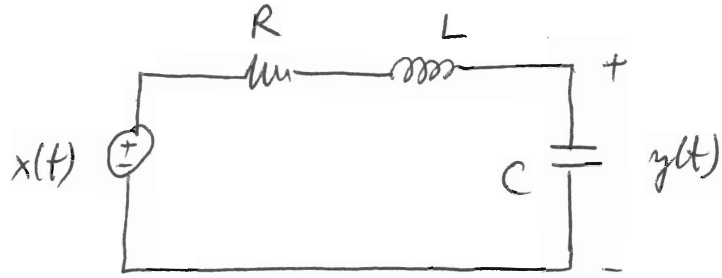
6.19



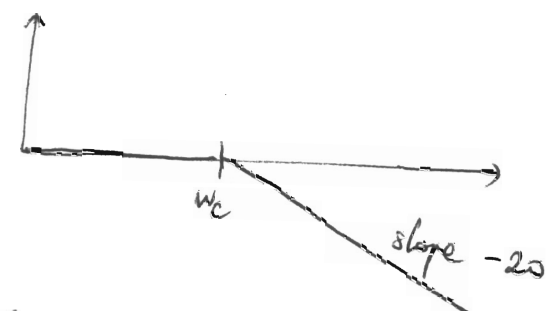
$u(t) \longrightarrow \boxed{RLC} \longrightarrow$ can this oscillate?

$$\hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{\text{impedance at } C}{\text{total impedance}} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + (j\omega)^2 LC + 1}$$

↓ Voltage division using Phasors

$j\omega RC + (j\omega)^2 LC + 1$ → 2nd order pole.

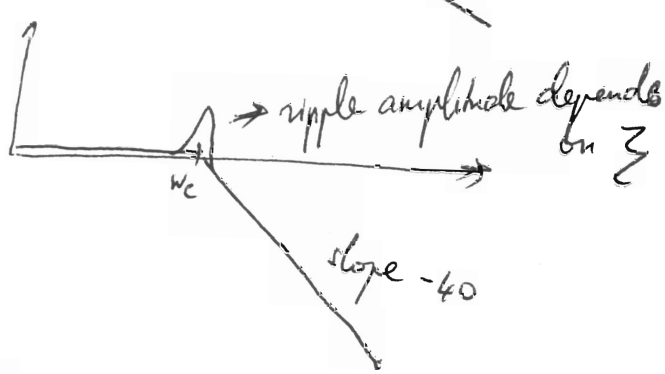
Bode Plot: 1st order pole:



2nd order pole:

$$\frac{1}{1 + 2(j\omega\tau\xi) + (j\omega\tau)^2}$$

τ → tau, ξ → xi



$$\tau = \sqrt{LC} \rightarrow 2\tau\xi = RC$$

$$\boxed{\xi = \frac{RC}{2\tau} = \frac{RC}{2\sqrt{LC}} = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$$y(t) = F^{-1}\left[\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)\underbrace{\frac{1}{j\omega RC + (j\omega\sqrt{LC})^2 + 1}}_{\text{has a ripple depending on } \xi}\right] \rightarrow \boxed{\text{oscillations}}$$

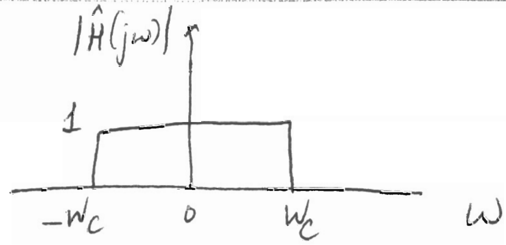
Can we adjust R, L, C such that the oscillation is eliminated?

Yes, since there is $\underline{\text{no ripple}}$ when $\xi \geq 1$ or $\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$

↳ no oscillation

or $\boxed{R \geq 2\sqrt{\frac{L}{C}}}$

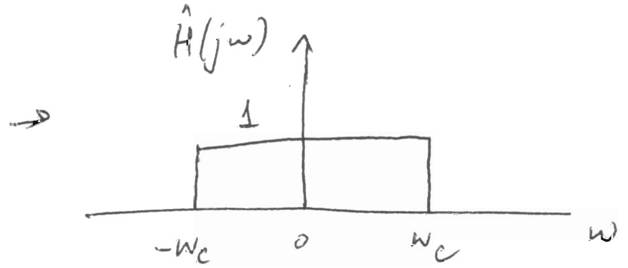
6.23 Low-pass filter:



Sketch impulse response for:

a) $\text{Phase}\left(\hat{H}(j\omega)\right) = 0$ or $\hat{H}(j\omega) = |\hat{H}(j\omega)| \cdot \underbrace{e^{j0}}_{1}$

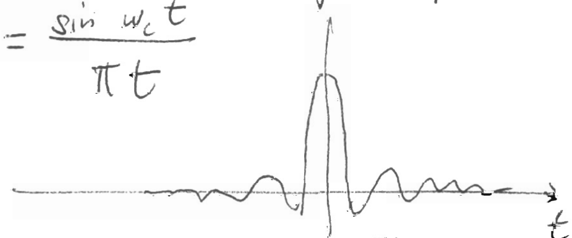
→



→ $h(t) = F^{-1}[\hat{H}]$

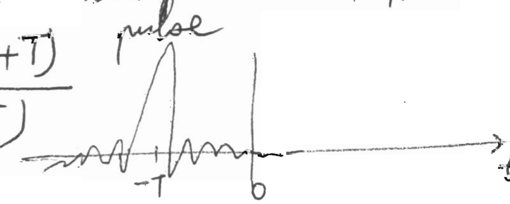
what is the inverse Fourier Transform of a rectangular pulse?

Table 4.2: Row #9: $h(t) = \frac{\sin w_c t}{\pi t}$

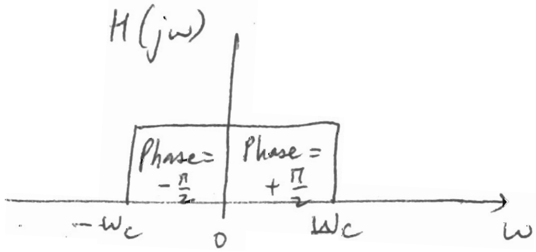


b) $\text{Phase}\left(\hat{H}(j\omega)\right) = \omega T \rightarrow \hat{H}(j\omega) = \underbrace{|\hat{H}(j\omega)|}_{\text{rectangular pulse}} \underbrace{e^{j\omega T}}$ ↳ time-shift

$h(t) = F^{-1}[\hat{H}] = \frac{\sin w_c (t+T)}{\pi (t+T)}$



(·) $\text{Phase}(H(j\omega)) = \begin{cases} \frac{\ }{2} & \\ -\frac{\pi}{2} & \omega < 0 \end{cases}$



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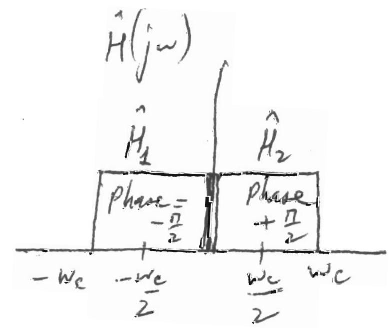
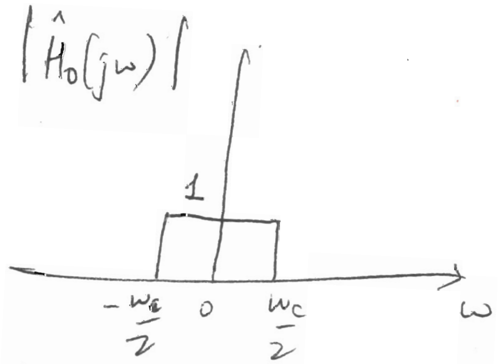


$$\hat{H}(j\omega) = \hat{H}_1(j\omega) + \hat{H}_2(j\omega) = -j\left|\hat{H}_0\left(j\left(\omega + \frac{\omega_c}{2}\right)\right)\right| + j\left|\hat{H}_0\left(j\left(\omega - \frac{\omega_c}{2}\right)\right)\right|$$

phase $= -\frac{\pi}{2}$ phase $= +\frac{\pi}{2}$



Shift in frequency: convolution with a delta function

$$\hat{H}(j\omega) = \underbrace{\left|\hat{H}_0(j\omega)\right|}_{\downarrow \text{ Table 4.2 row \#9}} * \underbrace{\left[-j\,\delta\left(\omega + \frac{\omega_c}{2}\right) + j\,\delta\left(\omega - \frac{\omega_c}{2}\right)\right]}_{\downarrow \text{ Table 4.2 row \#4}}$$

$$h(t) = \frac{\sin \frac{\omega_c}{2} t}{\pi t} \qquad \left(-\frac{1}{\pi} \sin \frac{\omega_c}{2} t\right)$$

$$h(t) = \left(-\frac{1}{\pi}\right) \frac{\sin^2 \frac{\omega_c}{2} t}{\pi t}$$

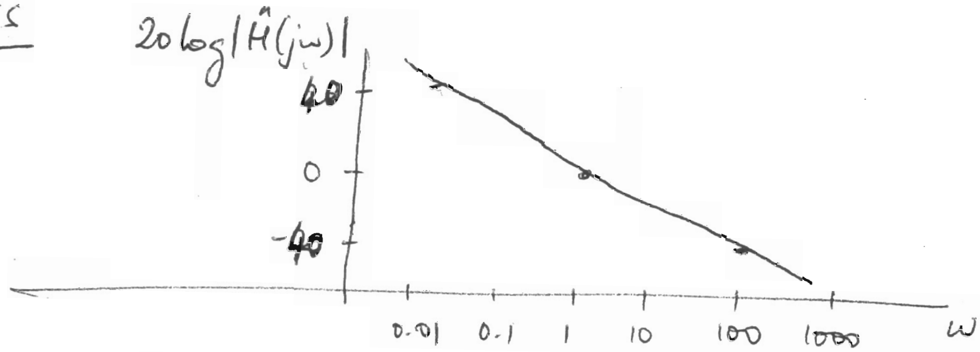
(6.31) Integrator: $\hat{H}(j\omega) = \frac{1}{j\omega} + \underbrace{\pi\,\delta(\omega)}$

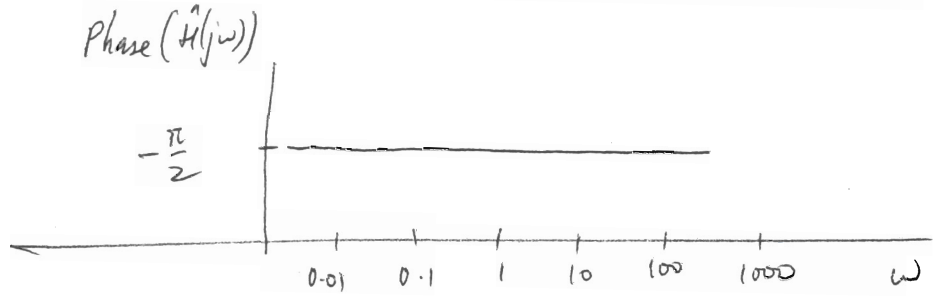
b/c integration of a constant from $t=-\infty$

$\hookrightarrow$ at $\omega = 0$

If only examine $\omega > 0$ $\rightarrow$ $\begin{cases} 20\log|\hat{H}(j\omega)| = 20\log\left|\frac{1}{j\omega}\right| = -20\log\omega \\ \text{Phase}(\hat{H}(j\omega)) = \text{Phase}(-j) = -\frac{\pi}{2} \end{cases}$

Bode Plots

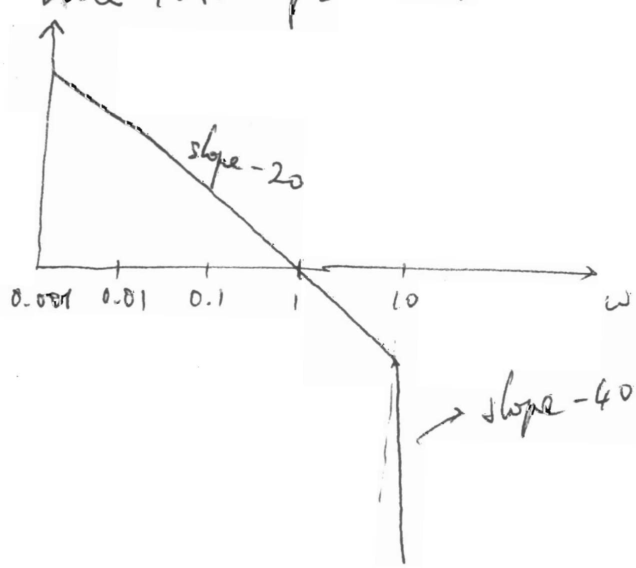


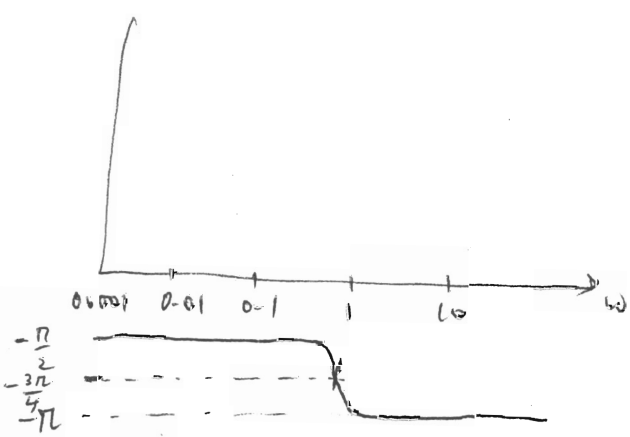


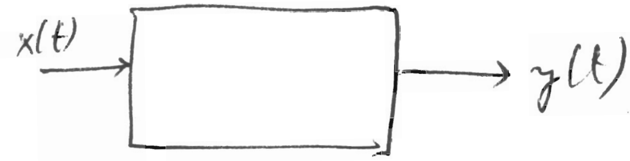
(a)

$\hat{H}(j\omega) = \frac{1}{j\omega\left(1 + j\frac{\omega}{10}\right)} + \pi\,\delta(\omega)$ (Electric motor)

Bode Plots for $\omega > 0.001$ $\rightarrow 0$







$$F\left\{ \frac{dy}{dt} + 2y = x(t) \right\} \rightarrow j\omega\hat{Y} + 2\hat{Y} = \hat{X}$$

a) $$\hat{H}(j\omega) = \frac{\hat{Y}(j\omega)}{\hat{X}(j\omega)} = \frac{1}{2+j\omega}$$

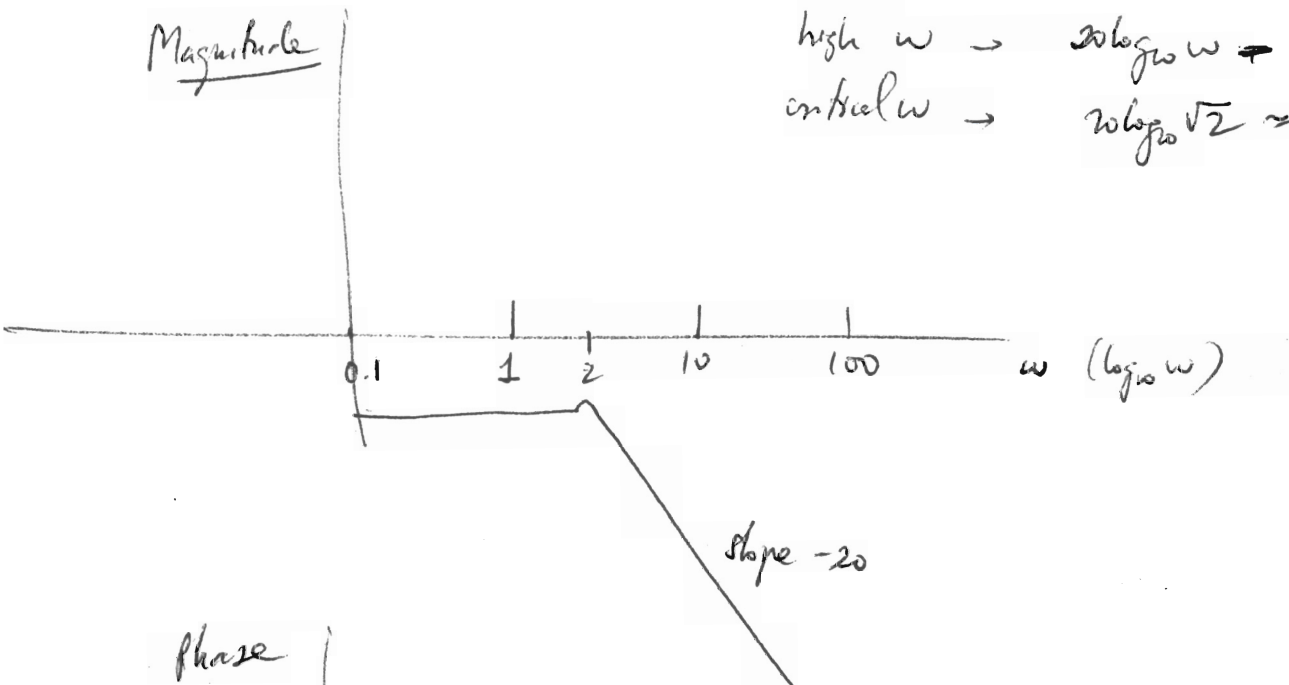
Bode plot: $$\boxed{\hat{H} = \frac{1/2}{1+j\frac{\omega}{2}}}$$

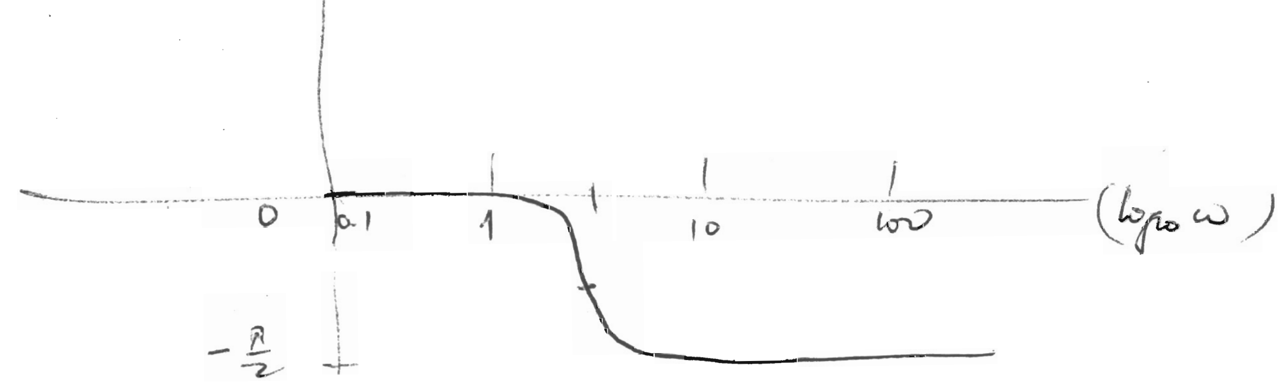
$$20\log_{10}|\hat{H}| = \underbrace{20\log_{10} 0.5} - \underbrace{20\log_{10}\left|1+j\frac{\omega}{2}\right|}$$

small $\omega \rightarrow 0$

high $\omega \rightarrow 20\log_{10}\omega - 20\log_{10} 0.5$

critical $\omega \rightarrow 20\log_{10}\sqrt{2} \simeq 0.3\,dB$





c) if $x(t) = e^{-t}u(t) \rightarrow \hat{Y}(j\omega)$?

$\downarrow$

$$\hat{X}(j\omega) = \frac{1}{1+j\omega} \rightarrow \hat{Y}(j\omega) = \hat{X}(j\omega)\,\hat{H}(j\omega)$$

$$= \frac{1}{1+j\omega} \cdot \frac{1/2}{1+j\frac{\omega}{2}}$$

d) $$y(t) = F^{-1}\left[\frac{a}{1+j\omega} + \frac{b}{1+j\frac{\omega}{2}}\right]$$

$$a + j\frac{\omega}{2}a + b + j\omega b = \frac{1}{2}$$

$$\begin{cases} a+b = \frac{1}{2} \\ \frac{a}{2} + b = 0 \end{cases}$$

$$\frac{a}{2} = \frac{1}{2} \rightarrow \boxed{a=1}$$

$$\boxed{b = -\frac{1}{2}}$$

$$y(t) = F^{-1}\left[\frac{1}{1+j\omega}\right] + \underbrace{F^{-1}\left[\frac{-\frac{1}{2}}{1+j\frac{\omega}{2}}\right]}_{F^{-1}\left[-\frac{1}{2+j\omega}\right]}$$

$$= e^{-t}u(t) - e^{-2t}u(t)$$

Ch 7 Sampling

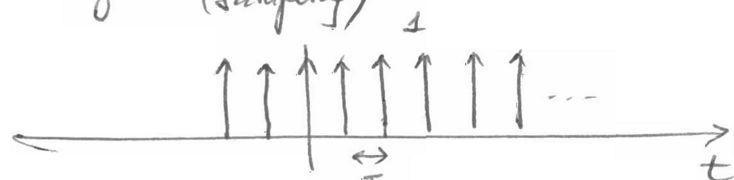
HW 7: 7.7; 7.9; 7.15; 7.24; 7.30; 7.31; 7.40; 7.45; 7.49; 7.50

Why sampling? → for signal processing using a computer
↳ @ at adequate rate (or frequency) to retain important information in signal but not to overload the computer

Sampled signal:

$$x_p(t) = \underbrace{x(t)}_{\text{original signal}} \cdot \underbrace{p(t)}_{\text{pulse (sampling)}}$$

↳ obtained by $\boxed{\text{multiplying}}$ the original signal by sampling pulse



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

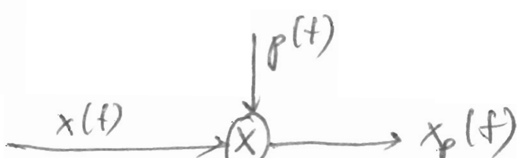


Diagram of a sampling

Example: what is the spectrum of a sampled signal of $x(t)$?

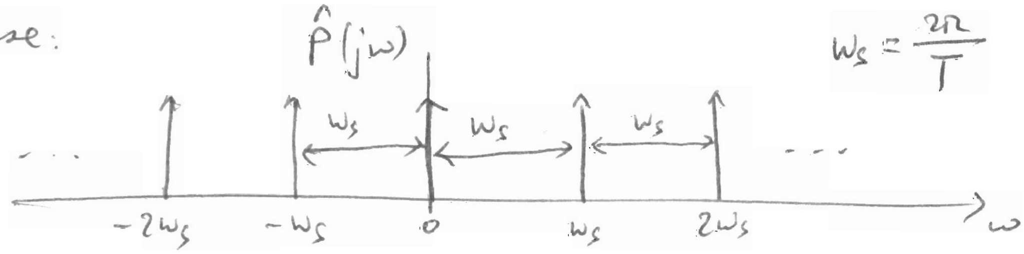
$$F\{\, x_p(t) = x(t) \cdot p(t) \,\}$$

$$\hat{X}_p = \frac{1}{2\pi} \hat{X} * \hat{P}$$

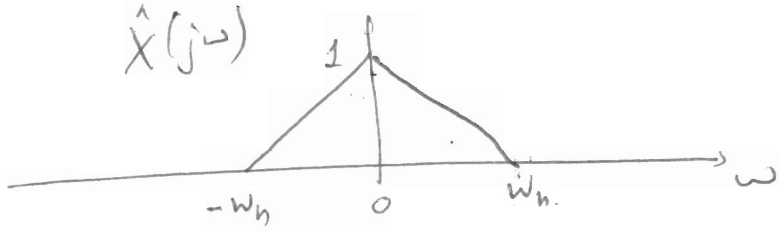
convolution

In our case:

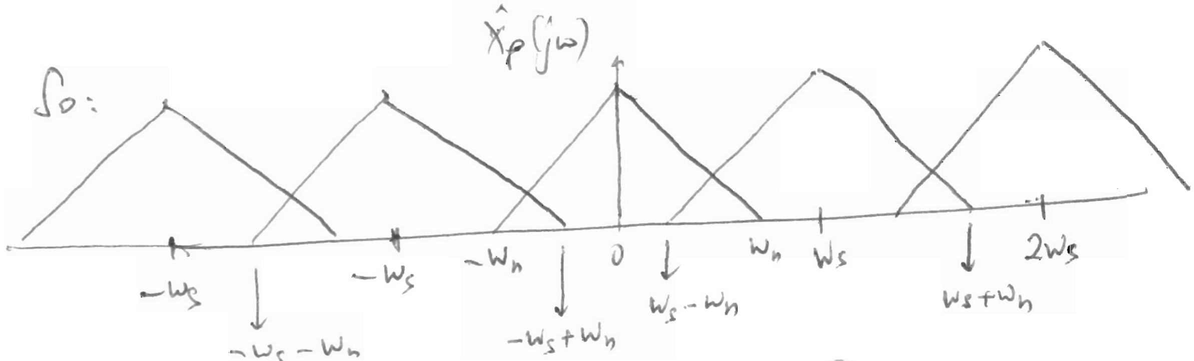
$$\omega_s = \frac{2\pi}{T}$$



<u>Assume</u> $x(t)$ is such that $\hat{X}$ is a triangular pulse:

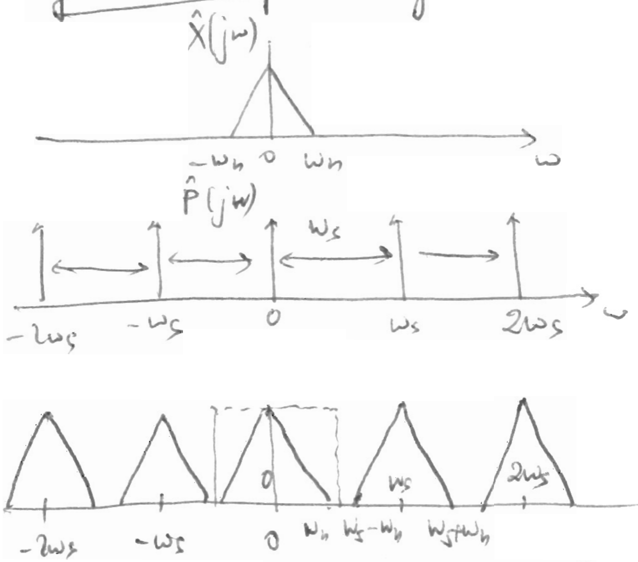


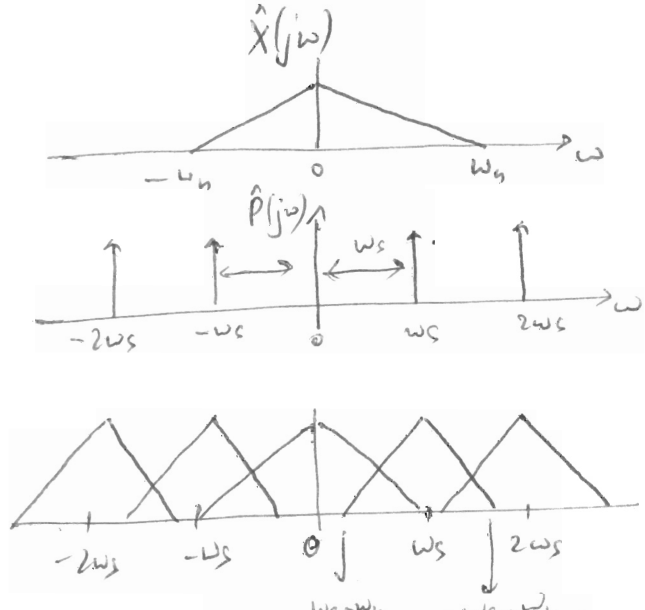
So:



The spectrum of ~~the~~ our sampled signal consists of a series of triangular pulses centered @ multiples of ω_s with a half base of ω_n

<u>Essential part of signal processing:</u> signal reconstruction; <u>reconstruct the original signal from the sampled signal.</u> For a complete reconstruction: we need to be able <u>to isolate the spectrum of the original signal from among the spectrum of the sampled signal</u>:





<u>can reconstruct original signal</u> cannot reconstruct original

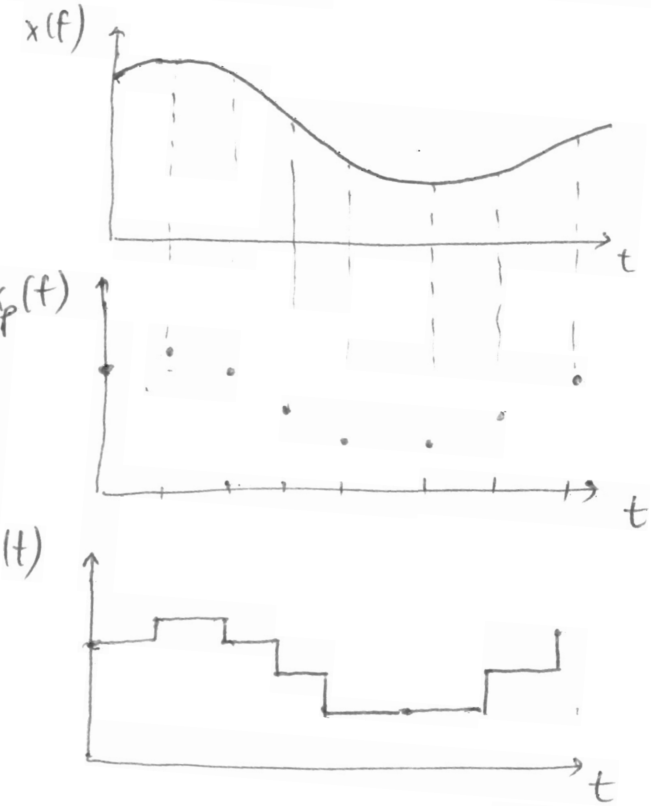
$\omega_s - \omega_n > \omega_n \Rightarrow$ $\boxed{\omega_s > 2\omega_n}$ $\boxed{\text{Nyquist Theorem}}$ signal completely from sampled signal

<u>Nyquist Theorem</u>: for complete reconstruction of original signal from a sampled signal (no lost of information): the sampling frequency w_s has to be equal or greater than 2 times the max. frequency w_n in the signal.

$$\boxed{w_s \geq 2w_n}$$

<u>Interpolation methods</u>:

a) "<u>Zero-order hold</u>" : hold value until next sample



original signal

sampled signal

Zero-order interpolation of the sampled signal

→ Same as doing a convolution of x_p with a rectangular pulse