% To visualize a second difference between continuous-time and discrete-time signals: for example, exp(j2t) is not the same as exp(j(2+2pi)t), while exp[j2n] is the same as exp[j(2+2*pi)t]

% Continuous-time signal with angular freq. omega1=2
omega1=2;
tc=0:.05:20;
figure(2), subplot(4,1,1), plot(tc,cos(omega1*tc))
title('continuous-time signal of period T=pi s; angular freq. =2')
xlabel('t (s)')
ylabel('f(t)')

% Continuous-time signal with angular freq. omega2=2+2*pi
omega2=2+2*pi;
subplot(4,1,2), plot(tc,cos(omega2*tc))
title('continuous-time signal of period T=2*pi/(2+2*pi) s; angular freq. =2+2*pi')
xlabel('t (s)')
ylabel('f(t)')

% Discrete-time signal with angular freq. omega1=2
td=0:1:20;
subplot(4,1,3), stem(td,cos(omega1*td))
title('discrete-time signal; angular freq. =2')
xlabel('time index n')
ylabel('f[n]')

% Discrete-time signal with angular freq. omega2=2+2*pi
subplot(4,1,4), stem(td,cos(omega2*td))
title('discrete-time signal; angular freq. =2+2*pi')
xlabel('time index n')
ylabel('f[n]')
continuous-time signal of period $T=\pi$ s; angular freq. $=2$

discrete-time signal; angular freq. $=2$

continuous-time signal of period $T=2\pi/(2+2\pi)$ s; angular freq. $=2+2\pi$

discrete-time signal; angular freq. $=2+2\pi$
(1.10) Determine fundamental period of \( x(t) = 2 \cos((10t+1)) - \sin(4t-1) \)

\[
\begin{align*}
\text{Periodic signal #1: } & \quad x_1(t+T_1) = x_1(t) \quad \Rightarrow \quad 10T_1 = 2\pi n_1 (n_1=2, 3, \text{etc.}) \\
\rightarrow & \quad T_1 = \frac{2\pi}{10} n_1 \\
\text{Periodic signal #2: } & \quad x_2(t+T_2) = x_2(t) \quad \Rightarrow \quad 4T_2 = 2\pi n_2 (n_2=1, 2, 3, \text{etc.}) \\
\rightarrow & \quad T_2 = \frac{2\pi}{4} n_2 \\
\end{align*}
\]

\( \rightarrow \) What is the period \( T \) of \( x(t) = 2x_1(t) - x_2(t) \)?

Both \( x_1 \) and \( x_2 \) repeat themselves \( @ T_1 = T_2 = \pi \rightarrow T \) will be the fundamental period of \( x(t) \rightarrow T = \pi \)

\( \star \) For the 1st time.

(1.11) Determine the fundamental period for \( x[n] = 1 + e^{j\frac{4\pi n}{7}} + e^{j\frac{2\pi n}{5}} \)

\[
\begin{align*}
\text{Periodic signal #1: } & \quad x[n+N_1] = x[n] \quad \Rightarrow \quad e^{j\frac{4\pi N_1}{7}} = 1 \rightarrow \frac{4\pi N_1}{7} = 2\pi n_1, \\
N_1 = \frac{7}{2} n_1 \rightarrow & \quad N_1 = \{ 7, 14, 21, 28, 35, 42, \ldots \} \\
\downarrow \quad \text{periodic} & \quad \uparrow \quad \text{periodic} \\
\text{Periodic signal #2: } & \quad x[n+N_2] = x[n] \quad \Rightarrow \quad e^{j\frac{2\pi N_2}{5}} = 1 \rightarrow \frac{2\pi N_2}{5} = 2\pi n_2, \\
N_2 = 5n_2 \rightarrow & \quad N_2 = \{ 5, 10, 15, 20, 25, 30, 35, 40, \ldots \} \\
\end{align*}
\]
The first time both \( x_1[n] \) & \( x_2[n] \) repeat themselves is when \( N_1 = N_2 = 35 \) → This is the fundamental period of

\[ x[n] = 1 + x_1[n] - x_2[n] \rightarrow N = 35 \]

\[ x(t) = e^{j\omega_0 t} \] periodic with fundamental freq. \( \omega_0 \)

fundamental period \( T_0 = \frac{2\pi}{\omega_0} \) look at the

discrete-time signal with \( t = nT \) (where \( T \) is the

time increment) \( \rightarrow x[n] = e^{j\omega_0 nT} \) is a discrete-time

signal

(a) Show that \( x[n] \) is periodic if & only if \( \frac{T}{T_0} \) is a

rational number (ratio of two integers):

A discrete-time sinusoid is periodic when the angular

frequency changes a factor of \( \pi \):

\[ \omega_0 T_e = k\pi \implies \frac{2\pi}{T_0} T = k\pi \] (k integer)

\[ \implies \frac{T}{T_0} = \frac{k}{2} \] (k integer) \( \Rightarrow \frac{T}{T_0} = \text{a rational number} \)

Suppose:

(b) \( x[n] \) is periodic \( \rightarrow \frac{T}{T_0} = \frac{p}{q} \) (p & q are integers)

→ what is the fundamental period \( N_0 \) for \( x[n] \) ?

If \( x[n] \) is periodic with fundamental period \( N_0 \):

\[ x[n + N_0] = x[n] \rightarrow e^{j\omega_0 (n+N_0)} \]

\[ \downarrow e^{j\omega_0 T_n} \]

\[ x[n] = e^{j\omega_0 T_n} \frac{x[n]}{e^{j\omega_0 T_n}} \]
\[ j\omegaTN_0 \]
\[ \Rightarrow \quad \omega = \frac{\pi}{T_0} \quad \Rightarrow \quad \omega TN_0 = \frac{\pi}{T_0} \cdot N_0 \quad \Rightarrow \quad \omega TN_0 = 2\pi m \quad (\text{m: integer}) \]

Since \( \omega = \frac{2\pi}{T_0} \),
\[ \Rightarrow \quad \frac{\pi}{T_0} N_0 \cdot \frac{T_0}{P} = \frac{P}{4} \]

\[ \Rightarrow \quad N_0 = m \cdot \frac{\frac{q}{p}}{P} \]

For example:

1. \( q = 7 \) and \( p = 3 \) \( \Rightarrow \) \( N_0 = m \cdot \frac{7}{3} \) since \( N_0 \) must be an integer \( \Rightarrow \) \( N_0 = 7 \) \( (m = 3) \)

2. \( q = 28 \) and \( p = 12 \) \( \Rightarrow \) \( N_0 = m \cdot \frac{28}{12} = m \cdot \frac{7}{3} \) \( \Rightarrow \) \( N_0 = 7 \)

\[ \Rightarrow \quad N_0 = q \quad \text{if} \quad \frac{q}{p} \quad \text{is irreducible} \quad \Rightarrow \quad N_0 = \frac{q}{\text{gcd}(q,p)} \]

\[ \text{gcd: "greatest common divisor" } \Rightarrow \text{check: } q = 28, \quad p = 12 \]
\[ \Rightarrow \quad \frac{q}{\text{gcd}(q,p)} = \frac{28}{\text{gcd}(28,12)} = \frac{28}{4} = 7 \]

Fundamental period is \( N_0 \) \( \Rightarrow \) fundamental freq is \( \omega_{0,D} = \frac{2\pi}{N_0} \)
\[ \omega_{0,D} = \frac{\frac{2\pi}{p}}{\text{gcd}(q,p)} = \frac{\frac{2\pi}{p}}{\text{gcd}(q,p)} \cdot \frac{q}{\text{gcd}(q,p)} \]

3. Assume \( \frac{T}{T_0} = \frac{p}{q} \) \( (p \& q \text{ are integers}) \) or \( x[n] \text{ is periodic} \)
\[ \Rightarrow \quad \text{how many periods } T_0 \text{ of } x(t) \text{ are needed to obtain samples that form the period } N_0 \text{ of } x[n] \]

One period of \( x[n] \) in index is \( N_0 \) \( \Rightarrow \) in time is \( \frac{N_0 T}{P} \)

This would contain \( \frac{N_0 T}{T_0} \) periods \( T_0 \) of \( x(t) \)

\[
\frac{N_0 T}{T_0} = \frac{4}{\gcd(q, p)} \frac{T}{T_0} = \frac{P}{q}
\]

This is how many periods of the original \( x(t) \) are contained in one period of \( x[n] \).
More on signals.

**Unit impulse:** δ(t)

A unit impulse is the limit of a Gaussian function when the width (variance; standard deviation) tends to zero. "Unit" when the maximum value is 1. The discrete-time version of it is δ[n], as shown in the figure below.

**Unit step function:** u(t) or u[n]
Relationship:
\[ \delta(t) = \frac{du}{dt} \]

Integrals involving impulse functions \( \delta(t) \):

\[ \int_{-\infty}^{\infty} f(t) \delta(t) = f(0) \]

Below is a impulse centered at 400

\[ \int_{-\infty}^{\infty} f(t) \delta(t - 400) = f(400) \quad \text{(impulse centered at 400)} \]

\[ \int_{-\infty}^{\infty} f(t) \delta(t + 400) = f(-400) \quad \text{(impulse centered at -400)} \]

\[ \int_{-3}^{3} f(t) \delta(t) = f(0) \]

\[ \int_{-3}^{3} f(t) \delta(t - 400) = 0 \]

From the relationship between impulse and the step functions:
\[ u_\Delta(t) = \int_{-\infty}^{t} d\tau \, \delta_\Delta(\tau), \quad u(t) = \lim_{\Delta \to 0} u_\Delta(t) \]

\( u_\Delta \) is a step function with a finite slope during \( \Delta \) at the transition from 0 to 1; when \( \Delta \to 0 \) the transition has an infinite slope, we get the actual step function \( u(t) \). Similarly \( \delta_\Delta \) is an impulse with a finite width \( \Delta \) starting from 0, when \( \Delta \to 0 \) we get back the actual impulse \( \delta(t) \).

The problem asks for this proof:
\[
\lim_{\Delta \to 0} [u_\Delta(t) \delta(t)] = 0
\]
This is obvious if we replace the definition of \( u_\Delta \):
\[
\lim_{\Delta \to 0} [u_\Delta(t) \delta(t)] = \delta(t) \lim_{\Delta \to 0} \int_{-\infty}^{t} d\tau \, \delta_\Delta(\tau), \quad \text{since when } t=0 \text{ the lim is 0; same when } t<0; \text{ when } t>0 \text{ the limit is nonzero however } \delta(t)=0, \text{ so the result is still 0.}
\]

The problem also asks for this proof:
\[
\lim_{\Delta \to 0} [u_\Delta(t) \delta_\Delta(t)] = \frac{1}{2} \delta(t)
\]