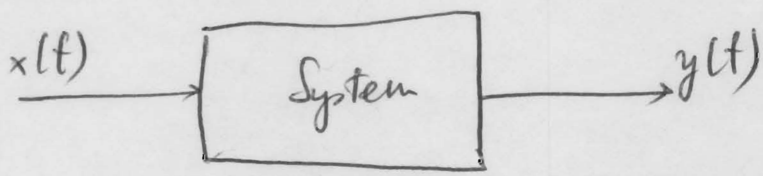
Introduction to Systems:

A system is represented with a box whose content we don't know. For us, a system is identified by the equation that relates the output $y(t)$ to the input $x(t)$:

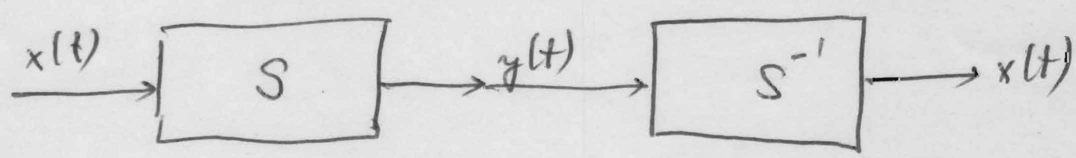


$$y(t) = 2x(t) - x^2(t+1)$$ → This is the "input - output" equation.

Properties of systems:

1) Memory: a system has memory when current output depends on previous or past inputs

2) Invertibility: a system S is invertible if we can find the inverse system S^{-1} such that:

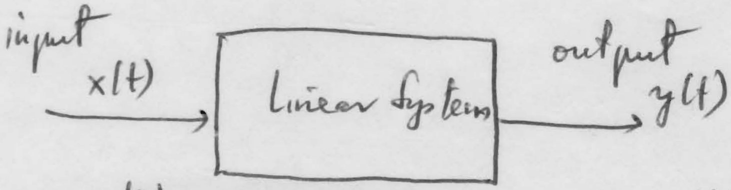


3) Causality: a system S is causal if its output only depends on current or past inputs

4) Stability: a system S is stable if it will not produce extremely large output for normal input

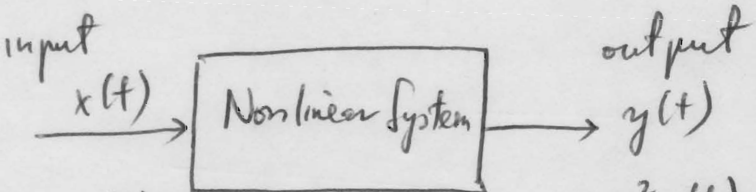
5) Time-invariance: a system S is time-invariant when it produces same output for an input regardless of when it is applied.

6) Linearity: a system S is linear if:



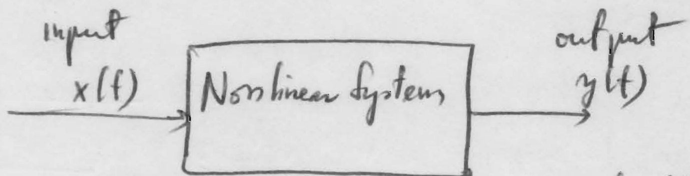
$a\,x(t)$ $\qquad$ $a\,y(t)$

$x_1(t)+x_2(t)$ $\qquad$ $y_1(t)+y_2(t)$



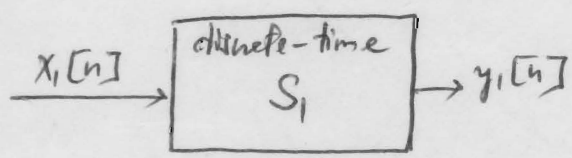
$a\,x(t)$ $\qquad$ $a^2 y(t)$

$x_1 + x_2$ $\qquad$ $y_1 + y_2 + 2\sqrt{y_1 y_2}$

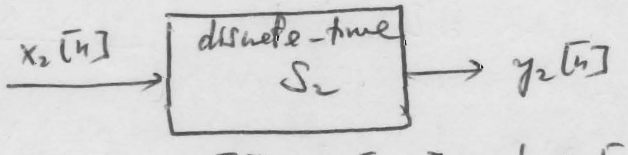


$a\,x(t)$ $\qquad$ $\sin(a\,x(t)) \neq a\,y(t)$

1.15

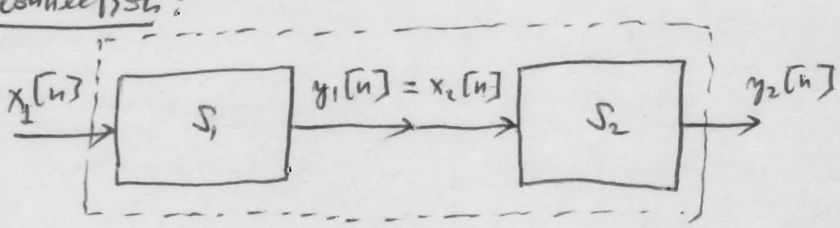


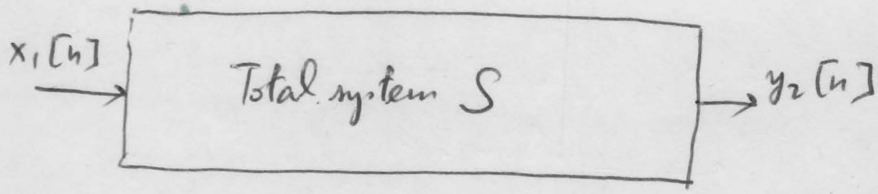
$$y_1[n] = 2x_1[n] + 4x_1[n-1]$$



$$y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$$

<u>Series connection:</u>





a) Input-Output equation for S = how to get $y_2[n]$ from $x_1[n]$

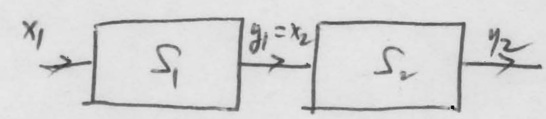
Use: $y_1[n] = x_2[n]$ or output to S_1 is input to S_2 in a series connection:

$$y_2[n] = y_1[n-2] + \tfrac{1}{2} y_1[n-3]$$
$$\overset{S_1}{=} 2x_1[n-2] + \underbrace{4x_1[n-3] + \tfrac{1}{2}2x_1[n-3]} + \tfrac{1}{2}4x_1[n-4]$$

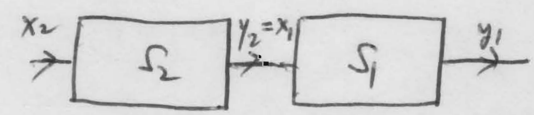
$$\boxed{y_2[n] = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]}$$

↳ input-output equation for system S, which is the series combination of S_1 & S_2

b)

$S =$ 

$\rightarrow y_2[n] = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]$

if $S =$ 

?

↳ use $y_2[n] = x_1[n]$

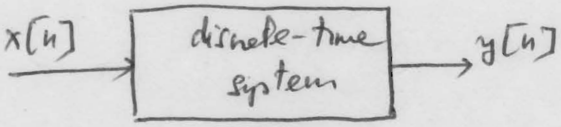
$$y_1[n] = 2y_2[n] + 4y_2[n-1]$$
$$\overset{S_2}{=} 2x_2[n-2] + \underbrace{2\tfrac{1}{2}x_2[n-3] + 4x_2[n-3]} + 4\tfrac{1}{2}x_2[n-4]$$
$$= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4]$$

→ Same system if order of connection is switched!

→ same input-output equation:

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

1.16



$$y[n] = x[n]\,x[n-2]$$

a) Is this system memoryless? → No, it has memory since current ouput depend on past input $x[n-2]$

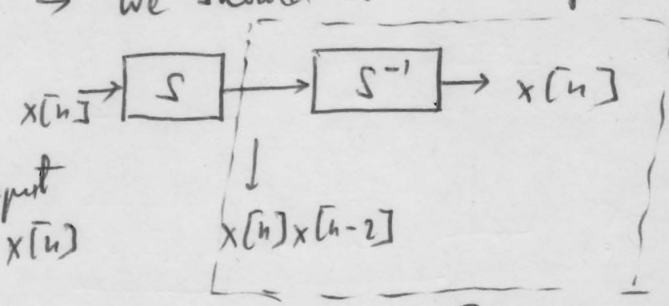
b) What is $y[n]$ if $x[n] = A\,\delta[n]$; A any real or complex number

$$y[n] = A\,\delta[n]\,A\,\delta[n-2] = A^2\,\delta[n]\,\delta[n-2] = 0$$

since $\begin{cases} \delta[n] \text{ is centered @ } n=0 \rightarrow \text{ it is 1 @ } n=0 \text{ \& 0 otherwise} \\ \delta[n-2] \text{ is centered @ } n=2 \rightarrow \text{ it is 1 @ } n=2 \text{ \& 0 otherwise} \end{cases}$

c) Is this system invertible?

→ If it is → we should be able to find S^{-1} such that:



?

only way: $y[n] = x[n]\,\dfrac{1}{x[n-2]}$

→ division: this will blowup with any zero in the input → Not possible.

→ This system is not invertible.

1.27 a) $y(t) = x(t-2) + x(2-t)$

1) Memory? Yes

2) Time invariant? $t \to t+a$ $\begin{cases} x(t-2) \to x(t+a-2) \\ x(2-t) \to x(2-t-a) \end{cases}$

$y(t+a) = x(t+a-2) + x(2-t-a)$ → Yes!

($x(t) \to x(t+a)$ then $y(t) \to y(t+a)$)

3) Linear? Yes

4) Causal? (if current output depends on previous inputs only).

$y(0) = \underbrace{x(-2)}_{\text{previous}} + \underbrace{x(2)}_{\text{future}}$ → not causal

$y(1) = x(-1) + x(1)$ → causal

$y(2) = x(0) + x(0)$ → causal

→ system is causal if $t \geq 1$

5) Stable? Yes (For normal input system won't produce ∞ output)

(b) $y(t) = (\cos 3t)\, x(t)$

1)

2)

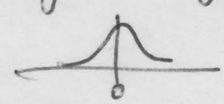
3)

4)

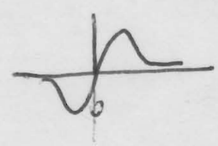
5)

$$f(t) \text{ any function} \longrightarrow \begin{cases} \text{Even} = Ev[f(t)] = \dfrac{f(t)+f(-t)}{2} \\ \text{Odd} = Odd[f(t)] = \dfrac{f(t)-f(-t)}{2} \end{cases}$$

a function $g(t)$ is even when $g(-t) = g(t)$ or $g(t)$ is symmetric w.r.t. the origin.



a function $g(t)$ is odd when $g(-t) = -g(t)$ or $g(t)$ is antisymmetric wrt the origin



$\rightarrow$ $Ev[f(t)]$ is even or symmetric w.r.t. origin

$Odd[f(t)]$ is odd or antisymmetric w.r.t. origin.

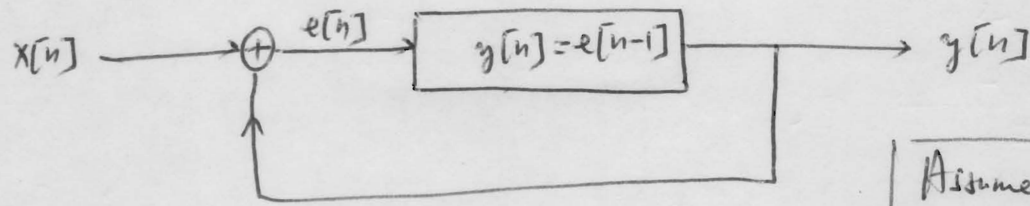
1.28 d): $y[n] = Ev[x[n-1]] = \frac{1}{2}(x[n-1] + x[1-n])$

- Memory ✓
- Time invariance : $n \to n+k$ $\quad y[n+k] = \frac{1}{2}(x[n+k-1] + x[1-n-k])$ ✓
- Causality ; $n=0$ $\quad y[0] = \frac{1}{2}[x[-1] + x[1]]$ Not causal $n \geq 1 \to$ Causal.
- Linearity : ✓
- Stability : ✓

e) $y[n] = \begin{cases} x[n] & n \geq 1 \\ 0 & n = 0 \\ x[n+1] & n \leq -1 \end{cases}$

- Memory : NO
- Time inv: ✓
- Causality : NO
- linearity ✓
- Stability : ✓

1.46



Assume
$y[n] = 0 \quad \forall n < 0$

a) Sketch $y[n]$ when $x[n] = \delta[n]$ b) when $x[n] = u[n]$

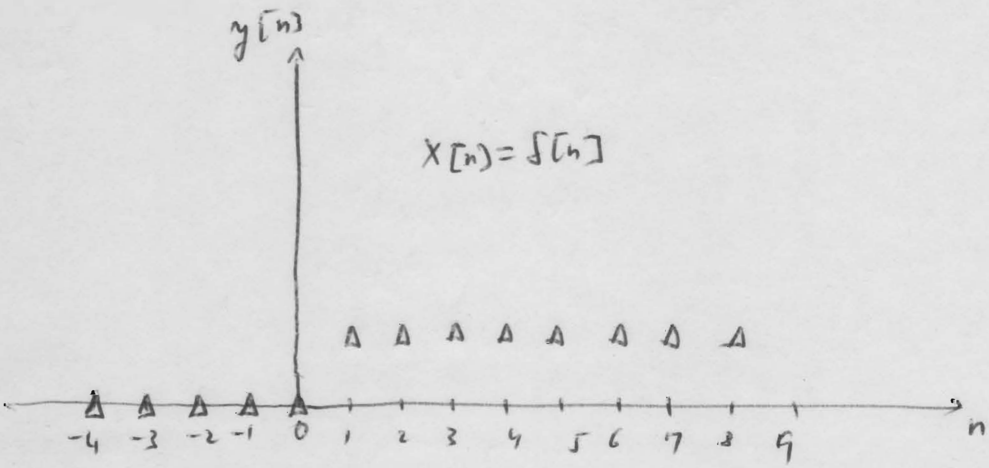
Find $y[n]$ ~~of~~ in term of $x[n]$ → or the input-output equation.

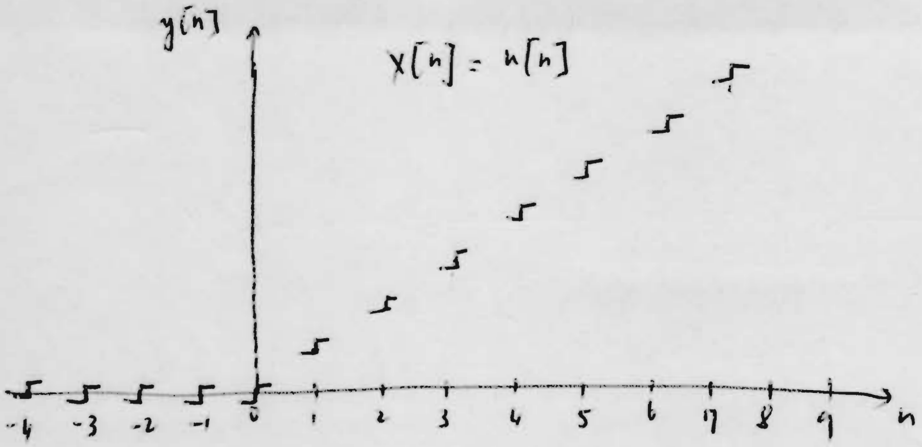
$e[n] = x[n] + y[n]$

$\boxed{y[n] = x[n-1] + y[n-1]}$

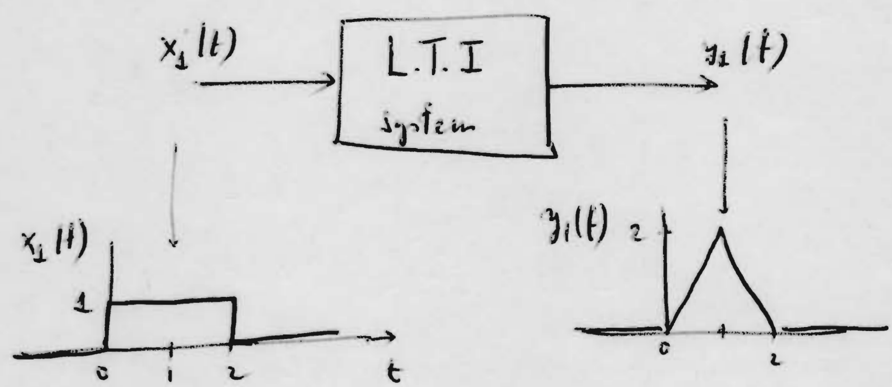
↳ $y[n] = 0 \quad \forall n < 0$

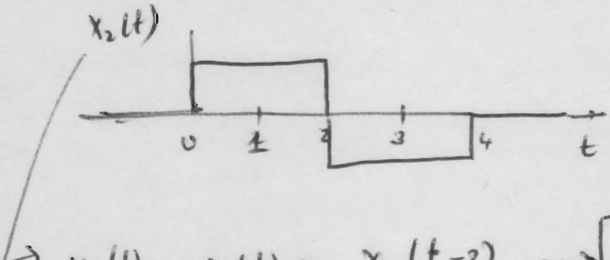
	delta $x[n] = \delta[n]$	step $x[n] = u[n]$
$y[0] = x[-1] + \underbrace{y[-1]}_{0} = x[-1]$	$= \delta[-1] = 0$	$= u[-1] = 0$
$y[1] = x[0] + y[0]$	$= 1 + 0 = 1$	$= 1 + 0 = 1$
$y[2] = x[1] + y[1]$	$= 0 + 1 = 1$	$= 1 + 1 = 2$
$y[3] = x[2] + y[2]$	$= 0 + 1 = 1$	$= 1 + 2 = 3$
$y[4] = x[3] + y[3]$	$= 0 + 1 = 1$	$= 1 + 3 = 4$



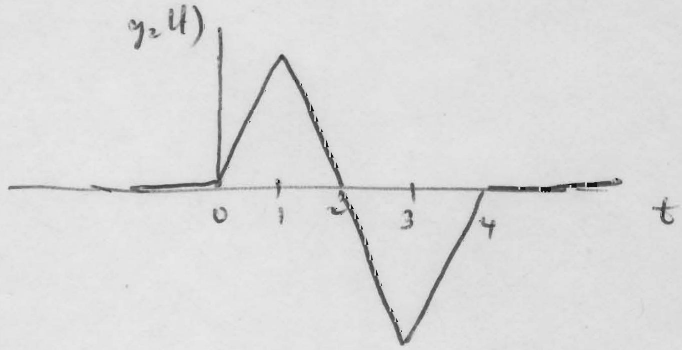


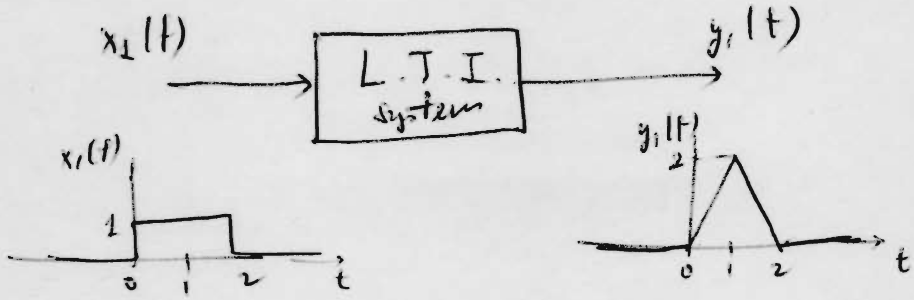
1.31

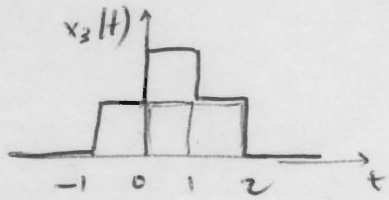




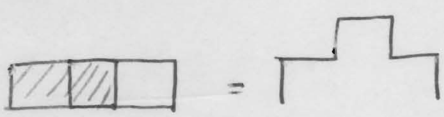
$$x_2(t) = x_1(t) - x_1(t-2) \longrightarrow \boxed{\text{LTI}} \longrightarrow y_2(t) = y_1(t) - y_1(t-2)$$

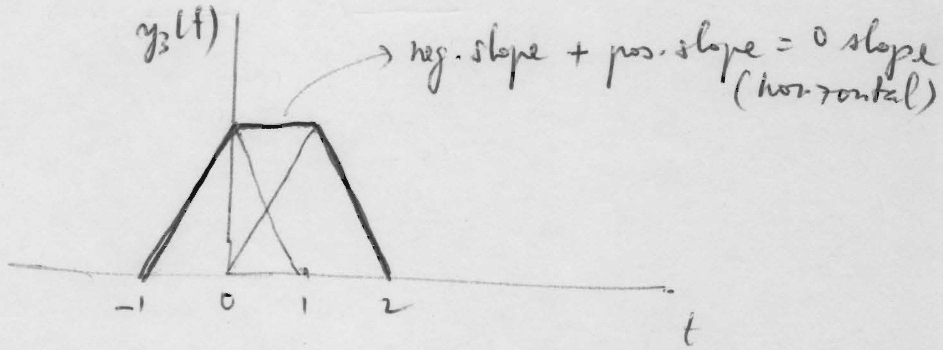






$x_3(t) = x_1(t+1) + x_1(t) \longrightarrow \boxed{L.T.I} \longrightarrow y_1(t+1) + y_1(t)$





Ch2: Linear Time-Invariant Systems

HW 2: 2.7; 2.8; 2.11; 2.17; 2.22; 2.24; 2.31; 2.40; 2.47; 2.61

Convolution: "$*$" is a mathematical operation that allows us to calculate $y[n]$ from $x[n]$ using impulse response $h[n]$

$$x[n] \longrightarrow \boxed{\begin{matrix}\text{System}\\ h[n]\end{matrix}} \longrightarrow y[n]$$

impulse response $h[n]$ is the output of a $\delta[n]$ input

$$\delta[n] \longrightarrow \boxed{\text{System}} \longrightarrow h[n]$$

→ $h[n]$ can be used to characterize the system.

$y[n] = x[n] * h[n]$ ("$y[n]$ is the convolution of $x[n]$ and $h[n]$")

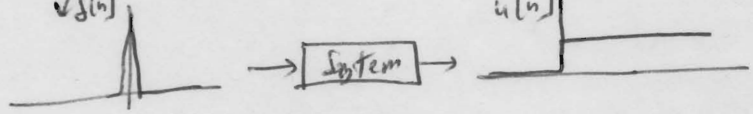
$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

↳ simple multiplication

Example: Find $y[n]$ from input $x[n] = \alpha^n u[n]$ → step function

and $\boxed{\text{impulse response } h[n] = u[n]}$

α a constant



Switch: when you turn the switch @ $t=0$; i.e. the $\delta[n]$ input, the light turns on and stays on; i.e; the step function.