Wigner Function for a Quantum Wire with an Impurity

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Abstract
We present the Wigner function for a Gallium Arsenide (GaAs) quantum wire subjected to a magnetic field with an off-center donor impurity. The Wigner function is more sensitive to detecting changes than the variational wavefunction for smaller scale differences in the radial position of the impurity and the quantum wire. We used an infinite potential at the boundary of the quantum wire.

Introduction
A system consisting of an electron bound to a donor ion, with infinite potential barrier at the surface is present in a magnetic field parallel to the wire axis. Its Hamiltonian is given by

\[ \mathbf{H} = \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{\Lambda}^2}{2a^2} \mathbf{R} \left(1 - \mathbf{\Lambda}^2 \mathbf{R}^2 \right) \]

where

- \(\mathbf{\Lambda}^2 = \frac{\pi^2}{a^2} \)
- \(a\) is the electron Bohr radius
- \(R\) is the impurity ion position
- \(\mathbf{\Lambda}^2 = \frac{\pi^2}{a^2} \)

For a magnetic field parallel to the wire axis \(\mathbf{\Lambda}^2 = \frac{\pi^2}{a^2} \)

Following Branis, Li, and Bajaj1 and Brown and Spector2, we can get wavefunction

\[ \psi = A \exp \left( - \frac{1}{2} \left( \frac{a^2}{2} \right) \right) \]

where

- \(\psi\) is the electron wavefunction
- \(A\) is a variational parameter
- \(\mathbf{\Lambda}^2 = \frac{\pi^2}{a^2} \)

The binding energy \(E_b(R,B)\) of the hydrogenic impurity is

\[ E_b(R,B) = -\frac{\hbar^2}{2m} \left( 1 - \frac{2e^2}{aR} \left( 1 + \frac{1}{2} \right) \right) \]

For computational purposes, we normalize the expression for the binding energy \(E_b(R,B)\) in units of impurity Rydberg:

\[ E_b(R,B) = -\frac{\hbar^2}{2m} \left( 1 - \frac{2e^2}{aR} \left( 1 + \frac{1}{2} \right) \right) \]

Sensitivity of Wigner Function
When the donor impurity is placed at different positions off the quantum wire axis, the wavefunction will change. As we can see in the figure, Wigner Function is very sensitive to detecting changes at smaller scale differences.

In the way binding energy is

\[ E_b(R,B) = -\frac{\hbar^2}{2m} \left( 1 - \frac{2e^2}{aR} \left( 1 + \frac{1}{2} \right) \right) \]

Where

\[ C = -\frac{\hbar^2}{2m} \left( 1 - \frac{2e^2}{aR} \left( 1 + \frac{1}{2} \right) \right) \]

Conclusions
The Wigner function is more sensitive to detecting changes than the wavefunction for smaller scale differences in a quantum wire with an off-center impurity. Also, varying the relative separation along the quantum wire axis between the impurity ion and the electron just changes the scale of the wavefunction.

References