Physics 640
October 7, 2011

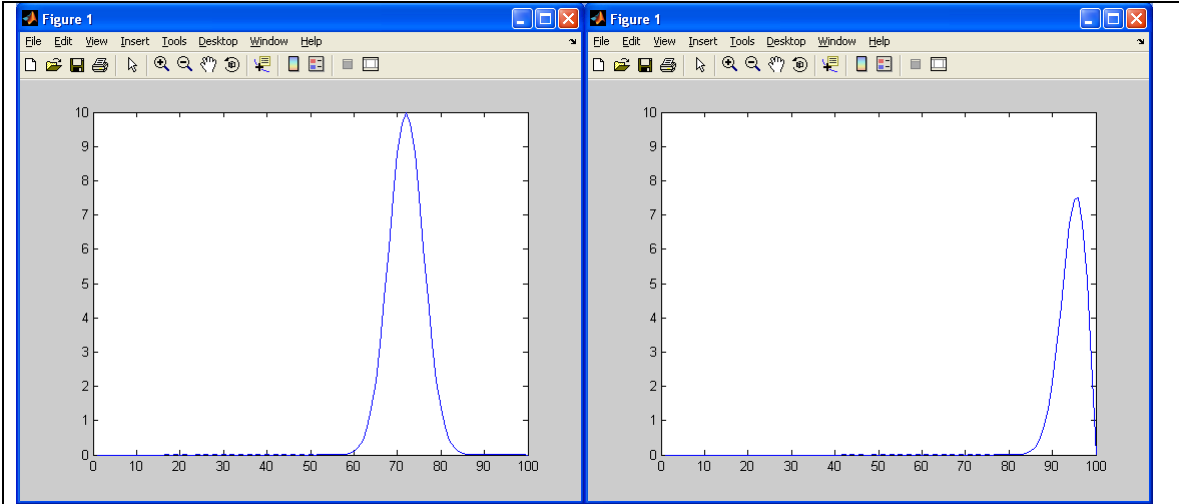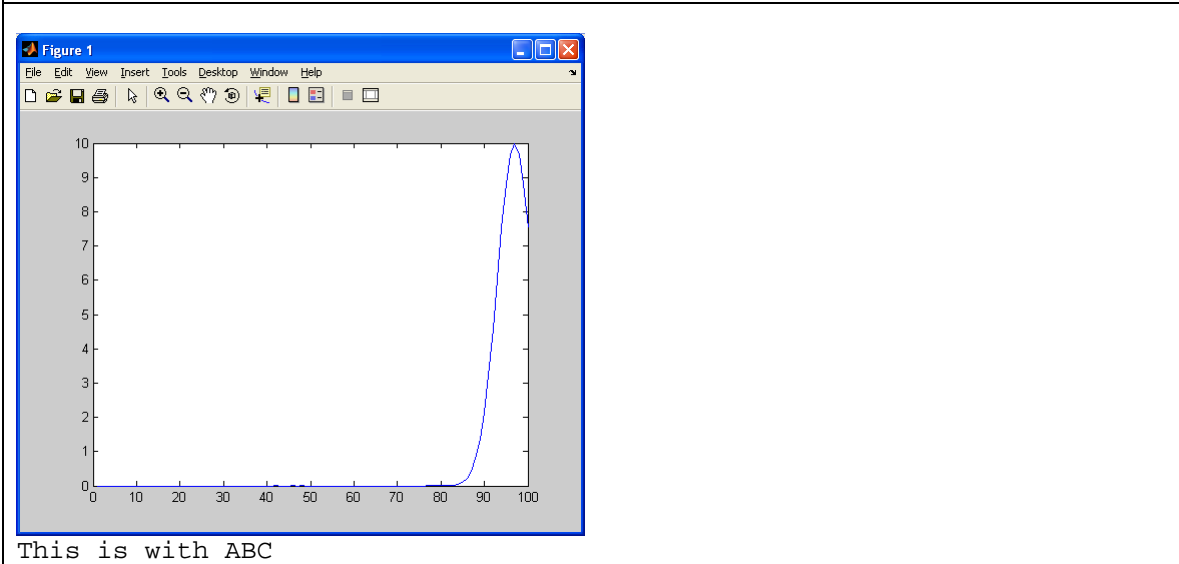**ABSORBING BOUNDARY CONDITIONS** (ABC)

In the Project 2 code, for the propagation algorithm, we used space indices i=2,nx-1, this means fx(nx,k) is always zero. Our Gaussian pulse dies out as it reaches the right boundary. For the pulse to go through the right boundary as if it was transparent, we need to apply some type of ABC

This is without ABC

This is with ABC

1) Boundary conditions:

EM waves: Mur 1st order, 2nd order ABC's, etc.
Surface waves/ string waves: constant flux

Mur ABC 1st order:

$$f(\vec{r},t) = f_0\, e^{j(\omega t - \vec{k}\cdot\vec{r})}\left[\frac{A(\theta,\varphi)}{r} + ...\right]$$

At boundary x=0, we have

$$\left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)f\bigg|_{x=0} = 0$$

Approximate this using finite differences:

$$f^{n+1}\left(0, j, k + \frac{1}{2}\right) = f^n\left(1, j, k + \frac{1}{2}\right) + \frac{c\delta t - \delta}{c\delta t + \delta}\left[f^{n+1}\left(1, j, k + \frac{1}{2}\right) - f^n\left(0, j, k + \frac{1}{2}\right)\right]$$

This ABC is used in the example code FDTD2D

Surface waves/ string waves: **constant flux (what happens at the next spatial point was at the previous point in the previous time iteration)**

$$f(t)\big|_{x=(N+1)\Delta x} = f(t-T)\big|_{x=N\Delta x}$$

2) In 1D the EM wave equations were very similar to the transverse wave in a string:

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} ;$$

In 2D, water waves and EM waves have different equations:

This is a set of 6 Maxwell's equations since there are 3 components to each vector

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \sigma E_x\right)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(-\frac{\partial H_z}{\partial x} - \sigma E_y\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right)$$

Above we have ignored variations in z for 2D EM waves in x and y

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \quad \left.\right\} TM$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \sigma E_x\right) \quad \left.\right\} TE$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(-\frac{\partial H_z}{\partial x} - \sigma E_y\right)$$

In 2D, EM waves decouple into Transverse Magnetic (TM) or Transverse Electric (TE) modes with 3 equations each to solve. While the equation for water/surface waves in 2D is:

$$\frac{\partial^2 y(x,z,t)}{\partial x^2} + \frac{\partial^2 y(x,z,t)}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 y(x,z,t)}{\partial t^2}$$

2D:
EM: TM or TE mode, coupled set of differential equations
Surface: direct extension of FDTD in 1D

Project 3 options (all with animations): a) and b) or a) and c)

a) 1)Run FDTD2D for some input file
2)Implement FDTD1D for EM waves with ABC and implement two and three media, should see reflection(s) and transmission(s) (FDTD can handle any number of media we may have). Implement a "perfectly absorbing layer".

b) Use existing FDTD2D with Mur ABC but implement a correct mode for waveguide coupling. You can implement two parallel waveguides into the FDTD code, then feed in a correct mode

c) Extend your FDTD1D with ABC to 2D, implement and visualize in Matlab

What is the waveguide?
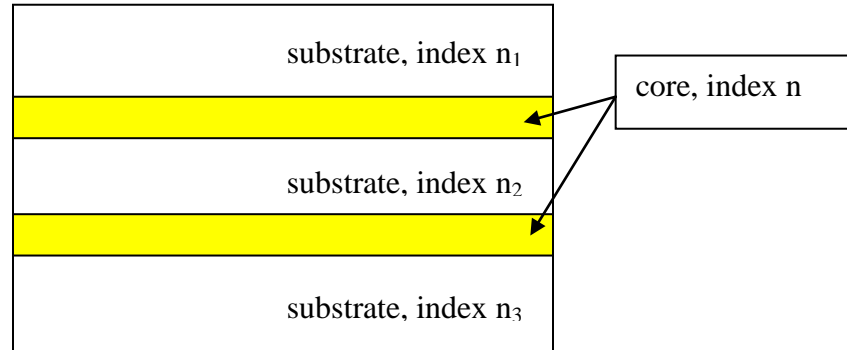
Physics requires only certain modes to exist within the two core layers (continuity of Maxwell equations at the separations). These modes can be solved using Mathematica to simplify a symbolic equation, or we can provide you with the numbers. Once you have the mode, the FDTD2D example code will propagate it properly within the core layers.

What is the coupling?
For example at initial time, more energy is fed into one waveguide, after sufficient time evolution, the energy goes into the other waveguide or "it is coupled into the other waveguide"

Visualization: in 2D we can do intensity plot (using color code to express intensity along the third direction), raised surface plot (using the third dimension to express the third direction)