2.7

$$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n]$$

g

input/output difference equation

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$$

$$g[n] = u[n] - u[n-4]$$

a) Find $y[n]$ when $x[n] = \delta[n-1]$

Strategy: plug $x[n]$ into the input/output difference equation:

$$y[n] = \sum_{k=-\infty}^{\infty} \underbrace{\delta[k-1]}_{\substack{\text{centered @} \\ k=1}} \cdot \left\{ u[n-2k] - u[n-4-2k] \right\}$$

$$\overset{k=1}{=} u[n-2] - u[n-6]$$

b) Find $y[n]$ when $x[n] = \delta[n-2]$

$$y[n] = u[n-4] - u[n-8]$$

c) Is this system L.T.I.? NO

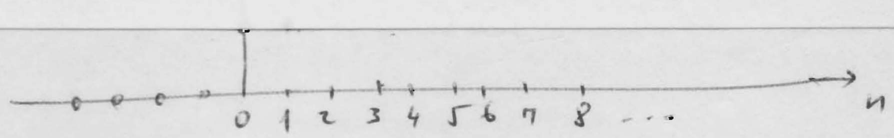
Time invariant: $x[n] \longrightarrow \boxed{} \longrightarrow y[n]$

$x[n-1]$ $\qquad$ $y[n-1]$

In this system: $\delta[n-1] \longrightarrow \boxed{} \longrightarrow y[n] = u[n-2] - u[n-6]$

$\delta[n-2]$ $\qquad$ $u[n-4] - u[n-8]$

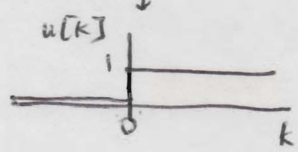
Not time invariant!



$$\boxed{y[n] = 2u[n] - \delta[n] - \delta[n-1]}$$

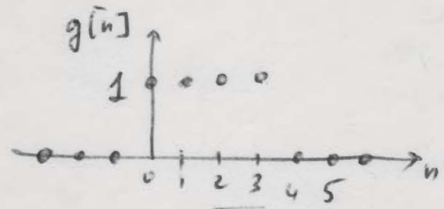
d) Find $y[n]$ when $x[n] = u[n]$.

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, g[n-2k] = \sum_{k=0}^{\infty} g[n-2k]$$



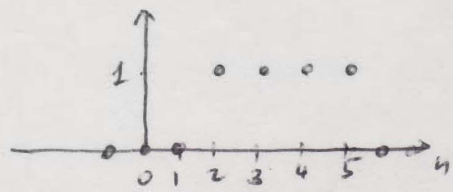
$k=0$

$g[n-0] = u[n] - u[n-4]$



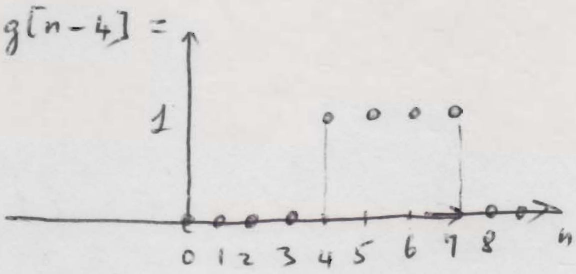
$k=1$

$g[n-2]$



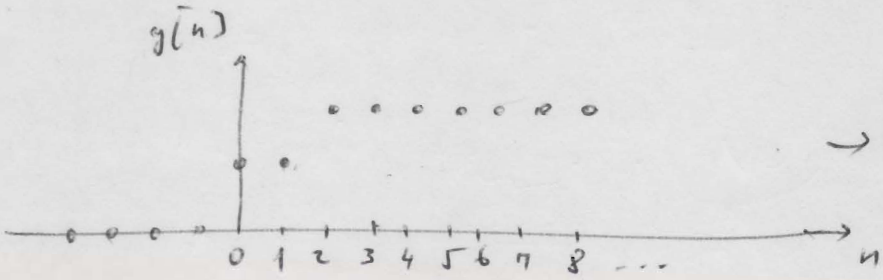
$k=2$

$g[n-4] =$



etc...

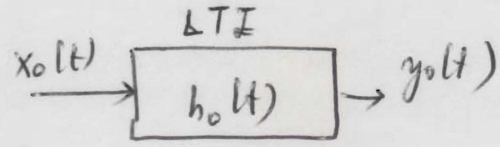
Need to add these signals to get $y[n] = \begin{cases} 1 & n = 0, 1 \\ 2 & n = 2, 3, 4, 5, \text{etc...} \\ 0 & n < 0 \end{cases}$

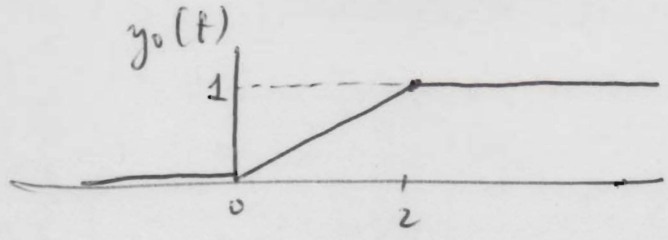


→ Mathematically:

$$\boxed{y[n] = 2u[n] - \delta[n] - \delta[n-1]}$$

2.47



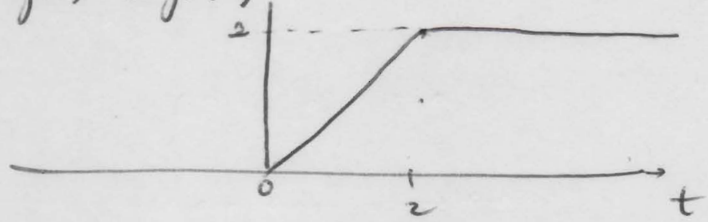


$\leftrightarrow y_o(t) = \int_{-\infty}^{\infty} d\lambda \, x_o(\lambda) h_o(t-\lambda)$

a) $x(t) = 2x_o(t)$ & $h(t) = h_o(t)$

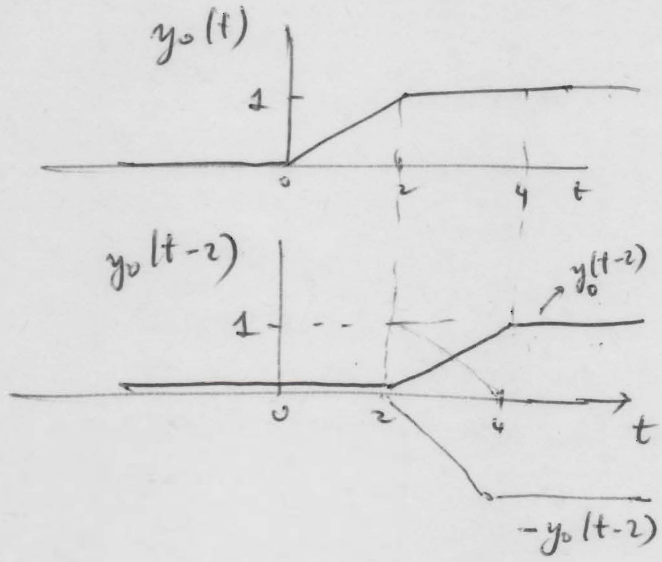
Can we determine $y(t)$? Yes $\rightarrow$ since system LTI

$y(t) = 2y_o(t)$

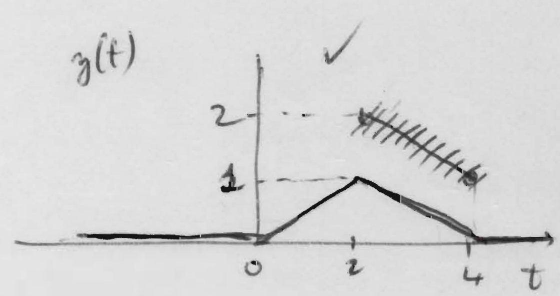


b) $x(t) = x_o(t) - x_o(t-2)$ & $h(t) = h_o(t)$

$y(t)$? Yes LTI $\rightarrow$ $y(t) = y_o(t) - y_o(t-2)$



$\Rightarrow$



c) $\boxed{x(t) = x_0(t-2) \quad ; \quad h(t) = h_0(t+1)}$

$y(t)$? $\rightarrow$ $y(t) = \int_{-\infty}^{\infty} d\lambda \; x(\lambda)\, h(t-\lambda) = \int_{-\infty}^{\infty} d\lambda \; x_0(\lambda-2)\, h_0(t-\lambda+1)$

? ? Not given

Alternative: Can we use LTI as in a) & b) ?

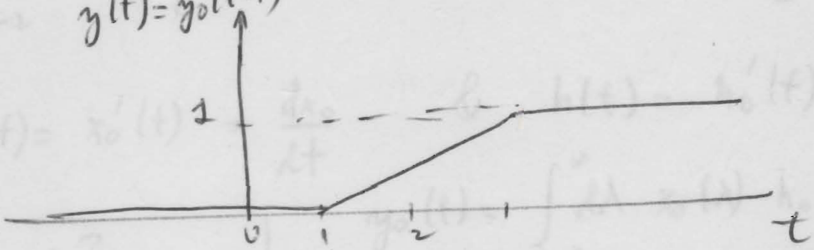
$x(t) = x_0(t-2) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = y_0(t-2)$

Time inv.

$x(t) = x_0(t-2) \longrightarrow \boxed{h(t+1)} \longrightarrow y(t) = y_0(t-1)$

Time inv

$y(t) = y_0(t-1)$



d) $x(t) = x_0(-t)$; $h(t) = h_0(t)$ $\rightarrow \boxed{y(t) = \int_{-\infty}^{\infty} d\lambda \; x_0(-\lambda)\, h_0(t-\lambda)}$

time reversal.

Can we use $\underbrace{\text{L.T.I}}_{\text{does not include time reversal}}$? $\rightarrow$ need to use convolution but x_0 & h_0 are not given $\rightarrow$ not enough information to get $y(t)$

No connection with $y_0(t) = \int_{-\infty}^{\infty} d\lambda \; x_0(\lambda)\, h_0(t-\lambda)$

e) $x(t) = x_0(-t)$; $h(t) = h_0(-t)$

Can't use $\underbrace{\text{LTI}}_{\text{no time reversal}}$

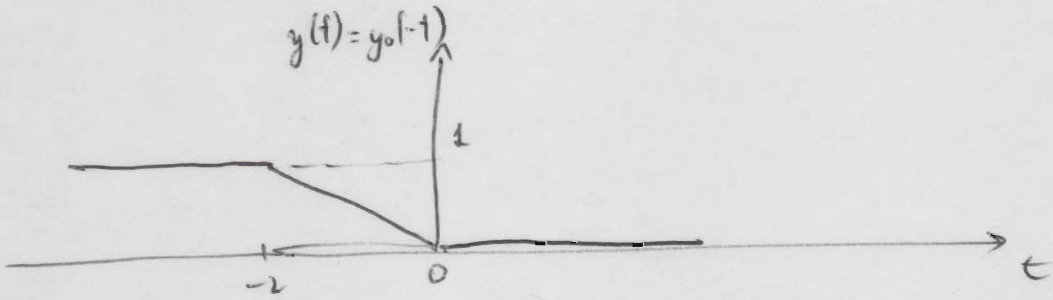
$y(t) = \int_{-\infty}^{\infty} d\lambda \; x_0(-\lambda)\, h_0(-t+\lambda) = -\int_{\infty}^{-\infty} d\rho \; x_0(\rho)\, h_0(-t-\rho) = \int_{-\infty}^{\infty} d\rho \; x_0(\rho)\, h_0(-t-\rho)$

$-\lambda \rightarrow \rho$

$$y(t) = \int_{-\infty}^{\infty} d\lambda\; x_0(\lambda)\; h_0(-t-\lambda) \quad = y_0(-t)$$

comparing with:

$$y_0(t) = \int_{-\infty}^{\infty} d\lambda \quad x_0(\lambda)\; h_0(t-\lambda)$$



f) $\quad x(t) = x_0'(t) = \dfrac{dx_0}{dt} \qquad \& \qquad h(t) = h_0'(t) = \dfrac{dh_0}{dt}$

$\rightarrow y(t)$?

$$y_0(t) = \int_{-\infty}^{\infty} d\lambda\; x_0(\lambda)\; h_0(t-\lambda)$$

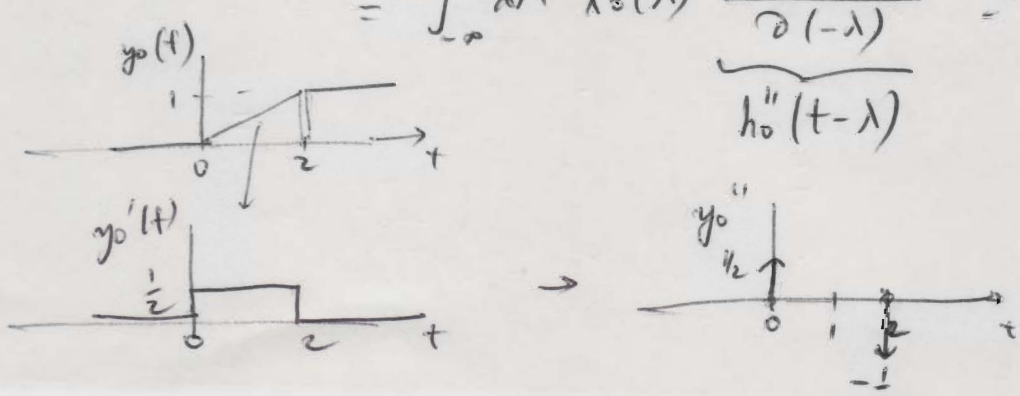
$$\boxed{y(t) = \int_{-\infty}^{\infty} d\lambda\; \underbrace{x_0'(\lambda)}_{\frac{\partial x_0}{\partial \lambda}}\; h_0'(t-\lambda)}$$

$$y_0'(t) = \frac{dy_0}{dt} = \int_{-\infty}^{\infty} d\lambda\; x_0(\lambda)\; h_0'(t-\lambda)$$

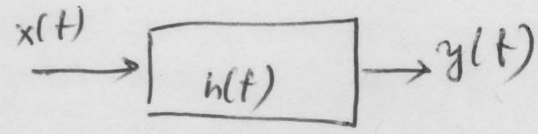
integration by parts: $\quad \int u\,dv = uv - \int v\,du$

$$y(t) = -\int_{-\infty}^{\infty} d\lambda\; x_0(\lambda)\; \frac{\partial h_0'(t-\lambda)}{\partial \lambda}$$

$$= \int_{-\infty}^{\infty} d\lambda\; x_0(\lambda)\; \underbrace{\frac{\partial h_0'(t-\lambda)}{\partial(-\lambda)}}_{h_0''(t-\lambda)} = \boxed{y_0''(t)}$$

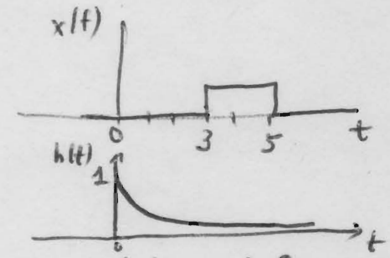


2.11



$x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t} u(t)$

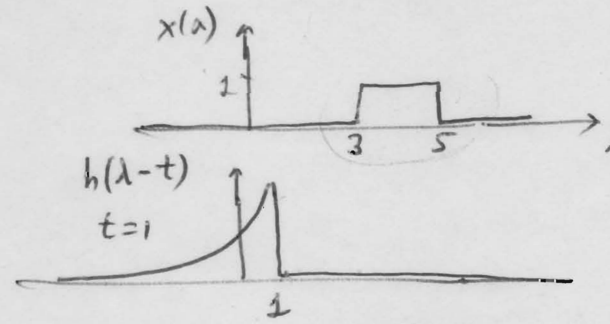


a) $y(t)\ ? = x(t) * h(t) = \int_{-\infty}^{\infty} d\lambda\ x(\lambda) . h(t-\lambda)$ ① ← time reverse and shift $h(\lambda)$

definition of convolution

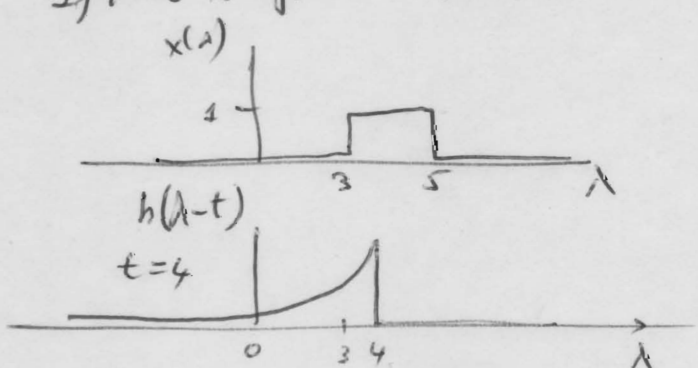
$= h(t) * x(t) = \int_{-\infty}^{\infty} d\lambda\ h(\lambda)\ x(t-\lambda)$ ② ← time reverse and shift $x(\lambda)$

① (i) If we time ~~reverse~~ reverse $h(t)$ and shift $t\ \forall -\infty < t < 3$



No overlap → zero product.
→ no integral
→ no convolution ∴ $y(t) = 0$

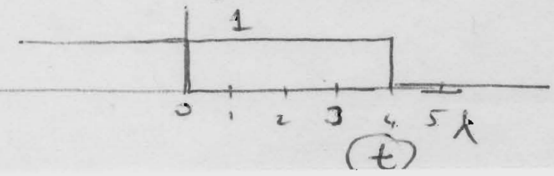
(ii) If time shift $t\ \ \forall\ 3 \leq t \leq 5$



Overlap b/w 3 & t

$\hookrightarrow y(t) = \int_3^t d\lambda . 1 . e^{-3(t-\lambda)} u(t-\lambda) = \int_3^t d\lambda\ e^{-3(t-\lambda)}$

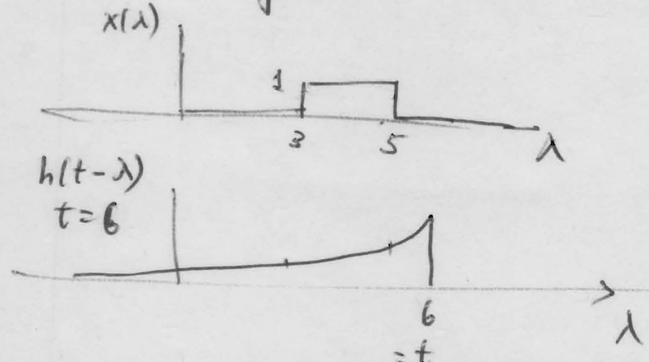
$u(t-\lambda) = 1$



$= \left[\frac{e^{-3(t-\lambda)}}{3} \right]_{\lambda=3}^{\lambda=t}$

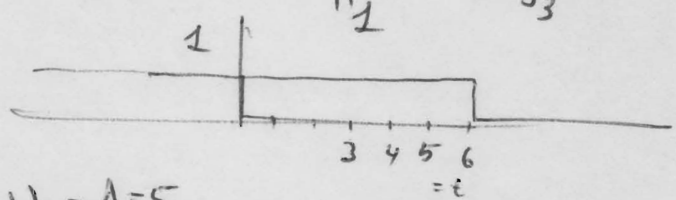
$\boxed{y(t) = \frac{1 - e^{-3(t-3)}}{3}, \quad 3 \leq t \leq 5}$

(iii) If time shift t $\forall$ $5 < t < +\infty$



overlap b/w 3 & 5

$$y(t) = \int_3^5 d\lambda\ 1 \cdot e^{-3(t-\lambda)} \underbrace{u(t-\lambda)}_{=1} = \int_3^5 d\lambda\, e^{-3(t-\lambda)}$$



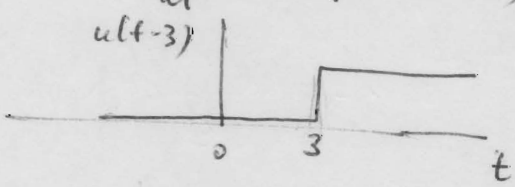
$$y(t) = \left[\frac{e^{-3(t-\lambda)}}{3} \right]_{\lambda=3}^{\lambda=5} = \frac{e^{-3(t-5)} - e^{-3(t-3)}}{3} \quad (5<t<\infty)$$

$$\Rightarrow \boxed{y = x * h = \begin{cases} 0 & -\infty < t < 3 \\ \dfrac{1 - e^{-3(t-3)}}{3} & 3 \le t \le 5 \\ \dfrac{e^{-3(t-3)}\left[e^{6} - 1\right]}{3} & 5 < t < \infty \end{cases}}$$

b) Compute: $g(t) = \dfrac{dx}{dt} * h =$

$x(t) = u(t-3) - u(t-5) \rightarrow \dfrac{dx}{dt} = \delta(t-3) - \delta(t-5)$

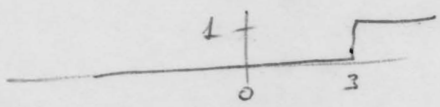
$\dfrac{dx}{dt}$ is slope of $x(t) \rightarrow \infty$ @ 3, 0 elsewhere

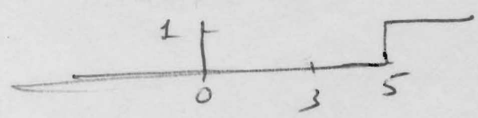


$$\rightarrow g(t) = \int_{-\infty}^{\infty} d\lambda \; \frac{dx}{dt}(\lambda) \; h(t-\lambda) = \int_{-\infty}^{\infty} d\lambda \left[\delta(\lambda-3) - \delta(\lambda-5)\right] e^{-3(t-\lambda)} u(t-\lambda)$$

$$\boxed{\text{Recall: } \int_{-\infty}^{\infty} d\alpha \; f(\alpha) \; \underbrace{\delta(\alpha - p)}_{\text{centered @ } p} = f(p)}$$

$$\rightarrow g(t) = e^{-3(t-3)} \underbrace{u(t-3)}_{} - e^{-3(t-5)} \underbrace{u(t-5)}_{}$$





$$\hookrightarrow g(t) = \begin{cases} 0 & -\infty < t < 3 \\ e^{-3(t-3)} & 3 \le t \le 5 \\ \boxed{e^{-3(t-3)} - e^{-3(t-5)}} & 5 < t < \infty \end{cases}$$

$$\hookrightarrow e^{-3(t-3)} \left[1 - e^{6}\right]$$

c) Note:

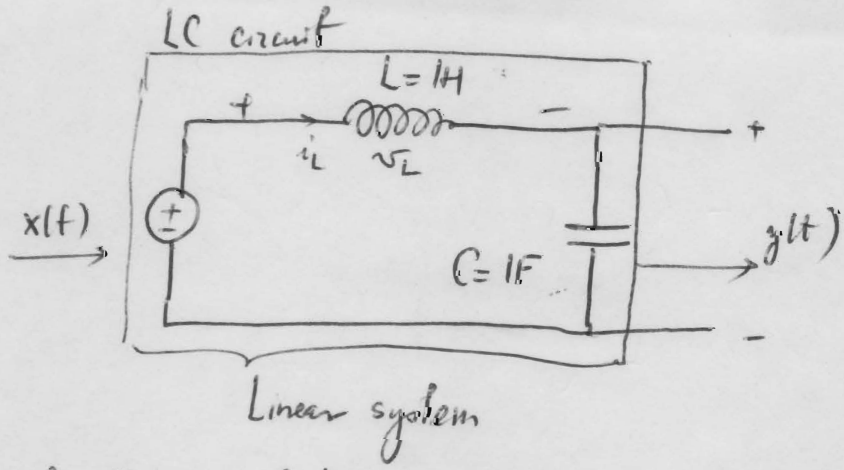
$$\frac{dy}{dt} = \begin{cases} 0 & -\infty < t < 3 \\ e^{-3(t-3)} & 3 \le t \le 5 \\ e^{-3(t-3)} \left[1 - e^{6}\right] & 5 < t < \infty \end{cases}$$

$$\hookrightarrow \frac{d}{dt} y(t) = \underbrace{\frac{dx}{dt} * h}_{g(t)}$$

If $y(t) = x(t) * h(t) \quad \longleftrightarrow \quad \boxed{\frac{dy}{dt} = \frac{dx}{dt} * h(t)}$

$$z(t) = x(t) \cdot h(t) \quad \longleftrightarrow \quad \frac{dz}{dt} = \frac{dx}{dt} h + x \frac{dh}{dt}$$

2.61)



1) Write the input/output D.E. relating x(t) & y(t)

$$x(t) - v_L - y = 0$$

$$\boxed{x(t) - L\frac{di_L}{dt} - \underbrace{\frac{1}{C}\int i_L\,dt}_{y} = 0}$$

$$\frac{dy}{dt} = \frac{1}{C} i_L \rightarrow \frac{d^2y}{dt^2} = \frac{1}{C}\frac{di_L}{dt} \rightarrow \boxed{\frac{di_L}{dt} = C\frac{d^2y}{dt^2}}$$

$$x(t) - LC\frac{d^2y}{dt^2} - y(t) = 0 \rightarrow \boxed{LC\frac{d^2y}{dt^2} + y(t) = x(t)}$$

2nd order D.E in y, where x is the non-homogeneous term.

2) Show that this D.E (homogeneous part) has as solution

$$K_1 e^{j\omega_1 t} + K_2 e^{j\omega_2 t}$$

(specify values for ω_1 & ω_2)