Chapter 7

7.1.1

a) $\bar{X} = E\{M\,\mathrm{rect}(t/2T)\} = \bar{M}\,\mathrm{rect}(t/2T) = 6\,\mathrm{rect}(t/2T)$

b) $X(f) = \mathcal{F}\{M\,\mathrm{rect}(t/2T)\} = 2TM\,\mathrm{sinc}(2Tf)$

c) $\bar{X}(f) = E\{2TM\,\mathrm{sinc}(2Tf)\} = 12T\,\mathrm{sinc}(2Tf)$

d) $\lim_{T\to\infty} 2TM\,\mathrm{sinc}(2Tf) = \lim_{T\to\infty} M\,\frac{\sin 2\pi f T}{\pi f} = M\,\delta(f)$

7.2.1

a) $\mathcal{F}^{-1}\left[8\,\frac{\sin 4w}{4w}\right] = \mathrm{rect}(t) = 1 \quad |t| \le 4$
$\quad = 0 \quad$ otherwise

$\mathcal{F}^{-1}\left[16\,\frac{\sin 8w}{8w}\right] = 1 \quad |t| \le 8$
$\quad = 0 \quad$ otherwise.

$\therefore \int_{-\infty}^{\infty} \frac{\sin 4w}{4w}\,\frac{\sin 8w}{8w}\,dw = \frac{2\pi}{128}\int_{-4}^{4} 1\,dt = \frac{\pi}{8}$

b) $\frac{1}{w^4+5w^2+4} = \frac{1}{(w^2+1)(w^2+4)}$

$\mathcal{F}^{-1}\left[\frac{1}{w^2+1}\right] = \frac{1}{2}e^{-|t|}, \quad \mathcal{F}^{-1}\left[\frac{1}{w^2+4}\right] = \frac{1}{4}e^{-2|t|}$

$\int_{-\infty}^{\infty} \frac{1}{w^4+5w^2+4} = \frac{2\pi}{8}\int_{-\infty}^{\infty} e^{-|t|}\cdot e^{-2|t|}\,dt = \frac{\pi}{2}\int_{0}^{\infty} e^{-3t}\,dt = \frac{\pi}{6}$

7.2.2

$\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(w)\,dw = \frac{1}{2\pi}\int_{-8\pi}^{8\pi}\left(1-\frac{|w|}{8\pi}\right)dw$

$= 4$

7.2.3

$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(w)\,dw = 4$

a) $\frac{1}{2\pi}\int_{-\infty}^{\infty} 4\,S_X(w)\,dw = 4 \times 4 = 16$

b) $\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(4w)\,dw = \frac{1}{4}\,\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(4w)\,d(4w) = \frac{1}{4}\cdot 4 = 1$

c) $\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X\left(\frac{w}{4}\right)dw = 4\,\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X\left(\frac{w}{4}\right)d\left(\frac{w}{4}\right) = 4\cdot 4 = 16$

d) $\frac{1}{2\pi}\int_{-\infty}^{\infty} 4\,S_X(4w)\,dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(4w)\,d(4w) = 4$

7.3.1

a) No. Not even

b) Yes.

c) Yes.

d) No. Might be negative

e) Yes.

f) No. Not even

7.3.2

a) $E[X] = E(M) = 3$

b) $\overline{X^2} = E(M^2) + \frac{1}{2}5^2 + \frac{1}{2}10^2 = 21 + \frac{25}{2} + \frac{100}{2} = 82.5$

c) $S_X(w) = 2\pi\cdot 21\cdot\delta(w) + \frac{\pi}{2}5^2\delta(w-10) + \frac{\pi}{2}5^2\delta(w+10)$
$\quad + \frac{\pi}{2}10^2\delta(w-5) + \frac{\pi}{2}10^2\delta(w+5)$
$\quad = 42\pi\,\delta(w) + 12.5\pi\left[\delta(w-10) + \delta(w+10)\right] + 50\pi\left[\delta(w-5) + \delta(w+5)\right]$

7.3.3

a) $\bar{X} = \sqrt{32\pi/2\pi}\,\sqrt{16} = 4$

b) $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\,d\omega = 56$

$\sigma_X^2 = 56 - 16 = 40$

c) $0, \pm 6, \pm 12$

7.3.4 Let Y = pulse height, 0 or 2 with equal probability

a) $\bar{Y} = 1$

b) $\overline{Y^2} = 2 \Rightarrow \sigma_Y^2 = 1$

c) Use (7-25) $F(\omega) = \int_{-0.25}^{0.25} 1 \cdot e^{-j\omega t}\,dt = 0.05\left[\frac{\sin 0.025\omega}{0.025\omega}\right]$

$|F(\omega)|^2 = 2.5 \times 10^{-3}\left[\frac{\sin \omega/40}{\omega/40}\right]^2$

$\therefore S_X(\omega) = 2.5\times10^{-3}\left[\frac{\sin \omega/40}{\omega/40}\right]^2\left[10 + 200\pi\sum_{n=-\infty}^{\infty}\delta(\omega - 20\pi n)\right]$

*** 7.4.1**

a) $S_X(\omega) = \frac{16(\omega^2+36)}{\omega^4+13\omega^2+36} \Rightarrow S_X(s) = \frac{16(-s^2+36)}{s^4-13s^2+36}$

b) $S_X(s) = \frac{16(s+6)(-s+6)}{(s-2)(s+2)(s-3)(s+3)} \Rightarrow$ Poles: $\pm 2, \pm 3$; Zeros: ± 6

c) When $s = 1$

$S_X(2\pi) = \frac{16((2\pi)^2+36)}{(2\pi)^4+13(2\pi)^2+36} = 0.573$

d) Let $s' = s/100$

$S_X(s') = \frac{16\left(\frac{s}{100}+6\right)\left(\frac{-s}{100}+6\right)}{\left(\frac{s}{100}-2\right)\left(\frac{s}{100}+2\right)\left(\frac{s}{100}-3\right)\left(\frac{s}{100}+3\right)}$

$= \frac{16\times10^4(s+600)(-s+600)}{(s-200)(s+200)(s-300)(s+300)}$

7.4.2

a) $S_X(s) = \frac{A(s+5)(s-5)}{(s+2)(s-2)(s-j4)(s+j4)(s+6)(s-6)(s+j3)(s-j3)}$

$= \frac{A(s^2-25)}{(s^2-4)(s^2+16)(s^2-36)(s^2+9)}$

$\because S_X(0) = 10 = \frac{A(-25)}{(-4)(16)(-36)(9)} \Rightarrow A = -8294.4$

$\therefore S_X(s) = \frac{-8294.4(s^2-25)}{(s^2-4)(s^2+16)(s^2-36)(s^2+9)}$

b) $s^2 = -\omega^2$

$S_X(\omega) = \frac{8294.4(\omega^2+25)}{(\omega^2+4)(\omega^2-16)(\omega^2+36)(\omega^2-9)}$

c) $S_X(1) = \frac{-8294.4(-24)}{(-3)(15)(-35)(10)} = 12.64$

7.5.1

a) $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\omega^2+2\omega+1}\,d\omega = \frac{1}{2\pi}\,\frac{2}{\sqrt{3}}\tan^{-1}\frac{2\omega+2}{\sqrt{3}}\Big|_{-\infty}^{\infty}$

$= 1/\sqrt{3}$

b) $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\omega^2+4}{\omega^4-4\omega^2+1}\,d\omega$

$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{-s^2+4}{s^4+4s^2+1}\,ds = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{(s+2)(-s+2)}{(-s^2-2-\sqrt{3})(s^2-2+\sqrt{3})}\,ds$

$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{(s+2)(-s+2)}{[s+(-2-\sqrt{3})^{1/2}][s+(-2+\sqrt{3})^{1/2}][-s+(-2-\sqrt{3})^{1/2}][-s+(-2+\sqrt{3})^{1/2}]}\,ds$

$\therefore c(s) = s+2$

$d(s) = [s+(-2-\sqrt{3})^{1/2}][s+(-2+\sqrt{3})^{1/2}] = s^2+[(-2-\sqrt{3})^{1/2}+(-2+\sqrt{3})^{1/2}]s+1$

$\overline{X^2} = I_2 = \frac{1+4}{2\cdot[(-2-\sqrt{3})^{1/2}+(-2+\sqrt{3})^{1/2}]} = 1.0\ldots/j = -j\,1.0\ldots$, impossible!!

Therefore, $\frac{\omega^2+4}{\omega^4-4\omega^2+1}$ can't be a spectral density function

7.5.2 From problem 7.4.1

$$S_X(\omega) = \frac{16(\omega^2+36)}{\omega^4+13\omega^2+36}$$

$$S_X(s) = \frac{16(-s^2+36)}{s^4-13s^2+36} = \frac{16(-s+6)(s+6)}{(-s^2+9)(-s^2+4)} = \frac{16(-s+6)(s+6)}{(-s+3)(s+3)(-s+2)(s+2)}$$

a) By table 7-1

$c(s) = 4s+24$

$d(s) = (s+3)(s+2) = s^2+5s+6$

$$\overline{X^2} = I_2 = \frac{4^2\times 6 + 24^2\times 1}{2\times 1\times 5\times 6} = 11.2$$

b) Poles on LHP are $s=-3$, $s=-2$

$$K_{-3} = \left[(s+3)S_X(s)\right]_{s=-3} = \left[\frac{16(-s+6)(s+6)}{(-s+3)(-s+2)(s+2)}\right]_{s=-3} = \frac{16\times 9\times 3}{6\times 5\times(-1)} = -14.4$$

$$K_{-2} = \left[(s+2)S_X(s)\right] = \left[\frac{16(-s+6)(s+6)}{(-s+3)(s+3)(-s+2)}\right]_{s=-2} = \frac{16\times 8\times 4}{(5)\times 1\times(4)} = 25.6$$

$\therefore \overline{X^2} = -14.4+25.6 = 11.2$

7.5.3

$$S_X(s) = \frac{(s)(-s)}{(s^2-36)(s^2-16)} = \frac{(s)(-s)}{(s+6)(s-6)(s+4)(s-4)}$$

$\therefore c(s) = s$

$d(s) = s^2+10s+24$

$$\therefore \overline{X^2} = \frac{1\cdot 24 + 0\cdot 1}{2\cdot 24\cdot 10\cdot 1} = 0.05$$

7.5.4

$$\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\omega^2+10}{\omega^4+5\omega^2+4}\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[8\pi\delta(\omega)+2\pi\delta(\omega-3)+2\pi\delta(\omega+3)\right]d\omega$$

$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{-s^2+10}{s^4-5s^2+4}\,ds + \frac{1}{2\pi}\left[8\pi+2\pi+2\pi\right]$$

$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{(s+\sqrt{10})(-s+\sqrt{10})}{(s+1)(s+2)(s-2)(s-1)}\,ds + 6$$

$$= \frac{1^2\cdot 2 + 10\cdot 1}{2\cdot 2\cdot 3\cdot 1} + 6 = 7$$

7.6.1

$$R_X(\tau) = 10\left[1-\frac{|\tau|}{0.05}\right]; \quad |\tau| \le 0.05$$

$= 0$; elsewhere

a) $\overline{X^2} = R_X(0) = 10$, $\bar{X} = R_X(\infty) = 0$

$\therefore \sigma_X^2 = 10$

b) $$S_X(\omega) = \mathcal{F}\left[R_X(\tau)\right] = \frac{10\cdot(0.05)\sin^2(\omega\cdot 0.05/2)}{(\omega\cdot 0.05/2)^2} = \frac{800\sin^2(0.025\omega)}{\omega^2}$$

c) At $\tau = \tau_1 = 0.05$ $\quad R_X(\tau_1) = 0$

At $f = f_1 = 20$ $\quad S_X(f) = 0$

$f_1 = 1/\tau_1$

7.8.1/ $S_X(\omega) = \frac{16}{\omega^2+16}$, $S_Y(\omega) = \frac{\omega^2}{\omega^2+16}$

a) $U(t) = X(t) + Y(t) \Leftrightarrow R_U(\tau) = R_X(\tau) + R_Y(\tau)$

$S_U(\omega) = \mathcal{F}[R_U(\tau)] = S_X(\omega) + S_Y(\omega) = 1$

b) Since X & Y are independent and X is zero mean

$\therefore R_{XY}(\tau) = 0 \Leftrightarrow S_{XY}(\omega) = 0$

c) $R_{UX}(\tau) = E\{X(t)[X(t+\tau) + Y(t+\tau)]\} = E[X(t)X(t+\tau)] + E[X(t)Y(t+\tau)]$

$= R_X(\tau) + R_{XY}(\tau) = R_X(\tau)$

$\therefore S_{XU}(\omega) = S_X(\omega) = 16/(\omega^2+16)$

7.8.2/ $V(t) = X(t) - Y(t)$

$R_{UV}(\tau) = E[(X(t)+Y(t))\cdot(X(t+\tau) - Y(t+\tau))]$

$= E[X(t)X(t+\tau)] + E[Y(t)X(t+\tau)] - E[X(t)Y(t+\tau)] - E[Y(t)Y(t+\tau)]$

$= R_X(\tau) + R_{YX}(\tau) - R_{XY}(\tau) - R_Y(\tau)$

$= R_X(\tau) - R_Y(\tau)$

$\therefore S_{UV}(\omega) = \mathcal{F}[R_{UV}(\tau)] = S_X(\omega) - S_Y(\omega) = \frac{16-\omega^2}{16+\omega^2}$

7.9.1/ $W_B(f) = \mathcal{F}\{\text{tri}(t)\} = \text{sinc}^2(f)$

$W_h(f) = \mathcal{F}\{[0.5 + 0.5\cos(\pi t)]\,\text{rect}(t)\}$

$= 0.5\,\text{sinc}(2f+1) + \text{sinc}(2f) + 0.5\,\text{sinc}(2f-1)$

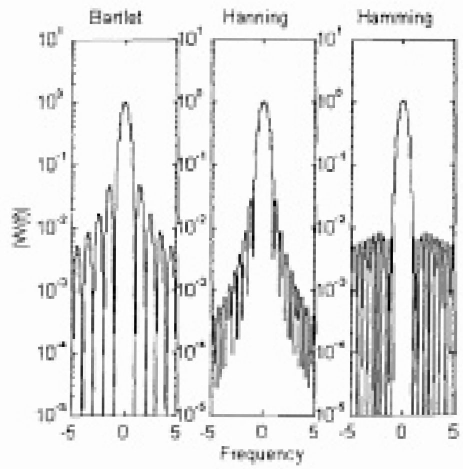
$W_H(f) = \mathcal{F}\{[0.46 + 0.54\cos(\pi t)]\,\text{rect}(t)\}$

$= 0.46\,\text{sinc}(2f+1) + 1.08\,\text{sinc}(2f) + 0.46\,\text{sinc}(2f-1)$

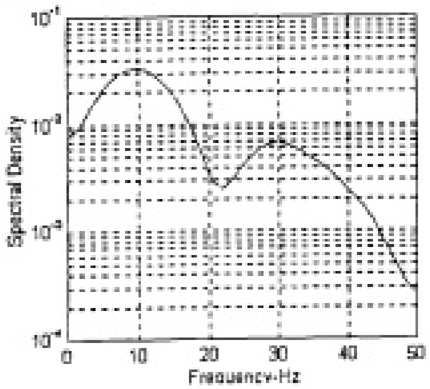
7-9.1 continued

```
%PR7_9_1.m
f=-5:.01:5;
W1=(sinc(f)).^2;
W2=sinc(2*f)+.5*sinc(2*f+ones(size(f)))+.5*sinc(2*f-ones(size(f)));
W3=1.08*sinc(2*f)+.46*sinc(2*f+ones(size(f)))+.46*sinc(2*f-ones(size(f)));
clg

axis([-5,5,1e-5,10]); whitebg
subplot(1,3,1);semilogy(f,abs(W1),'k');ylabel('|W(f)|')
title('Bartlet   ')
axis([-5,5,1e-5,10]);
subplot(1,3,2);semilogy(f,abs(W2),'k');
xlabel('Frequency');
title('Hanning   ');
axis([-5,5,1e-5,10]);
subplot(1,3,3);semilogy(f,abs(W3),'k')
title('Hamming   ');
axis([-5,5,1e-5,10]);
```



```
%PR7_9_2.m
x=[.19,.29,1.44,0.83,-.01,-1.23,-1.47,-1.24,-1.88,-.31,1.18,1.70,0.57,0.95,1.45,-0.82,-
0.25,0.23,-0.91,-0.19,0.24];
fs=100;   %Sampling frequency
M=8        %Number points in lag window (even)
[a,b]=size(x);
if a < b   % make x a column vector
 x=x';
 N=b;
 else N=a;
 end
x1=detrend(x,0);   %remove the dc component
x1(2*N-2)=0;       %zero pad to length of 2N-2
R1=real(ifft(abs(fft(x1)).^2));  %raw autocorrelation
    %compute weighted autocorrelation
W=triang(2*N-1);
R2=[R1(N:2*N-2);R1(1:N-1)]./((N)*W(1:2*N-2));
R3=R2(N-M:N+M-1);
H=hamming(2*M+1);
R4=R3.*H(1:2*M);
k=2^(ceil(log2(2*M)) +2);  %make length FFT power of 2 and add zeros
S1=abs((1/fs)*fft(R4,k));
f=0:fs/k:fs/2;
Scor=S1(1:k/2+1);   %positive frequency part of spectral density
semilogy(f,Scor);
grid;xlabel('Frequency-Hz'); ylabel('Spectral Density')
```

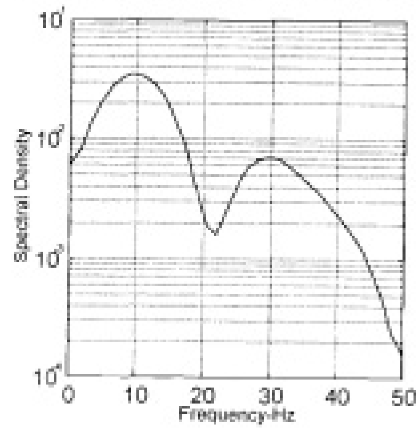


$\mathrm{var}[S_x] \approx \frac{M}{N} S_x(8\,\mathrm{Hz})$

$= \frac{8}{20}(8\times10^{-3})^2$

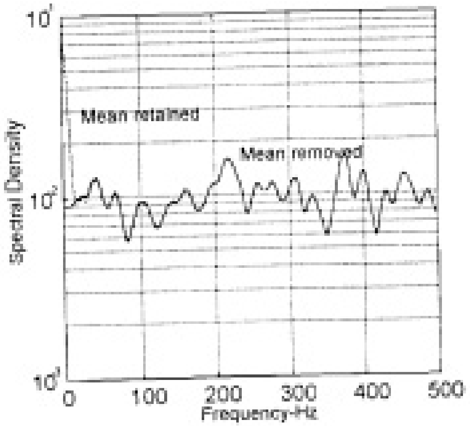
$= 0.00128$


```
%PR7_9_3.m
x=[.19,.29,1.44,0.83,-.01,-1.23,-1.47,-1.24,-1.88,-.31,1.18,1.70,0.57,0.95,1.45,-0.82,-
0.25,0.23,-0.91,-0.19,0.24];
fs=100;   %Sampling frequency
M=8;        %Number points in lag window (even)
[a,b]=size(x);
if a < b   % make x a column vector
 x=x';
 N=b;
 else N=a;
 end
x1=detrend(x,0);   %remove the dc component
x1(2*N-2)=0;       %zero pad to length of 2N-2
R1=real(ifft(abs(fft(x1)).^2));  %raw autocorrelation
W=triang(2*N-1);
R2=[R1(N:2*N-2);R1(1:N-1)]./((N)*W(1:2*N-2));
R3=R2(N-M:N+M-1);H=hamming(2*M+1);R4=R3.*H(1:2*M);
k=2^(ceil(log2(2*M)) +2);  %make length FFT power of 2 and add zeros
S1=abs((1/fs)*fft(R4,k));
f=0:fs/k:fs/2;
Scor=S1(1:k/2+1);   %positive frequency part of spectral density
semilogy(f,Scor,'k');whitebg
grid;xlabel('Frequency-Hz'); ylabel('Spectral Density')
```

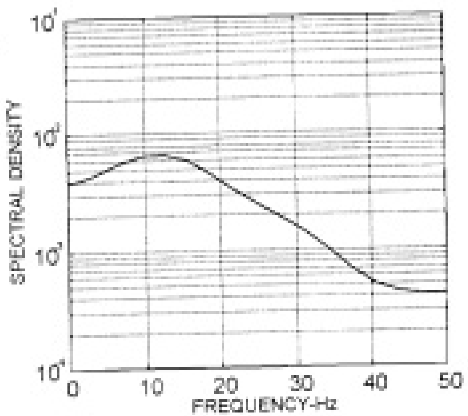


PR7-9.4

Use **corspec** with and without detrend.



PR7_10.1

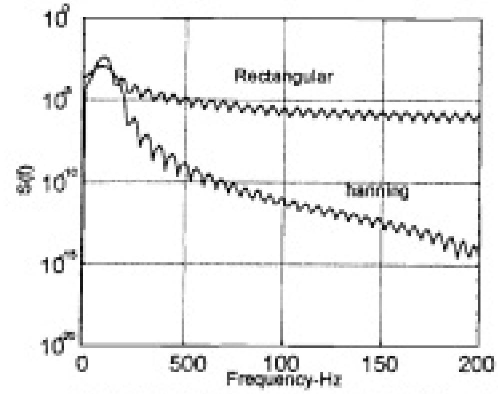


7.10.2

$$K_1 = \sqrt{\frac{\int_0^T [w_3(t)]^2 dt}{T}} = \sqrt{\frac{\int_{-T/2}^{T/2} \left(1 - \frac{|t|}{T/2}\right)^2 dt}{T}}$$

$$= \sqrt{\frac{2}{T}\left[t - \frac{4t^2}{2T} + \frac{4t^3}{3T^2}\right]_0^{T/2}} = \sqrt{.5} = .707$$

PR7_10_3



Rectangular window has sharper peak but very large sidelobes.

7.11.1

a) $f(t) = \frac{1}{t_1}\left(1 + \cos\frac{\pi t}{t_1}\right)$, $|t| \le t_1$

$\quad = 0$, $|t| > t_1$

$\Rightarrow \quad \mathcal{S}_s(\omega) = \frac{t_1}{\pi}\left[\frac{\sin \omega t_1}{\omega t_1}\right]^2 \left[\frac{\pi^2}{\pi^2 - (\omega t_1)^2}\right]^2$

b) $\left[\frac{1}{\omega_1 t_1}\right]^2 \left[\frac{\pi^2}{\pi^2 - (\omega_1 t_1)^2}\right]^2 \le 0.01$

$\Rightarrow \quad \omega_1 \simeq \frac{2.30}{t_1}$

c) When the duration of the pulses is doubled the bandwidth is reduced in half.

7.11.2

a) $d(s) = (s+10+j100)(s+10-j100)(s-10+j100)(s-10-j100)$

$= (s^2+20s+1.01\times10^4)(s^2-20s+1.01\times10^4)$

$= s^4 + 1.98\times10^4 s^2 + (1.01\times10^4)^2$

$d(\omega) = \omega^4 - 1.98\times10^4\omega^2 + (1.01\times10^4)^2$

Spectral density is maximum when $d(\omega)$ is minimum

$d(\omega)$ is minimum at $\omega_p = \sqrt{\frac{1.98\times10^4}{2}} = 99.5$

and $d(\omega_p) = 1.3\times10^8$

Solve for ω_1 for which $d(\omega_1) = 2.5\times10^8$

$\Rightarrow \omega_1^4 - 1.98\times10^4\omega_1^2 - 1.48\times10^8 = 0$

$\Rightarrow \omega_1 = 160$

$\therefore$ Half power bandwidth $= 2\times(160-99.5) = 121$ rad/s $= 19.2$ Hz

b) Solve for ω_2 for which $d(\omega_2) = 101\times10^8\times100$

$\Rightarrow \omega_2^4 - 1.98\times10^4\omega_2^2 - 1.33\times10^{10} = 0$

$\Rightarrow \omega_2 = 348.8$

$\therefore$ 1% bandwidth is $2\times(348.8-99.5) = 498.7$ rad $= 79.4$ Hz

---

7.11.3

$S_x(\omega) = \frac{1}{3600}\left[\frac{\sin \omega/3600}{\omega/3600}\right]^2 = 3600\,\frac{(\sin \omega/3600)^2}{\omega^2}$

$\frac{1}{\pi}\int_0^{\omega_1} S_x(\omega)\,d\omega = 0.9\times\frac{1}{\pi}\int_0^{\infty} S_x(\omega)\,d\omega = 0.9\times\frac{1}{\pi}\times\frac{1}{3600}\times[\ldots]$

$\frac{3600}{\pi}\int_0^{\omega_1}\frac{(\sin \omega/3600)^2}{\omega^2} = 0.9$

Solve for ω_1

7.11.4

$S_x(\omega) = \frac{1}{\left[1+\left(\frac{\omega}{2\pi B_1}\right)^2\right]^n}$

a) $\left[1+\left(\frac{\omega_b}{2\pi B_1}\right)^2\right]^n = 2 \Rightarrow \left(\frac{\omega_b}{2\pi B_1}\right)^2 = -1+\sqrt[n]{2}$

$\frac{\omega_b}{2\pi} = B_1\sqrt{\sqrt[n]{2}-1}$

$\therefore$ Half power bandwidth $= 2B_1\sqrt{\sqrt[n]{2}-1}$

b) $\left[1+\left(\frac{\omega_1}{2\pi B_1}\right)^2\right]^n = 100 \Rightarrow \left(\frac{\omega_1}{2\pi B_1}\right)^2 = \sqrt[n]{100}-1$

$\frac{\omega_1}{2\pi} = B_1\sqrt{\sqrt[n]{100}-1}$

$\therefore$ 1% bandwidth $= 2B_1\sqrt{\sqrt[n]{100}-1}$

Chapter 8

8.2.1/

a) $X(t) = M + B\cos(5t+\theta)$

$Y(t) = \int_{-\infty}^{t} x(\lambda)\,h(t-\lambda)\,d\lambda = \int_{-\infty}^{t}[M + B\cos(5\lambda+\theta)]\,10\,e^{-10(t-\lambda)}\,d\lambda$

$= M + \frac{B}{125}\left(10\cos(5t+\theta) + 5\sin(5t+\theta)\right)$

b) $\bar{Y} = E[Y(t)] = E_M\{E_B[E_\theta(Y(t))]\} = E_M[M] = 5$

c) $\overline{Y^2} = E[Y^2(t)] = E_M\{E_B[E_\theta(Y^2(t))]\} = E_M[M^2] = 89$

$\therefore\ \sigma_Y^2 = 89 - 5^2 = 64$

8.2.2/

a) $Y(t) = \int_{-\infty}^{t}[M + B\cos(5\lambda+\theta)][\delta(t-\lambda) - 10e^{-10(t-\lambda)}]\,d\lambda$

$= M + B\cos(5t+\theta) - M - \frac{B}{5}[\cos(5t+\theta) + 2\sin(5t+\theta)]$

$= \frac{4}{5}B\cos(5t+\theta) - \frac{2}{5}B\sin(5t+\theta)$

b) $\bar{Y} = E[Y(t)] = E_M\{E_B[E_\theta(Y(t))]\} = 0$

c) $\overline{Y^2} = E[Y^2(t)] = 0 \Rightarrow \sigma_Y^2 = 0$

8.3.1/

a) $\bar{Y} = \bar{X}\int_0^{\infty} h(\lambda)\,d\lambda = 0$

b) $\overline{Y^2} = S_0\int_0^{\infty} h^2(\lambda)\,d\lambda = 10\int_0^{0.5} 1^2\,d\lambda = 5.0\ V^2$

c) $\sigma_Y^2 = 5 - 0^2 = 5$

8.3.2/

a) $\bar{Y} = \bar{X}\int_0^{\infty} h(\lambda)\,d\lambda = 0$

b) $\overline{Y^2} = \int_0^{0.5}\int_0^{0.5} 16\,e^{-8|\lambda_2-\lambda_1|}\,d\lambda_1\,d\lambda_2 = \int_0^{0.5} 16\left[\int_0^{\lambda_2} e^{-8(\lambda_2-\lambda_1)}\,d\lambda_1 + \int_{\lambda_2}^{0.5} e^{-8(\lambda_1-\lambda_2)}\,d\lambda_1\right]d\lambda_2$

$= 8\int_0^{0.5} 2 - e^{-8\lambda_2} - e^{8(\lambda_2-0.5)}\,d\lambda_2$

$= 8e^{-1}$

c) $\sigma_Y^2 = \overline{Y^2} = 8e^{-1}$

8.3.3/

a) $\bar{Y} = \bar{X}\int_0^{\infty} h(\lambda)\,d\lambda = 16\int_0^{\infty}\delta(\lambda) - 2e^{-2\lambda}\,d\lambda = 16(1-1) = 0$

b) $\overline{Y^2} = \int_0^{\infty}\int_0^{\infty}\left(16e^{-8|\lambda_2-\lambda_1|} + 16\right)[\delta(\lambda_1) - 2e^{-2\lambda_1}][\delta(\lambda_2) - 2e^{-2\lambda_2}]\,d\lambda_1 d\lambda_2$

$= 16\int_0^{\infty}\int_0^{\infty} e^{-8|\lambda_2-\lambda_1|}\delta(\lambda_1)\delta(\lambda_2) + \delta(\lambda_1)\delta(\lambda_2) - 2e^{-8|\lambda_2-\lambda_1|}e^{-2\lambda_2}\delta(\lambda_1)$

$- 2e^{-2\lambda_2}\delta(\lambda_1) - 2e^{-8|\lambda_2-\lambda_1|}e^{-2\lambda_1}\delta(\lambda_2) - 2e^{-2\lambda_1}\delta(\lambda_2)$

$+ 4e^{-8|\lambda_2-\lambda_1|}e^{-2\lambda_1}e^{-2\lambda_2} + 4e^{-2\lambda_1}e^{-2\lambda_2}\,d\lambda_1 d\lambda_2$

$= 16\left[1 + 1 - \int_0^{\infty} 2e^{-10\lambda_2}\,d\lambda_2 - \int_0^{\infty} 2e^{-2\lambda_2}\,d\lambda_2 - \int_0^{\infty} 2e^{-10\lambda_1}\,d\lambda_1\right.$

$- 2\int_0^{\infty} e^{-2\lambda_1}\,d\lambda_1 + 4\left(\int_0^{\infty} e^{-2\lambda_1}\,d\lambda_1\right)^2$

$\left.+ 4\int_0^{\infty}\left(\int_0^{\lambda_2} e^{-8(\lambda_2-\lambda_1)}e^{-2\lambda_1}e^{-2\lambda_2}\,d\lambda_1 + \int_{\lambda_2}^{\infty} e^{-8(\lambda_1-\lambda_2)}e^{-2\lambda_1}e^{-2\lambda_2}\,d\lambda_1\right)d\lambda_2\right]$

$= 64\int_0^{\infty}\lambda_2 e^{-4\lambda_2} + \frac{1}{4}e^{-4\lambda_2}\,d\lambda_2 = 1/8$

c) $\sigma_Y^2 = \overline{Y^2} = 1/8$

8.3.4/ Since output bandwidth is 2×10^6 rad/sec while signal is at 2000π rad/sec, the output capacitance can be neglected.

Output signal $= 0.1\times\frac{1}{2}\times 0.01\times 5000 = 2.5$ V. (peak value)

Impulse response to V_N is

$H(S) = g_m\frac{R_L\left(\frac{1}{SC}\right)}{R_L + \frac{1}{SC}} = \frac{10^8}{S + 2\times10^6}\quad\therefore\ h(t) = 10^8 e^{-2\times10^6 t}u(t)$

Spectral density of noise component at V_N is

$2\times10^{-16}(5000)^2 = 5\times10^{-9}$

$\therefore\ \overline{V_0^2} = 5\times10^{-9}\int_0^{\infty} 10^{16}e^{-4\times10^6 t}\,dt = 0.5$

$\Rightarrow\ (SNR) = \frac{\frac{1}{2}(2.5)^2}{0.5} = 6.25$

★ 8.4.1

$H(s) = \frac{1000}{0.1(s+10000)} = \frac{10^4}{s+10^4} \Rightarrow h(t) = 10^4 e^{-10^4 t} u(t)$

a) $R_Y(\tau) = S_0 \int_0^\infty h(\lambda) h(\lambda+\tau)\, d\lambda = 10^{-4} \times 10^8 \int_0^\infty e^{-10^4 \lambda} e^{-10^4(\lambda+\tau)}\, d\lambda$

$= 0.5\, e^{-10^4 \tau} \quad ; \quad \tau \geq 0$

By symmetry $R_Y(\tau) = 0.5\, e^{-10^4 |\tau|}$

b) $\overline{Y^2} = R_Y(0) = 1/2$

8.4.2

a) $R_Y(\tau) = 2\times10^8 \int_0^\infty \int_0^\infty e^{-5000|\lambda_2-\lambda_1-\tau|} e^{-10^4\lambda_1} e^{-10^4\lambda_2}\, d\lambda_1\, d\lambda_2$

$= 2\times10^8 \int_\tau^\infty \Big[\int_0^{\lambda_2-\tau} e^{-5000(\lambda_2-\lambda_1-\tau)} e^{-10^4\lambda_1} e^{-10^4\lambda_2}\, d\lambda_1 \qquad \text{for } \tau \geq 0$

$+ \int_{\lambda_2-\tau}^\infty e^{5000(\lambda_2-\lambda_1-\tau)} e^{-10^4\lambda_1} e^{-10^4\lambda_2}\, d\lambda_1 \Big] d\lambda_2$

$= 0$

By symmetry $R_Y(\tau) = 0$

b) $\overline{Y^2} = R_Y(0) = 0$

8.4.3

$g(t) = \frac{t-a}{c-a} \cdot h \qquad a \leq t \leq c$

$= \frac{b-t}{b-c} \cdot h \qquad c \leq t \leq b$

$\int_{-\infty}^{\infty} g^2(t)\, dt = \int_a^c \frac{h^2}{(c-a)^2}(t-a)^2\, dt + \int_c^b \frac{h^2}{(b-c)^2}(b-t)^2\, dt$

$= \frac{h^2}{(c-a)^2}\left(\frac{1}{3}c^3 - a c^2 + a^2 c - \frac{1}{3}a^3\right) + \frac{h^2}{(b-c)^2}\left(\frac{1}{3}b^3 - b^2 c + b c^2 - \frac{1}{3}c^3\right)$

$= \frac{1}{3}h^2(c-a) + \frac{1}{3}h^2(b-c)$

$= \frac{1}{3}h^2(b-a)$

★ 8.4.4

$R_Y(\tau) = \int_0^\infty \int_0^\infty [2\delta(\lambda_2-\lambda_1-\tau) + 9] \cdot h(\lambda_1) h(\lambda_2)\, d\lambda_2\, d\lambda_1$

$= 2\int_0^1 \int_0^1 \delta(\lambda_2-\lambda_1-\tau)(1-\lambda_1)(1-\lambda_2)\, d\lambda_2\, d\lambda_1 + 9\left[\int_0^1 (1-\lambda_1)\, d\lambda_1\right]^2$

$= 2 \cdot \int_\tau^1 (1-\lambda_1)(1-\lambda_1-\tau)\, d\lambda_1 + 9 \cdot \left(\frac{1}{2}\right)^2$

$= \frac{2}{3} - \tau + \frac{1}{3}\tau^3 + \frac{9}{4} = \frac{35}{12} - \tau + \frac{1}{3}\tau^3 \qquad 0 \leq \tau \leq 1$

a) $\bar{Y} = \frac{3}{2}$

b) $\overline{Y^2} = \frac{35}{12}$

c) $R_Y(\tau) = \frac{35}{12} - |\tau| + \frac{1}{3}|\tau|^3 \quad ; \quad -1 \leq \tau \leq 1$

$= \frac{9}{4} \quad ; \quad \text{otherwise}$

8.5.1

$R_{XY}(\tau) = \int_0^\infty R_X(\tau-\lambda) h(\lambda)\, d\lambda = 10 \int_0^{0.5} 1\, d\lambda = 5$

$R_{YX}(\tau) = R_{XY}(-\tau) = 5$

8.5.2

$R_{XY}(\tau) = \int_0^{0.5} 16\, e^{-2|\tau-\lambda|}\, d\lambda \qquad \tau' = \tau - \lambda$

or $R_{XY}(\tau) = \int_{\tau-0.5}^{\tau} 16\, e^{-2|\tau'|}\, d\tau'$

$= \begin{cases} 8\, e^{-2\tau}[e-1] & \text{for } \tau \geq \frac{1}{2} \\ 8\, e^{2\tau}[1-e^{-1}] & \text{for } \tau \leq 0 \\ 8\,[2 - e^{-2\tau} - e^{2\tau-1}] & \text{for } 0 \leq \tau \leq \frac{1}{2} \end{cases}$

$R_{YX}(\tau) = R_{XY}(-\tau)$

8.5.3

$R_{XY}(\tau) = \int_0^1 [2\delta(\tau-\lambda)+9](1-\lambda)\,d\lambda$

$= 2(1-\tau) + \frac{9}{2} = \frac{13}{2} - \tau$

$R_{YX}(\tau) = \frac{13}{2} + \tau$

8.5.4

$H_{XY}(s) = \frac{1000}{1000 + \frac{1}{10^{-6}s}} = \ldots \Rightarrow h_{XY}(t) = \delta(t) - 1000\, e^{-1000t} u(t)$

$H_{XZ}(s) = \frac{1/10^{-6}s}{1000 + 1/10^{-6}s} = \ldots \Rightarrow h_{XZ}(t) = 1000\, e^{-1000t} u(t)$

$Y(t) = \int_0^\infty X(t-\lambda)\, h_{XY}(\lambda)\,d\lambda$

$Z(t) = \int_0^\infty X(t-\lambda)\, h_{XZ}(\lambda)\,d\lambda$

$\Rightarrow R_{YZ}(\tau) = E[\,Y(t)\,Z(t+\tau)]$

$= E\left[\int_0^\infty\int_0^\infty X(t-\lambda_1)X(t+\tau-\lambda_2)\,h_{XY}(\lambda_1)\,h_{XZ}(\lambda_2)\,d\lambda_1\,d\lambda_2\right]$

$= \int_0^\infty\int_0^\infty R_X(\tau+\lambda_1-\lambda_2)\,h_{XY}(\lambda_1)\,h_{XZ}(\lambda_2)\,d\lambda_1\,d\lambda_2$

$\because R_X(\tau) = 0.1\,\delta(\tau)$

$\Rightarrow R_{YZ}(\tau) = 0.1\int_0^\infty h_{XY}(\lambda_1)\,h_{XZ}(\tau+\lambda_1)\,d\lambda_1$

$= \begin{cases} 50\, e^{-1000\tau} & \text{for } \tau \geq 0 \\ -50\, e^{1000\tau} & \text{for } \tau \leq 0 \end{cases}$

8.6.1

a) $\overline{Y^2} = \int_0^T\int_0^T R_X(\lambda_2-\lambda_1)\left[\frac{1}{T}\right]^2 d\lambda_1\,d\lambda_2$

$= \frac{1}{3}A^2$

b) $R_Y(\tau) = \int_0^T\int_0^T R_X(\lambda_2-\lambda_1-\tau)\left[\frac{1}{T}\right]^2 d\lambda_1\,d\lambda_2$

$= \begin{cases} \int_0^T\int_0^T A^2\left[1-\frac{\lambda_2-\lambda_1-\tau}{T}\right]\frac{1}{T^2}\,d\lambda_1\,d\lambda_2 & ; \quad \tau < -T \\ \int_0^T\int_0^T A^2\left[1+\frac{\lambda_2-\lambda_1-\tau}{T}\right]\frac{1}{T^2}\,d\lambda_1\,d\lambda_2 & ; \quad \tau > T \\ \int_0^T \frac{A^2}{T^2}\left[\int_0^{\lambda_2-\tau}\left(1-\frac{\lambda_2-\lambda_1-\tau}{T}\right)d\lambda_1 + \int_{\lambda_2-\tau}^{T}\left(1+\frac{\lambda_2-\lambda_1-\tau}{T}\right)d\lambda_1\right]d\lambda_2 & -T \leq \tau \leq T \end{cases}$

8.6.1 cont

$= \begin{cases} \int_0^T\int_0^T \frac{A^2}{T^2}\left[1-\frac{\lambda_2-\lambda_1}{T}\right]d\lambda_1\,d\lambda_2 + \int_0^T\int_0^T \frac{A^2}{T^2}\left[\frac{\tau}{T}\right]d\lambda_1\,d\lambda_2 & ; \quad \tau < -T \\ \int_0^T\int_0^T \frac{A^2}{T^2}\left[1+\frac{\lambda_2-\lambda_1}{T}\right]d\lambda_1\,d\lambda_2 - \int_0^T\int_0^T \frac{A^2}{T^2}\left[\frac{\tau}{T}\right]d\lambda_1\,d\lambda_2 & ; \quad \tau > T \\ \int_0^T \frac{A^2}{T^2}\left[-\frac{\lambda_2^2}{T} + \frac{2\lambda_2\tau}{T} - \frac{\tau^2}{T} + \frac{T}{2} + \lambda_2 - \tau\right]d\lambda_2 & ; \quad -T \leq \tau \leq T \end{cases}$

$= \begin{cases} \frac{7}{6}A^2 + \frac{\tau}{T}A^2 & ; \quad \tau \leq -T \\ \frac{7}{6}A^2 - \frac{\tau}{T}A^2 & ; \quad \tau \geq T \\ \frac{2}{3}A^2 - \frac{\tau^2}{T^2}A^2 & ; \quad -T \leq \tau \leq T \end{cases}$

8.6.2

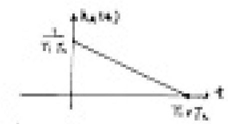
Assume input is zero mean $\Rightarrow$ output is zero mean

$\Rightarrow$ Variance = mean-square value.

The equivalent impulse response $h_e(t) = h_1(t) * h_2(t)$

$\therefore \sigma_Y^2 = \overline{Y^2} = 10^{-3}\int_0^{T_1+T_2} h_e^2(t)\,dt$

$= \frac{1}{3}10^{-3}\left(\frac{1}{T_1T_2}\right)^2(T_1+T_2)$



a) $T_1 = T_2 = 10^{-1}$

$\sigma_Y^2 = \frac{2}{3}$

b) $T_1 = 0.1 \quad T_2 = 0.01 \qquad \sigma_Y^2 = \frac{11}{3}\times 10^{1}$

c) $T_1 = 0.1 \quad T_2 = 1.0 \qquad \sigma_Y^2 = \frac{11}{3}\times 10^{-2}$

8.6.3

Let $\underline{X} = \begin{bmatrix} X_1 - \bar{X} \\ X_2 - \bar{X} \\ \vdots \\ X_n - \bar{X} \end{bmatrix}$

a. $E[\underline{X}\,\underline{X}^T] = \begin{bmatrix} E[(X_1-\bar{X})(X_1-\bar{X})] & E[(X_1-\bar{X})(X_2-\bar{X})] & \cdots \\ E[(X_2-\bar{X})(X_1-\bar{X})] & E[(X_2-\bar{X})(X_2-\bar{X})] & \\ \vdots & & \ddots \\ & & E[(X_n-\bar{X})(X_n-\bar{X})] \end{bmatrix}$

$= \begin{bmatrix} R_X(0) & 0 & \\ & R_X(0) & 0 \\ 0 & & \ddots \quad R_X(0) \end{bmatrix}$

8.6.3 cont.

b) $E[\underline{X}\,\underline{X}^T] = \begin{bmatrix} R_x(0) & R_x(\Delta t) & R_x(2\Delta t) \cdots & R_x(n\Delta t) \\ R_x(\Delta t) & R_x(0) & & \\ \vdots & & \ddots & R_x(\Delta t) \\ R_x(n\Delta t) & & R_x(\Delta t) & R_x(0) \end{bmatrix}$

8.6.4

a) The maximum value of the impulse response is 0.1839 at $t = 1/20$

$\therefore\ 10t\,e^{-20t} \geq 0.02 \times 0.1839$

$\Rightarrow t = 6.02/20 = 3.01 \times 10^{-1}$

$\Delta t = 3.01\times10^{-1}/60 = 5.02 \times 10^{-3}$ second

b) Variance $\leq (0.01 \times 0.1839)^2 = 3.38 \times 10^{-6}$

$\int_0^\infty h^2(t)\,dt = \int_0^\infty 100t^2 e^{-40t}\,dt = \frac{1}{1.6}\sqrt{\frac{\pi}{40}}$

$N \geq \frac{2 \times \frac{1}{1.6}\sqrt{\frac{\pi}{40}}}{3.38\times10^{-6}} = 2.65 \times 10^5$

c) For one percent error $10t\,e^{-20t} \geq 0.01 \times 0.1839$

$\Rightarrow t = 7.64/20 = 3.82 \times 10^{-1}$ sec.

$\therefore\ \Delta t = 3.82\times10^{-1}/60 = 6.37 \times 10^{-3}$ sec.

Total time $= \Delta t \cdot N = 6.37\times10^{-3} \times 2.65\times10^5 = 1.69 \times 10^3$ sec.

* 8.7.1

a) $\frac{V_o}{V_{in}} = \frac{10\times10^3 + \frac{1}{sC}}{5\times10^3 + \frac{1}{sC}} = \frac{s+1}{3s+1} = H(s)$

b) $Y(s) = X(s)H(s) = \frac{s(s+1)}{(3s+1)(s+4)}$

$|Y(s)|^2 = Y(s)\cdot Y(-s) = \frac{s(s+1)(-s)(-s+1)}{(3s+1)(s+4)(-3s+1)(-s+4)}$

$= \frac{s^2(s^2-1)}{(9s^2-1)(s^2-16)}$

8.7.2

$H(s) = \frac{k}{(s+6)(s+3+j3\sqrt{3})(s+3-j3\sqrt{3})} = \frac{k}{s^3+12s^2+72s+216}$

$H(0) = 1 \Rightarrow k = 216$

a) $H(\omega) = \frac{216}{-j\omega^3 - 12\omega^2 + 72j\omega + 216}$

b) $|H(\omega)|^2 = \frac{216^2}{(216-12\omega^2)^2 + (72\omega-\omega^3)^2} = \frac{46656}{\omega^6 + 46656}$

c) $|H(s)|^2 = \frac{46656}{-s^6 + 46656}$

8.8.1

a) $S_x(\omega) = 0.5$

$S_Y(\omega) = S_x(\omega)|H(\omega)|^2 = \frac{\omega^2(\omega^2+1)}{2(9\omega^2+1)(\omega^2+16)}$

b) $S_x(\omega) = \omega^2/\omega^4+5\omega^2+4$

$S_Y(\omega) = \frac{\omega^2(\omega^2+1)\quad \omega^2}{2(9\omega^2+1)(\omega^2+16)(\omega^2+1)(\omega^2+4)} = \frac{\omega^4}{2(9\omega^2+1)(\omega^2+16)(\omega^2+4)}$

8.8.2

a) $R_x(\tau) = 10\,e^{-|\tau|} \Rightarrow S_x(\omega) = \frac{20}{\omega^2+1}$

$S_Y(\omega) = S_x(\omega)|H(\omega)|^2$

$= \frac{20 \times 216^2}{(\omega^2+1)(\omega^6+46656)}$

b) $S_Y(0) = \frac{20 \times 46656}{46656} = 20$

8.8.3 a) $S_Y(s) = \dfrac{1.2\,(s)(-s)}{(s^2+15s+50)(s^2-15s+50)}$

b) $\overline{Y^2} = \dfrac{1}{2\pi j}\displaystyle\int_{-j\infty}^{j\infty} \dfrac{1.2\,(s)(-s)}{(s^2+15s+50)(s^2-15s+50)}\,ds$

$c(s) = \sqrt{1.2}\,s \qquad d(s) = s^2+15s+50$

$\therefore \overline{Y^2} = \dfrac{1.2 \times 50 + 0}{2 \times 50 \times 15 \times 50} = 0.04\ V^2$

8.8.4 $\overline{Y^2} = \dfrac{1}{2\pi j}\displaystyle\int_{-j\infty}^{j\infty} \dfrac{0.8 \times 216^2}{-s^2+216^2}\,ds$

$= \dfrac{1}{2\pi j}\displaystyle\int_{-j\infty}^{j\infty} \dfrac{0.8 \times 216^2}{(s+216)(-s+216)}\,ds$

$c(s) = \sqrt{0.8} \times 216 \qquad d(s) = s + 216$

$\therefore \overline{Y^2} = \dfrac{0}{2 \cdot 216\,(-216)} = 0$

8.9.1 $S_{XY}(s) = H(s)\,S_X(s) = \dfrac{216 \times 20}{(s^3+12s^2+72s+216)(-s^2+1)}$

$S_{YX}(s) = H(-s)\,S_X(s) = \dfrac{216 \times 20}{(-s^3+12s^2-72s+216)(-s^2+1)}$

8.9.2 $R_{YZ}(\tau) = E[Y(t)\,Z(t+\tau)] =$

$= E\left[\int_0^\infty h_Y(\lambda_1)\,n(t-\lambda_1)\,d\lambda_1 \int_0^\infty h_Z(\lambda_2)\,n(t+\tau-\lambda_2)\,d\lambda_2\right]$

$= \int_0^\infty \int_0^\infty R_n(\lambda_2-\lambda_1-\tau)\,h_Y(\lambda_1)\,h_Z(\lambda_2)\,d\lambda_1\,d\lambda_2$

$S_{YZ}(\omega) = \mathcal{F}[R_{YZ}(\tau)] = \int_0^\infty \int_0^\infty S_n(\omega)\,h_Y(\lambda_1)\,h_Z(\lambda_2)\,e^{-j\omega(\lambda_2-\lambda_1)}\,d\lambda_1\,d\lambda_2$

$= S_n(\omega)\,H_Y(-\omega)\,H_Z(\omega)$

Similarly $S_{ZY}(\omega) = S_n(\omega)\,H_Z(-\omega)\,H_Y(\omega)$

8.9.2 cont. $S_n(\omega) = 1$

$S_{YZ}(s) = H_Y(-s) \cdot H_Z(s)$

$S_{ZY}(s) = H_Y(s) \cdot H_Z(-s)$

8.9.3 $H_Y(s) = 1/(s+1) \qquad H_Z(s) = s/(s+1)$

$S_{YZ}(s) = \dfrac{(s)}{(s+1)(-s+1)} = \dfrac{-s}{s^2-1}$

$S_{ZY}(s) = \dfrac{-s}{(s+1)(-s+1)} = \dfrac{s}{s^2-1}$

8.10.1 a) $H(s) = \dfrac{216}{s^3+12s^2+72s+216} \qquad B = \dfrac{1}{4\pi j\,|H(0)|^2}\displaystyle\int_{-j\infty}^{j\infty} H(s)H(-s)\,ds$

$H(0) = 1 \qquad c(s) = 216,\ d(s) = s^3+12s^2+72s+216$

$\therefore B = \dfrac{1}{2 \cdot (1)^2} \times \dfrac{216^2\,(12)\,(1)}{2\,(216)\,(1)\,(72 \times 12 - 216)} = 1$

b) $B_X = \dfrac{6}{2\pi} = 0.9549$

$\therefore B = 1.047\,B_X$

8.10.2 a) $B = \dfrac{\int_0^\infty h^2(t)\,dt}{2\left[\int_0^\infty h(t)\,dt\right]^2} = \dfrac{3}{16} = 0.1875$

b) $H(0) = \int_0^\infty h(t)\,dt = 4$

$\overline{Y^2} = 2 \times 2 \times 0.1875 \times 4^2 = 12$

c) $\overline{Y^2} = 4S_0\,B\,|H(0)|^2 = 4S_0 \dfrac{\int_0^\infty h^2(t)\,dt}{2\left[\int_0^\infty h(t)\,dt\right]^2} \cdot \left[\int_0^\infty h(t)\,dt\right]^2$

$= 2S_0 \int_0^\infty h^2(t)\,dt$

$= 2 \times 6 = 12$

8.10.3

a) $\left[X(s) - 5Y(s)\right] \cdot \frac{1}{s+4} = Y(s) \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{2s+4} = H(s)$

b) $S_Y(s) = S_X(s)\,|H(s)|^2 = \frac{10}{-s^2+4} \cdot \frac{1}{4(s+2)(-s+2)}$

$$\overline{Y^2} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{10}{4(s+\sqrt{2})(-s+\sqrt{2})(s+2)(-s+2)}\,ds$$

$c(s) = \frac{\sqrt{10}}{2}$ $\quad d(s) = s^2 + (2+\sqrt{2})s + 2\sqrt{2}$

$$\overline{Y^2} = \frac{\frac{10}{4}\times 1}{2\times 1\times(2+\sqrt{2})\times 2\sqrt{2}} = 0.13$$

c) The transfer function between $X(t)$ & $X(t) - Y(t)$ is $H_1(s)$

$$H_1(s) = \frac{X(s) - Y(s)}{X(s)} = 1 - H(s) = \frac{2s+3}{2s+4}$$

$$\therefore \overline{Y_1^2} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{10\,(2s+3)(-2s+3)}{(s+\sqrt{2})(-s+\sqrt{2})(2s+4)(-2s+4)}\,ds$$

$c(s) = \sqrt{10}\,(2s+3)$ $\quad d(s) = 2s^2 + (4+2\sqrt{2})s + 4\sqrt{2}$

$$\overline{Y_1^2} = \frac{40\times 4\sqrt{2} + 90\times 2}{2\times 2\times(4+2\sqrt{2})\times 4\sqrt{2}} = 2.63$$

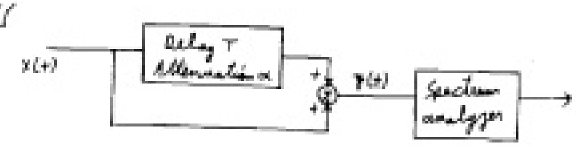
8.10.4

$B = \frac{\pi}{2} B_X = \frac{\pi}{2}\ \text{MHz}.$

$\overline{Y^2} = |a|^2 = 2S_0 B = 2S_0\,\frac{\pi}{2}\times 10^6$

$\therefore S_0 = \frac{1}{\pi}\times 10^{-8}\ V^2$

8.10.5



$Y(t) = X(t) + \alpha X(t+T)$

$R_Y(\tau) = E\left[Y(t)\cdot Y(t+\tau)\right]$

$= E\left[\left(X(t) + \alpha X(t+T)\right)\cdot\left(X(t+\tau) + \alpha X(t+\tau+T)\right)\right]$

$= E\left[X(t)X(t+\tau) + \alpha X(t)X(t+T+\tau) + \alpha X(t+T)X(t+\tau)\right.$
$\left. + \alpha^2 X(t+T)X(t+T+\tau)\right]$

$= R_X(\tau) + \alpha R_X(\tau+T) + \alpha R_X(\tau-T)$

α^2 is negligible

$S_Y(w) = S_X(w) + \alpha S_X(w)\cdot e^{jwT} + \alpha S_X(w)\cdot e^{-jwT}$

$= S_X(w) + \alpha S_X(w)\left(e^{jwT} + e^{-jwT}\right)$

$= S_X(w) + \alpha S_X(w)\,(2\cos wT)$

$= S_X(w)\left[1 + 2\alpha\cos wT\right]$

We can measure T by looking at the period of $S_Y(w)$

1. The receiver noise is omitted in here, and it will affect the measurement.
2. If two or more return signals are present, the measurement will be affected.

8.10.6/

$H(f) = e^{-f^2/2\sigma^2}$ $H(0) = 1$

$H(W) = e^{-W^2/2\sigma^2} = \frac{1}{2} \Rightarrow \sigma = W\sqrt{\frac{1}{2.386}}$

$B = \frac{1}{2|H(0)|^2}\int_{-\infty}^{\infty} |H(f)|^2 df = \frac{1}{2}\int_{-\infty}^{\infty} e^{-f^2/\sigma^2} df = \frac{1}{2}\sigma\sqrt{\pi} = \frac{W}{2}\sqrt{\frac{\pi}{1.386}}$

8.10.7/

a) $H(s) = \frac{V_0}{V_n} = \frac{1}{RCs+1}$

$\overline{V^2} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{2kTR}{(RCs+1)(-RCs+1)} ds = 2kTR \cdot \frac{1}{2RC}$

$= kT/C$ independent of R

b) As R changes, the dependence of the noise spectral density on R is cancelled by the dependence of the circuit bandwidth on R.

c) $B = \frac{1}{4\pi j|H(0)|^2}\int_{-j\infty}^{j\infty} H(s)H(-s)ds = \frac{1}{4\pi j}\int_{-j\infty}^{j\infty}\frac{1}{(RCs+1)(-RCs+1)}ds$

$= \frac{1}{4RC}$

For matched load

$\text{Power} = \frac{\overline{V^2}}{R} = \frac{kT}{RC} = kTB$

8.10.8/ For a system of bandwidth B and gain G,

$F = \frac{kTBG + P_n}{kTBG} = 1 + \frac{P_n}{kTBG}$

where P_n is the internal generated noise power

$\therefore P_n = kTBG(F-1)$

8.10.8/ cont.

a) The noise figure of two cascaded amplifiers is

$F = \frac{kTBG_1G_2 + kTBG_1G_2(F_1-1) + kTBG_2(F_2-1)}{kTBG_1G_2}$

$= F_1 + \frac{F_2-1}{G_1}$

b) $F = 13\text{ dB} = 20$ assume room temperature

$P_n = kTBG(F-1) = 1.38\times10^{-23}\times 300\times 100\times10^{6}\times\frac{\pi}{2}\times10^{10}\times(20-1)$

$= 0.123\ W$

$V_0^2 = P_n\times R = 6.3\times 10^{-1}$

$\Rightarrow V = 8.1\times10^{-1}\ V$

c) $P = 10\times 0.123 = 1.23\ W = \frac{1}{2}A^2$

$A = 1.57\ V$

7.11.? cont.

a) The noise figure of two cascaded amplifiers is

$$F = \frac{kTBG_1G_2 + kTBG_1G_2(F_1-1) + kTBG_2(F_2-1)}{kTBG_1G_2}$$

$$= F_1 + \frac{F_2 - 1}{G_1}$$

b) $F = 13\ dB = 20$ assume room temperature

$P_{in} = kTBG(F-1) = 1.38 \times 10^{-23} \times 300 \times 1 \times 10^{6} \times \frac{?}{?} \times 10^{10} \times (20-1)$

$= 0.125\ W$

$V_o^2 = P_{in} \times R \times G = 3.4, 8 \times 10^{?}$

$\Rightarrow V = 1.1 \times 10^{?}\ V$

c) $P = 10 \times 0.123 = 1.23\ W = \frac{1}{?} A^2$

$A = 1.57\ ?$

8.11.1

$H(\omega) = \frac{1}{j\omega + 100}$ $\qquad S_x(\omega) = 10^{-2}$

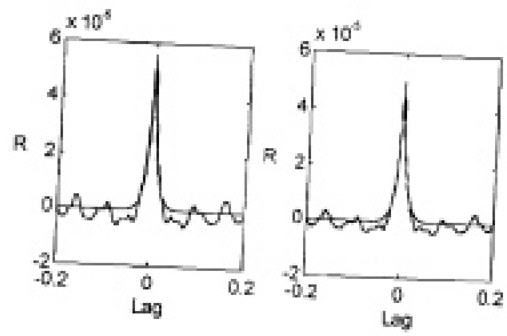
$S_y(\omega) = \frac{10^{-2}}{\omega^2 + 10^4}$

$R_y(\tau) = \mathcal{F}^{-1}\left\{\frac{10^{-2}}{\omega^2 + 10^4}\right\} = 5 \times 10^{-5} e^{-100|\tau|}$

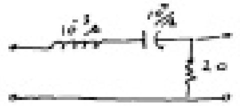
Simulation is carried out by computing a sample of $y(t)$ and then calculating $R_y(\tau)$. $y(t)$ will be computed using digital filtering and also by convolving with impulse response in following MATLAB m-files.


```
%PR8_11_1.m
randn('seed',1000)
x=sqrt(10)*randn(1,2000);
[b,a]=butter(1,1/(10*pi)); %must use digital filter with
   %bilinear transformation to minimize aliasing
y=.01*filter(b,a,x);  %adjust dc gain of filter
[lag,R]=corb(y,y,1000);
RR=5e-5*exp(-100*abs(lag));
N1=length(lag);
tt=round(N1/2)-200:round(N1/2)+200;
subplot(2,3,1);plot(lag(tt),R(tt),lag(tt),RR(tt))
ylabel('R');xlabel('Lag')


%alternative solution
randn('seed',1000)
xx=sqrt(10)*randn(1,2000);
t=0:.001:1;
h=exp(-100*t);
yy=.001*conv(xx,h);
[L1,R1]=corb(yy,yy,1000);
RR1=5e-5*exp(-100*abs(L1));
N=length(L1);
tt1=round(N/2)-200:round(N/2)+200;
subplot(2,3,2);plot(L1(tt1),R1(tt1),L1(tt1),RR1(tt1))
ylabel('R');xlabel('Lag')
```



8.11.21



$H(s) = \frac{20}{10^{-3}s + 10^{-7}/s + 20} = \frac{2\times10^{4}s}{s^2 + 2\times10^{4}s + 10^{10}}$

Max occurs at $10^{-3}(j\omega) + \frac{10^{7}}{j\omega} = 0 \Rightarrow \omega_0 = 10^5$

$B = \frac{1}{j4\pi|H(\omega_0)|^2}\int_{-j\infty}^{j\infty}\frac{(2\times10^{4}s)(-2\times10^{4}s)\,ds}{(s^2+2\times10^{4}s+10^{10})(s^2-2\times10^{4}s+10^{10})}$

$H(\omega_0) = \frac{2\times10^{4}(j10^{5})}{-10^{10} + 2\times10^{4}(j10^{5}) + 10^{10}} = 1$

$\therefore B = \frac{1}{2}\frac{1}{j2\pi}\int_{-j\infty}^{j\infty}\frac{(2\times10^{4}s)(2\times10^{4}s)\,ds}{(s^2+\dots)(s^2\dots)}$

Using Table 7.1

$I_2 = \frac{4\times10^{8}\times10^{10}}{2\times1(2\times10^{4}\times10^{10})} = 10^{4}$

$B = \frac{1}{2}\times10^{4} = 5000\ Hz$

For the numerical relation

$B = \frac{1}{4\pi|H(\omega_0)|^2}\int_{-\infty}^{\infty}|H(\omega)|^2$

$= \frac{1}{2\pi}\int_{0}^{\infty}\frac{4\times10^{8}\omega^2\,d\omega}{\omega^4 - 1.96\times10^{10}\omega^2 + 10^{20}}$

Because of slow roll-off must carry integration to high frequency for reasonable accuracy.

```
%H1.m
function x=H1(w)
bb=polyval([4e8,0,0],w);
aa=polyval([1,0,-1.96e10,0,1e20],w);
x=bb./aa;

B=(.5/pi)*quad8('H1',0,1e6)
 = 4.9359e+003
```



```
%PR8_11_3.m
fs=1e6; df=fs/400;f=0:df:fs/2;
Y=fft(y);
z=[0;hanning(18);0];z(length(y))=0;  %zero pad
Z=abs(fft(z));
plot(f,abs(Y(1:201)),f,10*Z(1:201));pause
%from graph select components out to f=1e5
% 40 fft components at origin and 39 at end of Y
%selection will be made using hanning window
w=hanning(80);
ww=[w(40:80);zeros(400-80,1);w(1:39)];
yy=real(ifft(Y.*ww'));
clg;subplot(2,2,1);plot(t,yy);grid,ylabel('Estimate')
subplot(2,2,3);plot(t,x);grid;xlabel('Time');ylabel('True')
```