tau)$$

where (f,t) are frequency and time, respectively. Since time information is retained, unlike the Fourier Transform, the Wigner function has been an option in time-frequency analysis of signals such as human speech. The resolution offered by the Wigner function is much better than that offered by a short-time Fourier transform.

TIME–FREQUENCY CONTENT IN THE WIGNER FUNCTION

Perturbation was added to the signal by knocking off some cycles of the signal and the Wigner function of the perturbed signal was computed and plotted using MATLAB codes. The signal and its Wigner function are shown in the figure below. We can clearly identify the position of the perturbation along the time axis by analyzing the Wigner function.

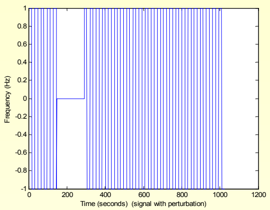

Figure 1: Signal with the perturbation.

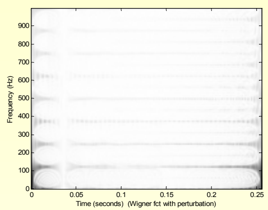

Figure 2: Wigner function of the signal in figure 1, we can clearly see the frequencies present in the signal and also the perturbation due to the missing cycles can be observed.

ANALYSIS OF NOISY SIGNALS USING THE WIGNER FUNCTION

The simulated signal consisted of simple harmonics of frequencies 500 Hz and 750 Hz, and was buried in random noise of equal amplitude to that of the signal or higher. We analyzed how increasing the number of points in the signal can enhance the signal's frequency content despite the noise.

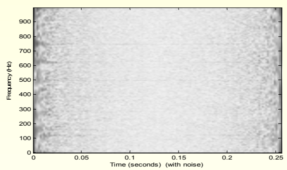

Figure 3 : The Wigner function with SNR 1:1 and N=1024 , we can clearly see the two frequencies 500 Hz and 700 Hz, also the other lines represent the beat frequencies which are generated from these two frequencies

As we increase the number of points (N) of the signal, the resolution along the frequency axis increases. The frequencies seem to disappear as we increase the number of points.

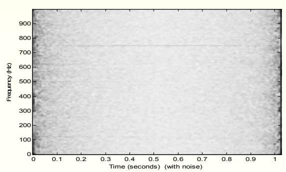

Figure 3 : The Wigner function with SNR 1:1 and N=1024 , we can clearly see the two frequencies 500 Hz and 700 Hz, also the other lines represent the beat frequencies which are generated from these two frequencies

In the above plot we can see that the frequency 625Hz is clearer as we increased the number of points from 1024 to 4096. The disappearance of the lines corresponding to the other frequencies can be explained on the basis of the thinning up of lines because of increased resolution.

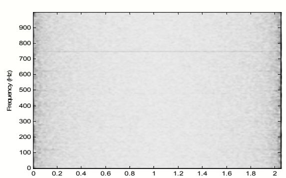

Figure 5: Wigner function for SNR 1:1, N=8191

Multi plots or data zooming

We divided the plots in to multiple figures so that each frequency component is magnified.

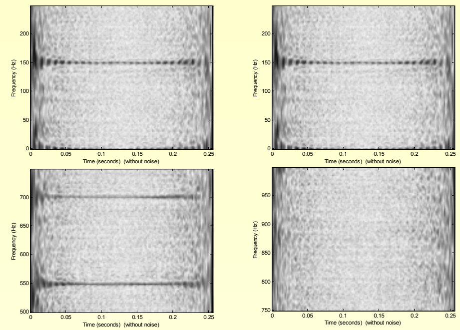

Figure 6: Wigner function (SNR 1:1, N=1024) is split into four different graphs so that we can clearly zoom in along the frequency axis and we can see better the frequency content.

Signal with higher noise contents

On dealing with signals with still higher noise contents, we have to go for higher and higher number of points in order to extract the true signal. This involves a lot of computing time. One way to work around this is to calculate the Wigner function only at some specific intervals. Since higher number of points corresponds to higher resolution in the frequency span, we can omit frequencies at specific intervals. This can be done by adding an increment to the FFT computation.

In this case a higher increment value corresponding to a coarser plot along the frequency axis. Since this works just the opposite way in increasing the number of points in the signal, we can have higher and higher increments as we go for higher and higher points.

Frequency plots with different frequency increments for a particular value of N.

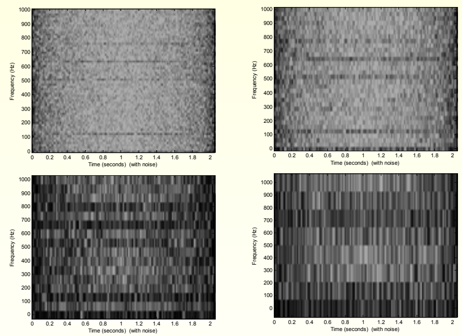

Figure 7: SNR 1:2, 8192 points, frequency increments: 64, 128, 256, 512

The plots clearly show the coarse structure as we increase the increment in frequency. From the above discussion we can conclude that, the resolution in the frequency for 'N' number of points in the signal with 'n' increment in the calculation of FFT, will be same as for '2N' points and '2n' increments. But in the latter case, the signal is better extracted from the background noise. Given below are the plots for 32,000 and 64,000 points for a signal with SNR=0.2. In the second figure we can see that the frequencies start showing up.

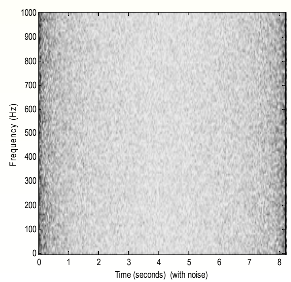

Figure 8: Wigner function for SNR 1:5, N=32,000. Frequency increment=128

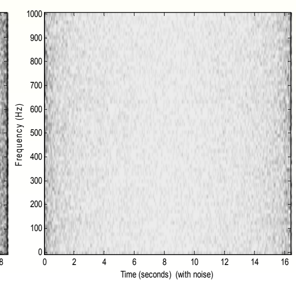

Figure 9: Wigner function for SNR 1:5, N=64,000. Frequency increment=512

CALCULATION OF THE CURRENT DENSITY FROM THE MAGNETIC FIELD AT A DISTANCE Z USING THE WIGNER FUNCTION

Introduction:

. Another type of noise is the "separation noise" that is present in a non-destructive measurement of the current in a nano sample by measuring the magnetic field it creates at certain distance z. Larger distance implies higher "separation noise", since at certain separation it is impossible to recover any current from a measured magnetic field. This current recovery method using the Fourier transform has been presented in a 1989 paper[7]. We will present results with a similar method using the Wigner transform instead of the Fourier transform. We use a random approximation to calculate the inverse Wigner function as needed in calculating the current from the field at a distance z. Other methods to calculate the inverse Wigner transform are described in the literature [8,9].

The simulated magnetic field B_y was calculated from a current density J_x with four parallel traces, each of 20μm width, whose intensity and profile plots are shown in Figs. 10 and 11, respectively. The magnetic field B_y at z=10μm is shown with an intensity plot in Fig. 12 and a profile in Fig. 13. The recovery of the current density starts with the Wigner transform of the magnetic field at z=10μm, which is shown in Figs. 14 and 15. Then the Wigner transform of the current density J_x was calculated and finally J_x was obtained by performing an inverse Wigner transform. The results are shown in Figs. 16 and 17.

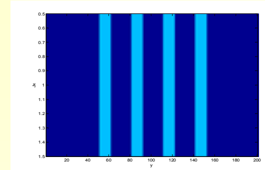

Figure 10: Simulated current densities Jx

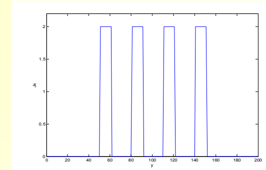

Figure 11: Profile of the simulated current densities Jx

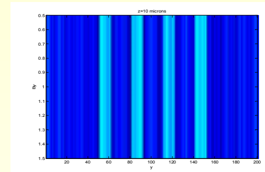

Figure 12: Simulated magnetic field By at z=10 μm

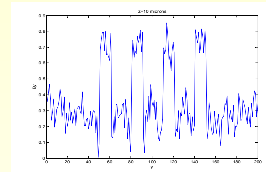

Figure 13: Profile of the simulated magnetic field By

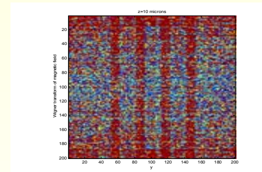

Figure 14: Wigner transform of the magnetic field By at z=10μm

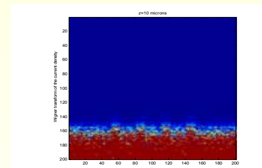

Figure 15: Wigner transform of the recovered current dens. Jx

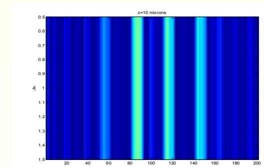

Figure 16: Recovered current density Jx

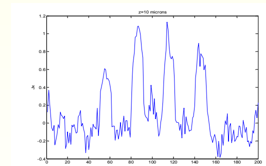

Figure 17: Profile of the recovered current density Jx

Conclusions

The Wigner function is the tool through which we can obtain the accurate frequency and temporal information, even with very high-noise signals. In the second part of our research we have applied the Wigner transform to the calculation of the current density from a simulated magnetic field at z=10 microns (high separation noise). The inverse Wigner transform was calculated using a random approximation method.

References

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