Week 3 & 4
-Introduction to numerical algorithms
-Finite-Difference in the Time-Domain (FDTD) and the numerical solution of wave equations in 1D
-Finite-Difference in the Time-Domain (FDTD) and the numerical solution of Maxwell's equations in 2D
-Implementation of a FDTD code in Fortran:

Project 2:
   a) Develop a Fortran code to propagate a sinusoid or a Gaussian pulse in 1D, save outputs into a file
   b) Show the animations using Matlab

*How do I create a movies in Matlab?
By plotting results for successive time iterations, and grabbing frames and store in a movies file (.avi) that can be played later on in windows.
*What is a numerical algorithm?
To develop a numerical simulation, we need to go through the following a steps:
a) Define the problem and find appriate equation to describe it (e.g. wave propagation in a string → wave equation in 1D
y(x,t)= A cos(kx –wt)
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$ v is the wave speed)
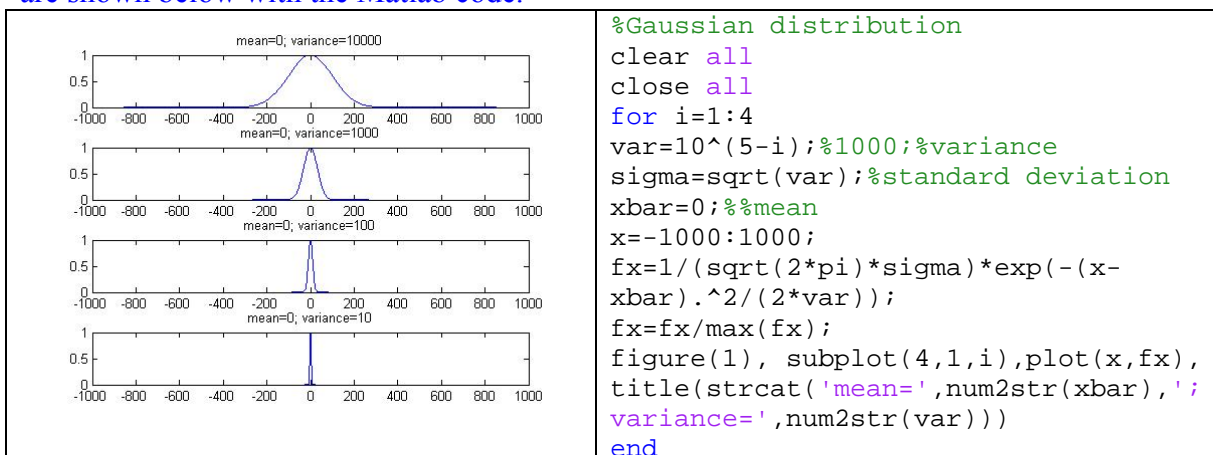b) Use a numerical scheme to obtain an algorithm, **appropriate discretization of the equation** to be used with a computer (in this Project #2 we will use the FDTD numerical scheme)
c) Implement the algorithm into a code using a computer language (for example we will use Fortran in this project)
d) Debug and plot results
e) Testing and validation against theoretical results, experimental values, or results from other establish numerical methods

*What is a gaussian wave packet? Examples of gaussian of different variances (widths) are shown below with the Matlab code.


```matlab
%Gaussian distribution
clear all
close all
for i=1:4
var=10^(5-i);%1000;%variance
sigma=sqrt(var);%standard deviation
xbar=0;%%mean
x=-1000:1000;
fx=1/(sqrt(2*pi)*sigma)*exp(-(x-
xbar).^2/(2*var));
fx=fx/max(fx);
figure(1), subplot(4,1,i),plot(x,fx),
title(strcat('mean=',num2str(xbar),';
variance=',num2str(var)))
end
```

Bilbiography:

1.- A. Taflove and S. C. Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed. Norwood, MA: Artech House, 2005.
2.- D.M. Sullivan, Electromagnetic Simulation Using the FDTD Method , IEEE Press
3.- K. S. Yee, ``Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media,'' *IEEE Trans. Antennas Propagat.*, vol. AP-14, no. 4, pp. 302--307, 1966.
4.- A. Taflove and M. E. Brodwin, "Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations," IEEE Trans. Microwave Theory and Techniques, vol. 23, pp. 623-630, Aug. 1975. (Download Paper2.pdf)
5.- A. Taflove and K. R. Umashankar, "Solution of Complex Electromagnetic Penetration and Scattering Problems in Unbounded Regions," pp. 83-113 in Computational Methods for Infinite Domain Media-Structure Interactions, American Society of Mechanical Engineers, AMD vol. 46 (1981).
6.-V. Sathiaseelan, B. B. Mittal, A. J. Fenn, and A. Taflove, "Recent Advances in External Electromagnetic Hyperthermia," Chap. 10 in Advances in Radiation Therapy, B. B. Mittal, J. A. Purdy, and K. K. Ang, eds. (Cancer Treatment and Research, S. T. Rosen, series ed.) Dordrecht, Netherlands: Kluwer (1998).

FDTD Method and the 1D Maxwell Equation:

Finite difference is an approximation to derivatives using Taylor's expansion of different orders. The Maxwell equations (governing behavior of electric and magnetic fields) can be written in differential form:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This is a set of 6 equations since there are 3 components to each vector

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right)$$

σ is the electrical conductivity
In 1D: suppose we work in x; variations in y and z are zero

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x}\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \sigma E_z\right)$$

In empty space (no materials) $\sigma=0$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial E_z}{\partial x}\right) \rightarrow \frac{\partial}{\partial x}\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial^2 E_z}{\partial x^2}\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial H_y}{\partial x}\right) \rightarrow \frac{\partial}{\partial t}\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial^2 H_y}{\partial t\partial x}\right) \rightarrow \frac{\partial^2 E_z}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E_z}{\partial x^2} = c^2\frac{\partial^2 E_z}{\partial x^2}$$

Using Taylor's expansion up to second order, derivatives can be approximated using finite differences: we need to discretize space and time derivatives.

$$\frac{\partial F^n(i)}{\partial x} \cong \frac{F^n\left(i+\frac{1}{2}\right) - F^n\left(i-\frac{1}{2}\right)}{\delta}$$

$$\frac{\partial F^n(i)}{\partial t} \cong \frac{F^{n+\frac{1}{2}}(i) - F^{n-\frac{1}{2}}(i)}{\delta t}$$

n is the time index; i is the space index; δt is the time increment; δ is the space increment.

Next step is to apply this numerical scheme (FDTD) to write the algorithm for the wave equation and proceed to its implementation using Fortran