Eng. ~~EngrA~~ Engin 322 Materdey

Chapter 6

6.1.1 / $\sigma_X^2 = R_X(0) = 5$

a) $b = \dfrac{R_X(\tau)}{\sigma_X^2} = \dfrac{R_X(-0.1)}{5} = 0.606$

b) $\sigma_Y^2 = \sigma_X^2 + 2b\,R_X(-0.1) + b^2\sigma_X^2 = 5 + \dfrac{2R_X^2(-0.1)}{5} + 0.606^2 \times 5 = 10.51$

c) When $b = -1$

$\sigma_Y^2 = 5 + 2R_X(-0.1) + 5 = 16.07$

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* **6.1.2 /** a) $R_X(t_1, t_2) = e^{t_1} e^{-t_2} = e^{(t_1 - t_2)}$ ⇒ wide-sense stationary

b) $R_X(t_1, t_2) = \cos t_1 \cos t_2 + \sin t_1 \sin t_2 = \cos(t_1 - t_2)$

⇒ wide-sense stationary

c) $R_X(t_1, t_2) = e^{(t_1^2 - t_2^2)}$ ⇒ not wide-sense stationary

d) $R_X(t_1, t_2) = \dfrac{\sin t_1 \cos t_2 - \cos t_1 \sin t_2}{t_1 - t_2} = \dfrac{\sin(t_1 - t_2)}{t_1 - t_2}$

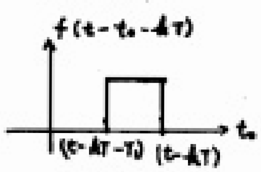
⇒ wide-sense stationary.

6.2.1 / $X(t) = \sum_{k=-\infty}^{\infty} A\, f(t - t_0 - kT)$ where $f(t) = u(t) - u(t - T_1)$

$A = 0$ or 1.

a) $\bar{X} = E\left[\sum_{k=-\infty}^{\infty} A\, f(t - t_0 - kT)\right] = \sum_{k=-\infty}^{\infty} E[A] \cdot E[f(t - t_0 - kT)]$

$E[A] = \frac{1}{2}$ $\qquad E[f(t - t_0 - kT)] = \displaystyle\int_{t-kT-T_1}^{t-kT} \frac{1}{T}\, dt_0 = \frac{T_1}{T}$



For a given t, only one k will yield a non zero value

$\Rightarrow \bar{X} = \frac{1}{2} \cdot \frac{T_1}{T} = T_1/2T$

6.2.1

a) cont. $\overline{X^2} = E[A^2]\cdot E[f^2(t-t_0-kT)]$ for any t

$E[A^2] = \frac{1}{2}$

Since $f^2(t-t_0-kT) = f(t-t_0-kT)$

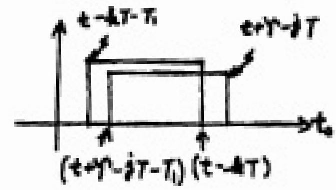
$E[f^2(t-t_0-kT)] = E[f(t-t_0-kT)] = \frac{T_1}{T}$

$\therefore\ \overline{X^2} = T_1/2T$

b) $R_X(\tau) = E[X(t)X(t+\tau)] = E\left[\sum_{k=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} A_k A_j f(t-t_0-kT)f(t+\tau-t_0-jT)\right]$

$E[A_k A_j] = \frac{1}{2}$ $\quad j=k$

$= \frac{1}{4}$ $\quad j\neq k$



For $\tau > 0$ $\quad E[f(t-t_0-kT)f(t+\tau-t_0-jT)]$

$= \int_{t+\tau-jT-T_1}^{t-kT} \frac{1}{T}\,dt_0 = \frac{T_1 - (\tau-(j-k)T)}{T}$

where $\tau-(j-k)T \leq T$

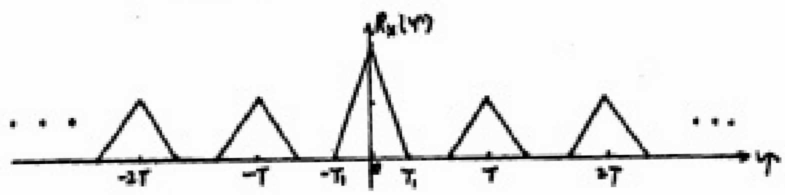
Similar for $\tau < 0$

For any τ, only one value of $(j-k)$ contribute

$\therefore\ R_X(\tau) = \frac{1}{2}\frac{T_1}{T}\left[1-\frac{|\tau|}{T_1}\right]$, $\quad j=k \quad |\tau| \leq T_1$

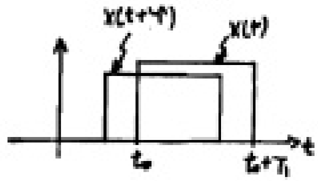
$= \frac{1}{4}\frac{T_1}{T}\left[1-\frac{|\tau-(j-k)T|}{T_1}\right]$, $\ j\neq k$, $\ |\tau-(j-k)T| \leq T_1$

$= 0$ $\quad$ elsewhere



6.2.2

$$R_X(\tau) = \lim_{V\to\infty} \frac{1}{2V}\int_{-V}^{V} x(t)\,x(t+\tau)\,dt$$



only ½ of the possible pulse positions have pulses in them

$$\Rightarrow R_X(\tau) = \frac{1}{2}\cdot\frac{T_1-\tau}{T}, \quad \tau>0, \quad \tau<T_1$$

By symmetry $R_X(\tau) = \frac{1}{2}\,\frac{T_1}{T}\left[1-\frac{|\tau|}{T_1}\right], \quad |\tau|\le T_1$

For τ near kT, only ¼ of possible pulse positions will have pulses in both waveforms

$$R_X(\tau) = \frac{1}{4}\,\frac{T_1}{T}\left[1-\frac{|\tau-kT|}{T_1}\right], \quad |\tau-kT|\le T_1$$

$$= 0 \quad \text{elsewhere}$$

***6.2.3**

$$R_X(\tau) = E[x(t)\,x(t+\tau)] = E\left[\sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} A_n A_k\, g(t-t_0-nT)\,g(t+\tau-t_0-kT)\right]$$

$$E[A_n A_k] = 0 \quad n\ne k$$
$$= 1 \quad n=k$$

For $n=k$

$$E[g(t-t_0-nT)\cdot g(t+\tau-t_0-kT)] = E[g(t-t_0-nT)\cdot g(t+\tau-t_0-nT)]$$

$$= \int_0^T \frac{1}{T}\, g(t-t_0-nT)\,g(t+\tau-t_0-nT)\,dt_0$$

$$= \int_{t-(n+1)T}^{t-nT} \frac{1}{T}\, g(t')\,g(t'+\tau)\,dt' \qquad \text{where } t' = t-t_0-nT$$

$$R_X = \sum_{n=-\infty}^{\infty} 1\cdot \int_{t-(n+1)T}^{t-nT} \frac{1}{T}\, g(t')\,g(t'+\tau)\,dt'$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} g(t')\,g(t'+\tau)\,dt' = \frac{1}{T}\,G(\tau)$$

6.3.1 a) It is not an even function

c) It is not an even function

d) $R_x(0) \ngeq |R_x(\tau)|$

* 6.3.2 a) This process is stationary but not ergodic.

b) $\overline{X(t)} = E[X(t)] = E[Y(\cos w_0 t \cos\theta - \sin w_0 t \sin\theta)]$

$= E(Y) \cdot \{ E[\cos w_0 t] \cdot E[\cos\theta] - E[\sin w_0 t] \cdot E[\sin\theta] \}$

$= 0$

$\overline{X^2(t)} = E[X^2(t)] = E[Y^2[\frac{1}{2}(1 + \cos 2w_0 t \cos 2\theta - \sin 2w_0 t \sin 2\theta)]]$

$= \frac{1}{2} E[Y^2] = 9$

c) $E[x(t)\, x(t+\tau)] = E[Y^2 \cos(w_0 t + \theta) \cos(w_0 t + w_0\tau + \theta)]$

$= E[Y^2] \cdot E[\cos w_0 t \cos(w_0 t + w_0\tau)\cos^2\theta + \sin w_0 t \sin(w_0 t + w_0\tau)\sin^2\theta$
$- \sin w_0 t \cos(w_0 t + w_0\tau)\sin\theta\cos\theta - \cos w_0 t \sin(w_0 t + w_0\tau)\cos\theta\sin\theta]$

$= E[Y^2] \cdot E[\frac{1}{2}\cos w_0 t \cos(w_0 t + w_0\tau) + \frac{1}{2}\sin w_0 t \sin(w_0 t + w_0\tau)$
$- \sin\theta\cos\theta[\sin(2w_0 t + w_0\tau)]]$

$= E[Y^2] \cdot E[\frac{1}{2}\cos w_0\tau]$

$= 18 \cdot \frac{1}{2} \cdot \frac{1}{6\tau} \sin 6\tau = \frac{3}{2\tau} \sin 6\tau$

6.3.3 $R_X(\tau) = 100\, e^{-\tau^2} \cos 2\pi\tau + 10 \cos 6\pi\tau + 36$

a) $\bar{X} = \pm 6$, $\overline{X^2} = R_X(0) = 146$, $\sigma_X^2 = 146 - 6^2 = 110$

b) Frequencies = 0; 1, 3, Hz

c) First zero crossing of $R_X(\tau)$

$R_X(\tau) = 0 \Rightarrow \tau = 0.318$ by trial and error.

6.3.4 $V(\tau) = 1 - \frac{|\tau|}{2} \qquad |\tau| \le T$

$= 0 \qquad |\tau| > T$

$\mathcal{F}[V(\tau)] = \int_{-\infty}^{\infty} V(\tau) e^{-j\omega\tau} d\tau \geq 0$

For $\tau \geq 0$

$\Rightarrow \int_0^T (1 - \frac{\tau}{2}) e^{-j\omega\tau} d\tau = \int_0^T e^{-j\omega\tau} d\tau - \frac{1}{2}\int_0^T \tau e^{-j\omega\tau} d\tau$

For $\tau < 0$

$\Rightarrow \int_{-T}^0 (1 + \frac{\tau}{2}) e^{-j\omega\tau} d\tau = \int_{-T}^0 e^{-j\omega\tau} d\tau + \frac{1}{2}\int_{-T}^0 \tau e^{-j\omega\tau} d\tau$

Only when $T = 2$ $\mathcal{F}(V(\tau)) \geq 0$

6.4.1 a) $\hat{X} = \frac{1}{21} \sum_{i=0}^{N} X_i = 0.0362 \qquad N = 20$

b) $\hat{R}(0.01n) = \frac{1}{N - n + 1} \sum_{k=0}^{N-n} X_k X_{k+n}$

$\hat{R}(0) = 1.002$, $\hat{R}(0.01) = 0.581$, $\hat{R}(0.02) = 0.1669$

$\hat{R}(0.03) = -0.0460$

c) $\hat{R}(0.01n) = \frac{1}{N+1} \sum_{k=0}^{N-n} X_k X_{k+1}$

$\hat{R}(0) = 1.002$, $\hat{R}(0.01) = 0.553$, $\hat{R}(0.02) = 0.151$, $\hat{R}(0.03) = -0.0387$

6.4.2

$$Var[\hat{R}(n\Delta t)] \le \frac{2}{N}\sum_{k=-M}^{M} R_x^2(k\Delta t)$$

a) $$Var(\hat{R}(n\Delta t)] \le 0.1\,[(1.002)^2 + 2\times(0.581)^2 + 2\times(0.1669)^2 + 2\times(-0.046)^2]$$

$$\le 0.1740$$

b) $$Var(\hat{R}(n\Delta t)] \le 0.1\,[(1.002)^2 + 2\times(0.543)^2 + 2\times(0.157)^2 + 2\times(-0.0787)^2]$$

$$\le 0.1664$$

6.4.3

$$R(\tau) = 10\,\mathrm{sinc}^2\,\tau$$

a) Zeros occur at $\tau = 1, 2 \ldots \Rightarrow \tau \le 2$

b) $\Delta\tau = 2/20 = 0.1$

c) $R_{max} = 10$ $\quad$ 5% of 10 = 0.5

$$(0.5)^2 = \frac{2}{N}\sum_{k=-20}^{20} 100\,\mathrm{sinc}^4(0.1k) = \frac{2\times 666.37}{N}$$

$$N = 5331$$

6.4.4

$$R(\tau) = A[1 + \frac{|\tau|}{T}] = A + \frac{A}{T}|\tau|, \quad |\tau| \le T$$

a) By (6-23)

$$\frac{A}{T} = \frac{n\sum_{i=0}^{n-1} X_i \cdot i - \sum_{i=0}^{n-1} X_i \cdot \sum_{i=0}^{n-1} i}{n\sum_{i=0}^{n-1} X_i^2 - (\sum_{i=0}^{n-1} X_i)^2}, \quad n = 4$$

$$= \frac{4\times 0.0178 - 1.7039\times 0.06}{4\times 14\times 10^{-4} - (0.06)^2} = \frac{-0.071}{0.002} = -35.52$$

$$A = \frac{\sum_{i=0}^{n-1} X_i - \frac{A}{T}\sum_{i=0}^{n-1} i}{4} = \frac{1}{4}(1.7039 + 35.52\times 0.06)$$

$$= 0.959 \qquad \Rightarrow T = 2.7\times 10^{-2}$$

$$\therefore R(\tau) = 0.959 - 35.52|\tau|$$

6.4.4 continued

b) $Var[\hat{R}_x(n\Delta t)] \le \frac{2}{T}\int_{-\infty}^{\infty} R_x^2(\tau)d\tau$

$\int_{-\infty}^{\infty} R_x^2(\tau)d\tau = 2\int_0^T R^2(\tau)\,d\tau = 2\int_0^{2.7\times10^{-2}}(0.969-35.82\tau)^2 d\tau$

$= 2\times 2.33\times10^{-2} = 4.66\times10^{-2}$

$\therefore Var[\hat{R}_x(n\Delta t)] \le \frac{2}{2.7}\times 4.66\times10^{-2} = 3.45\times10^{-2} = 0.0345$

The variance of estimate is smaller!

6.4.5 $R_x(\tau) = 10\,e^{-5|\tau|}\cos 20\tau$

$Var[\hat{R}_x(n\Delta t)] \le \frac{2}{T}\int_{-\infty}^{\infty} R_x^2(\tau)d\tau$

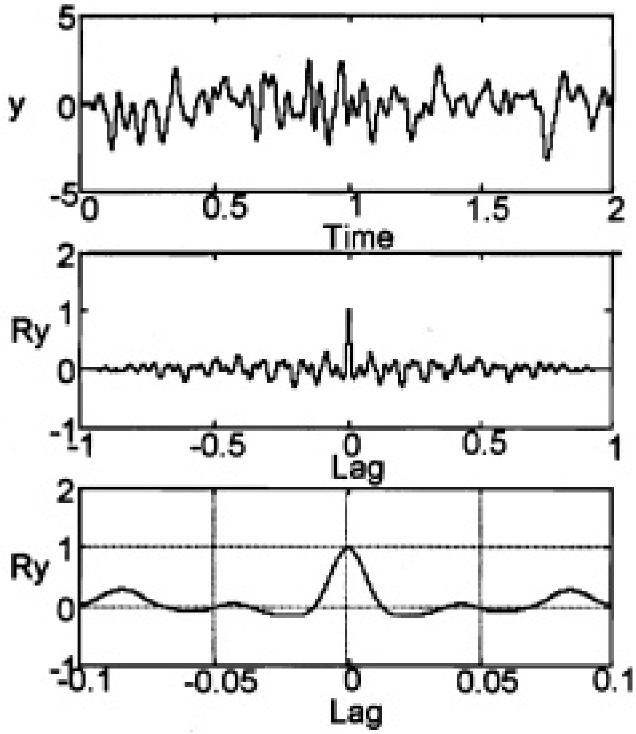
$\int_{-\infty}^{\infty} 100\,e^{-10|\tau|}\cos^2 20\tau = 2\int_0^{\infty} 100\,e^{-10\tau}\cos^2 20\tau\, d\tau$

$= 187/1700 = 9/85$

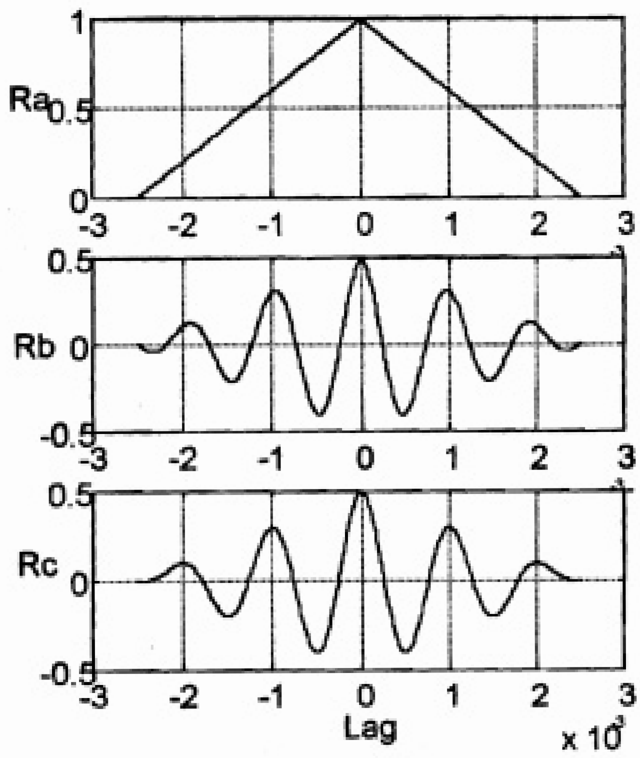
$[0.01\times 10]^2 \le \frac{2}{T}\times 9/85 \le \frac{18}{85N\cdot 0.01}$

$N \le 18/85\times0.01\times0.01 = 2117.6$

```
%PR6_4_6.m
x = randn(1,2000);
[b,a] = butter(4,20/500);
y = filter(b,a,x);
y = y/std(y);
t1= 0:.001:1.999;
[t2,R] = corb(y,y,2000);
clf
t3=t2(1800:2200);
subplot(3,1,1);plot(t1,y)
ylabel('y'); xlabel('Time')
subplot(3,1,2);plot(t2,R);grid
ylabel('Ry'); xlabel('Lag');grid
subplot(3,1,3);plot(t3,R(1800:2200))
ylabel('Ry'); xlabel('L);grid
```



```
%PR6_4_7.m
t=-1/800:1e-5:1/800;
x=ones(1,251);
[T,Rx]=corb(x,x,1e5);
y=sin(pi*t*2000);
[T,Ry]=corb(y,y,1e5);
z=cos(pi*t*2000);
[T,Rz]=corb(z,z,1e5);
subplot(3,1,1);plot(T,Rx);grid;ylabel('Ra')
subplot(3,1,2);plot(T,Ry);grid;ylabel('Rb')
subplot(3,1,3);plot(T,Rz);grid
xlabel('Lag');ylabel('Rc')
```



6.5.1 $R_X(\tau) = E[X(t)X(t+\tau)]$

When $|\tau|$ is greater than the time interval

$\Rightarrow X(t)$ & $X(t+\tau)$ are independent

$Pr[|\tau| \le \text{Time interval}] = e^{-\alpha|\tau|}$

and $X(t) = X(t+\tau)$ when $|\tau| \le$ time interval

$\therefore R_X(\tau) = A^2 e^{-\alpha|\tau|}$

6.5.2 When $|\tau|$ is greater than time interval

$\Rightarrow X(t)$ & $X(t+\tau)$ are independent

When $|\tau| \le$ time interval $X(t) = X(t+\tau)$

$Pr[|\tau| \le \text{Time interval}, \text{sample is } 0] = \frac{1}{2} e^{-\alpha|\tau|}$

$Pr[|\tau| \le \text{Time interval}, \text{sample is } 2A] = \frac{1}{2} e^{-\alpha|\tau|}$

$\therefore R_X(\tau) = \frac{1}{2}(2A)^2 e^{-\alpha|\tau|} = 2A^2 e^{-\alpha|\tau|}$

6.5.3 a) $R_X(\tau) = 10e^{-\tau^2}$, $\bar{X}^2 = R_X(\infty) = 0$, $\overline{X^2} = R_X(0) = 10$

$\sigma_X^2 = 100$

b) $R_X(\tau) = 10e^{-\tau^2}\cos 2\pi\tau^2$, $\bar{X}^2 = 0$, $\overline{X^2} = 10$

$\sigma_X^2 = 100$

c) $R_X(\tau) = 10\dfrac{\tau^2+8}{\tau^2+4}$, $\bar{X}^2 = 10$, $\overline{X^2} = 20$

$\sigma_X^2 = 10$

6.7.4

$$R_{\dot{X}Y}(\tau) = E\{\dot{X}(t)\,Y(t+\tau)\} = E\left[\lim_{\varepsilon\to 0} \frac{X(t+\varepsilon)-X(t)}{\varepsilon}\cdot Y(t+\tau)\right]$$

$$= \lim_{\varepsilon\to 0} \frac{R_{XY}(\tau-\varepsilon) - R_{XY}(\tau)}{\varepsilon} = -\frac{d}{d\tau}R_{XY}(\tau)$$

$$= -\left[-2(\tau-1)\cdot 16\,e^{-(\tau-1)^2}\right] = 32(\tau-1)\,e^{-(\tau-1)^2}$$

6.8.1

a) $R_X(\tau) = \frac{10^{-4}}{2}\cos 100\tau$, $R_N(\tau) = 10\,e^{-100|\tau|}$

$R_{X+N}(0) = R_X(0) + R_N(0) = 10.00005$

b) $\left|\frac{10^{-4}}{2}\cos 100\tau\right| = \left|10\cdot\%\,e^{-100|\tau|}\right|$

$\Rightarrow \tau = 0.146$

6.8.2

a) $E\{Z(t)\} = E\{Y(t)X(t)\} + E\{Y(t)N(t)\}$

$= Y(t)\,X(t) + E\{Y(t)\}\cdot E\{N(t)\} = Y(t)\,X(t)$

$= \frac{0.1}{2}\left\{\sin(2\omega t + \varphi + \theta) + \sin(\theta - \varphi)\right]$

After low pass filter

$E\{Z(t)\} = \frac{0.1}{2}\sin[(\theta-\varphi)]$

b) When $\theta + \frac{\pi}{2} = \varphi$ $E\{Z(t)\}$ is greatest.

6.8.4 $R_X(\tau) = 10e^{-2|\tau|} - 5e^{-4|\tau|}$

a) $\bar{X} = 0, \quad \overline{X^2} = 5, \quad \sigma_X^2 = 5$

b) For $\tau \geq 0$ $\quad \frac{d}{d\tau} R_X(\tau) = -20e^{-2\tau} + 20e^{-4\tau}$

For $\tau \leq 0$ $\quad \frac{d}{d\tau} R_X(\tau) = 20e^{2\tau} - 20e^{4\tau}$

$\therefore$ $\frac{d}{d\tau} R_X(\tau)$ is continuous at $\tau = 0$

$\Rightarrow$ Differentiable!

***6.7.1** $X(t)$ & $Y(t)$ are independent

a) $R_{X+Y}(\tau) = R_X(\tau) + R_Y(\tau) = 25e^{-10|\tau|}\cos 100\pi\tau + 16\frac{\sin 50\pi\tau}{50\pi\tau}$

b) $R_{X-Y}(\tau) = R_X(\tau) + R_Y(\tau) = 25e^{-10|\tau|}\cos 100\pi\tau + 16\frac{\sin 50\pi\tau}{50\pi\tau}$

c) Crosscorrelation of $X+Y$ and $X-Y$

$E[(X_1+Y_1)(X_2-Y_2)] = E[X_1X_2 + Y_1X_2 - X_1Y_2 - Y_1Y_2]$

$= R_X(\tau) - R_Y(\tau) = 25e^{-10|\tau|}\cos\pi\tau + 16\frac{\sin 50\pi\tau}{50\pi\tau}$

d) $R_{XY}(\tau) = R_X(\tau)\cdot R_Y(\tau) = 400\,e^{-10|\tau|}\cos 100\pi\tau\left[\frac{\sin 50\pi\tau}{50\tau\pi}\right]$

6.7.2 By equation 6-29

$|R_{(X+Y)(X-Y)}(\tau)| \leq [R_{X+Y}(0)\ R_{X-Y}(0)]^{\frac{1}{2}} = 41$

The actual maximum value is when $\tau = 0$

$R_{(X+Y)(X-Y)}(0) = 9$ which is smaller than the bound

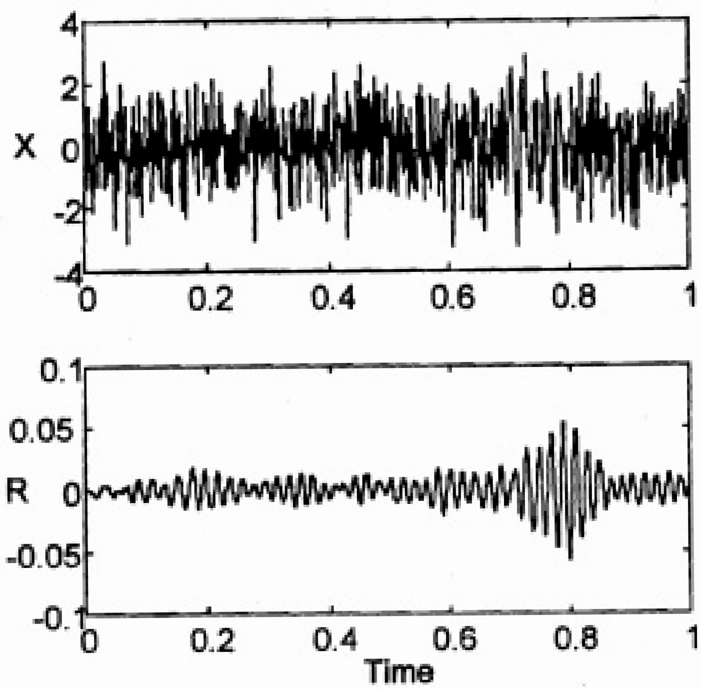
6.7.3 a) $R_{\dot{X}X}(\tau) = R_{X\dot{X}}(\tau) = \frac{d}{d\tau}R_X(-\tau) = \frac{\tau\cos(-\tau) + \sin(-\tau)}{\tau^2}$

b) $R_{\dot{X}}(\tau) = -\frac{d}{d\tau^2}R_X(\tau) = -\frac{d}{d\tau}\left[\frac{x\cos x - \sin x}{x^2}\right]$

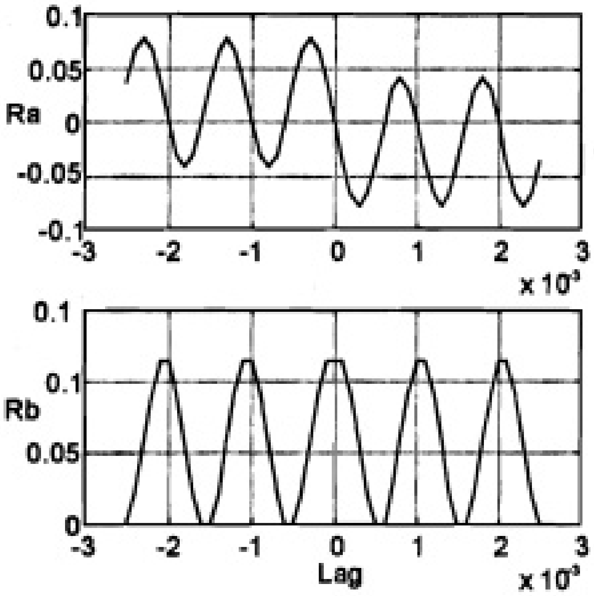
$= \frac{x^2\sin x + 2x^2\cos x - 2x\sin x}{x^4}$

```
%pr6_8_3
t1=0.0:0.001:0.099;
s1 = sin(100*pi*t1);
s = zeros(1,1000);
s(700:799) = s1;
randn('seed', 1000)
n1 = randn(1,1000);
x = s + n1;

y=fliplr(s1);
z=.001*conv(s1,x);
%tt=0:.001:.001*(length(z)-1);
tt=0:.001:0.999;
subplot(2,1,1); plot(tt,x);ylabel('X')
subplot(2,1,2); plot(tt,z(1:1000));xlabel('Time');ylabel('R')
```



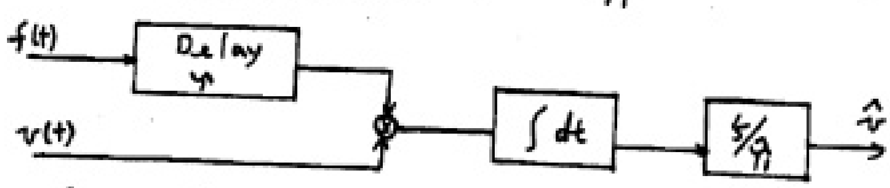
```
%P6_8_4.m
dt=1e-4; T=1/400;
t=-T/2:dt:T/2;
x1=ones(size(t));
y1=sin(2000*pi*t);
[t2,R1]=corb(x1,y1,10000);
x2=x1; y2=cos(2000*pi*t);
[t2,R2]=corb(x2,y2,10000);
subplot(2,1,1);plot(t2,R1);ylabel('Ra')
subplot(2,1,2);;plot(t2,R2);ylabel('Rb');xlabel('Lag')
```



6.8.5

a) τ_1 is time to move 5 m $\therefore$ $v = \frac{5}{\tau_1}$

b)



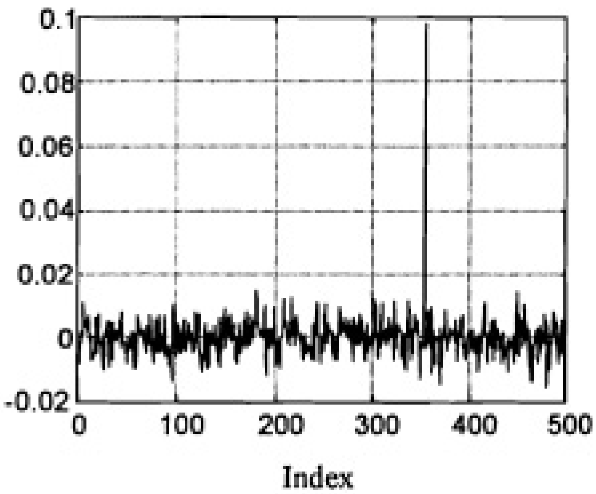
$\tau_{max} = \frac{5}{v_{min}} = 1$ sec.

$T_{min} = \frac{5}{v_{max}} = 0.1$ sec.

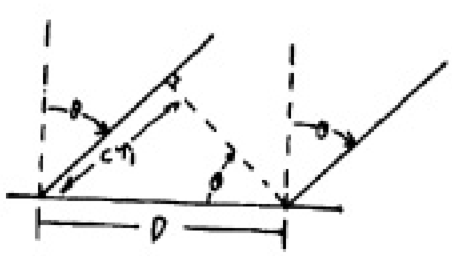
c) $\tau_{max} \to \infty$ when $v_{min} \to 0$

d) $v_{max} = \frac{5}{\tau_{min}} = \frac{5}{1/12} = 60$ m/sec. $\left(\Delta t = \frac{1}{12} = \tau_{min}\right)$

```
%PR6_8_6.m
randn('seed', 2000)
g=round(200*sqrt(pi));z=randn(1,10000 + g);
y=sqrt(0.1)*z(g:10000+g-1) + randn(1,10000);  %-10dB SNR
x=z(1:10000);
----------------------
xx=sign(x); yy=sign(y);
L=length(xx);
for k=1:500
  w(k)=(sum(xx(k:L) == yy(1:L-k+1))-.5*(L-k+1))/(L-k+1);
   % .5*(L-k+1) rqd to eliminate 0.5 offset
 end
plot(1:k,w)
[z,n]=max(w);
disp(['Empirical Delay = ',num2str(n)])
disp(['     True Delay = ',num2str(g)])
disp(['Empirical a = ', num2str(z)])
Empirical Delay = 354
      True Delay = 354
     Empirical a = 0.09791
```



6.8.7



c: Speed of light.

$\tau_1 = \frac{D\sin\theta}{c}$ $\quad\therefore\ \theta = \sin^{-1}\left[\frac{c\tau_1}{D}\right]$

$$d\theta = \frac{d\theta}{dD}\,dD + \frac{d\theta}{d\tau_1}\,d\tau_1$$

$$= -\frac{1}{D}\tan\theta\,dD + \frac{c}{D\cos\theta}\,d\tau_1$$

6.8.8| cont.

Assume dD & dT_1 are independent.

$$\Rightarrow \sigma_\theta^2 = \left(\frac{-1}{D}\tan\theta\right)^2 \sigma_D^2 + \left(\frac{c}{D\cos\theta}\right)^2 \sigma_{T_1}^2 \le (10^{-3})^2 = 10^{-6}$$

$$\Rightarrow \sigma_{T_1}^2 \le \left(\frac{500\cos\theta}{3\times10^8}\right)^2 \left[10^{-6} - \left(\frac{\tan\theta}{500}\right)^2 10^{-4}\right]$$

For $\theta = 0$ $\quad \sigma_{T_1}^2 = 2.778\times10^{-18}$

$\theta = 1.4$ $\quad \sigma_{T_1}^2 = 7.917\times10^{-20}$

$\Rightarrow \sigma_{T_1} \le 2.81\times10^{-10}$ or 0.281 nsec.

6.9.1| $X(t)$ is zero mean

$\Lambda_x = R_x$

$R_x(0.5n) = 36\, e^{-2|0.5n|} \cos \pi \tau$

$$\therefore \Lambda_x = \begin{bmatrix} 36 & 0 & -4.87 & 0 \\ 0 & 36 & 0 & -4.87 \\ -4.87 & 0 & 36 & 0 \\ 0 & -4.87 & 0 & 36 \end{bmatrix}$$

6.9.2|

$$\Lambda^{-1} = \begin{bmatrix} 1.5 & -1 & 0.5 \\ -1 & 2 & -1 \\ 0.5 & -1 & 1.5 \end{bmatrix}$$

$$\therefore E[X^T \Lambda^{-1} X] = E\left\{ (X_1, X_2, X_3) \begin{bmatrix} 1.5 & -1 & 0.5 \\ -1 & 2 & -1 \\ 0.5 & -1 & 1.5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \right\}$$

$$= E\left[1.5X_1^2 - X_1X_2 + \tfrac{1}{2}X_1X_3 - X_1X_2 + 2X_2^2 - X_2X_3 + \tfrac{1}{2}X_1X_3 - X_2X_3 + 1.5X_3^2\right]$$

$$= 1.5\times1 - \tfrac{1}{2} + 0.5\times0 - \tfrac{1}{2} + 2\times1 - \tfrac{1}{2} + \tfrac{1}{2}\times0 - \tfrac{1}{2} + 1.5\times1$$

$$= 3$$

6.9.3 a) $Y(t) = a_0 \cdot X(t) + a_1 \cdot X(t-\Delta t) + a_2 \cdot X(t-2\Delta t) + \cdots = \sum_{i=0}^{N} a_i \cdot X(t-i\Delta t)$

b) $E[Y(t)\,Y(t+\tau)] = E\left[\sum_{i=0}^{N} a_i \cdot X(t-i\Delta t) \sum_{j=0}^{N} a_j \cdot X(t-j\Delta t+\tau)\right]$

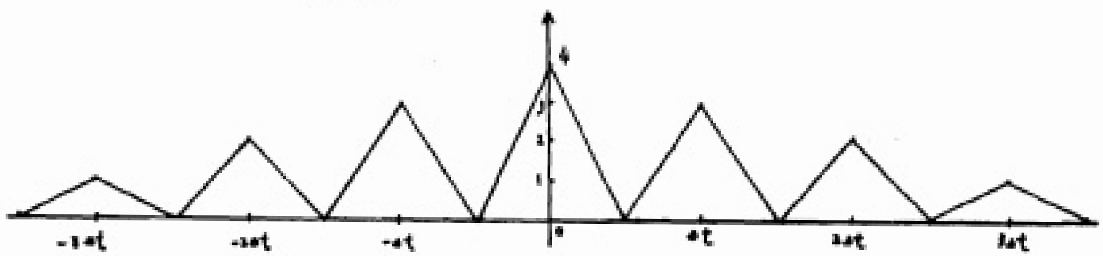
$= \sum_{i=0}^{N} \sum_{j=0}^{N} a_i a_j \, E[X(t-i\Delta t)\, X(t-j\Delta t+\tau)]$

$= \sum_{i=0}^{N} \sum_{j=0}^{N} a_i a_j \, R_X((i-j)\Delta t+\tau)$

6.9.4 $R_X(\tau) = 1 - \frac{|\tau|}{\Delta t} \qquad |\tau| \le \Delta t$

a) $N=3 \qquad a_0 = a_1 = a_2 = a_3 = 1$

$R_Y(\tau) = \sum_{i=0}^{3} \sum_{j=0}^{3} R_X((i-j)\Delta t+\tau)$



b) $N=3 \qquad a_0 = 4,\ a_1 = 3,\ a_2 = 2,\ a_3 = 1$

$R_Y(\tau) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_i a_j \, R_X((i-j)\Delta t+\tau)$