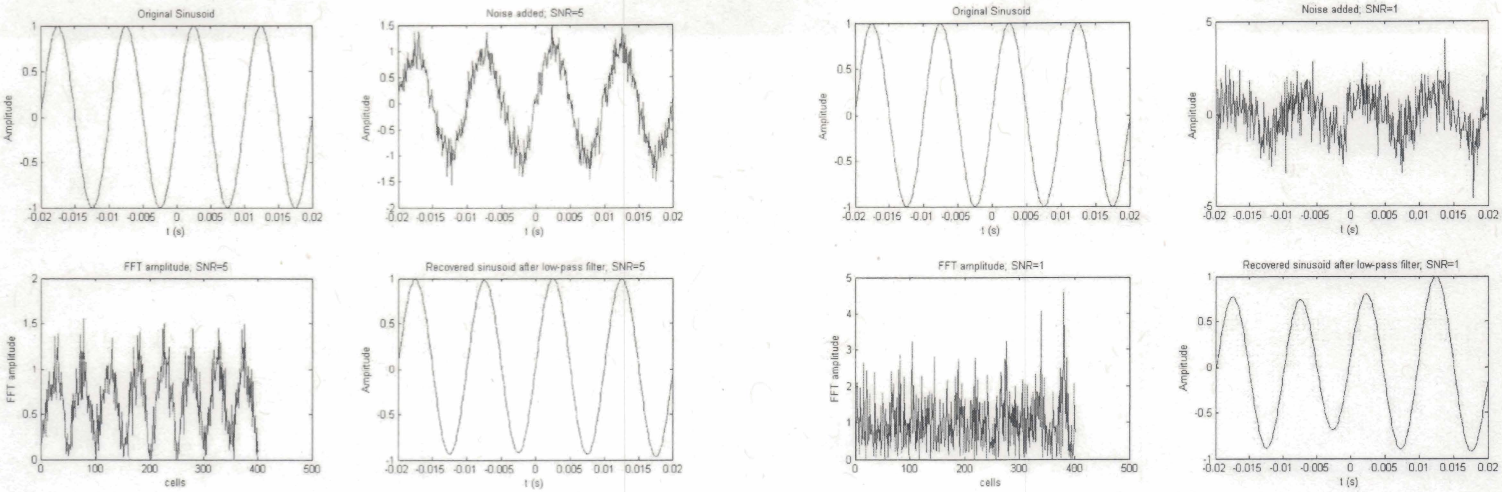

The Fourier Transform plays an essential role in the signal processing application of noise elimination

Fall '08
Engin 321
Linear System Theory I
Class #10027, 3 credits
Tu. & Th. 12:30-1:45, S-3-126
Prof. Tomas Materdey

The concepts of signals and systems arise in all areas of technology. This course provides an introduction to the analysis of linear systems in the time- and frequency-domain, e.g., what is the output of a system if we know the input and the transfer function of a system. Students will use the convolution theorem, and the continuous-time and discrete-time Fourier and Laplace transforms in different applications. They will also learn to write simple Matlab™ codes as related to signal processing as illustrated in the figures above and below. The prerequisites are Math 140 (Calculus I) and Engin 232 (Circuit Analysis II) or by permission of instructor.

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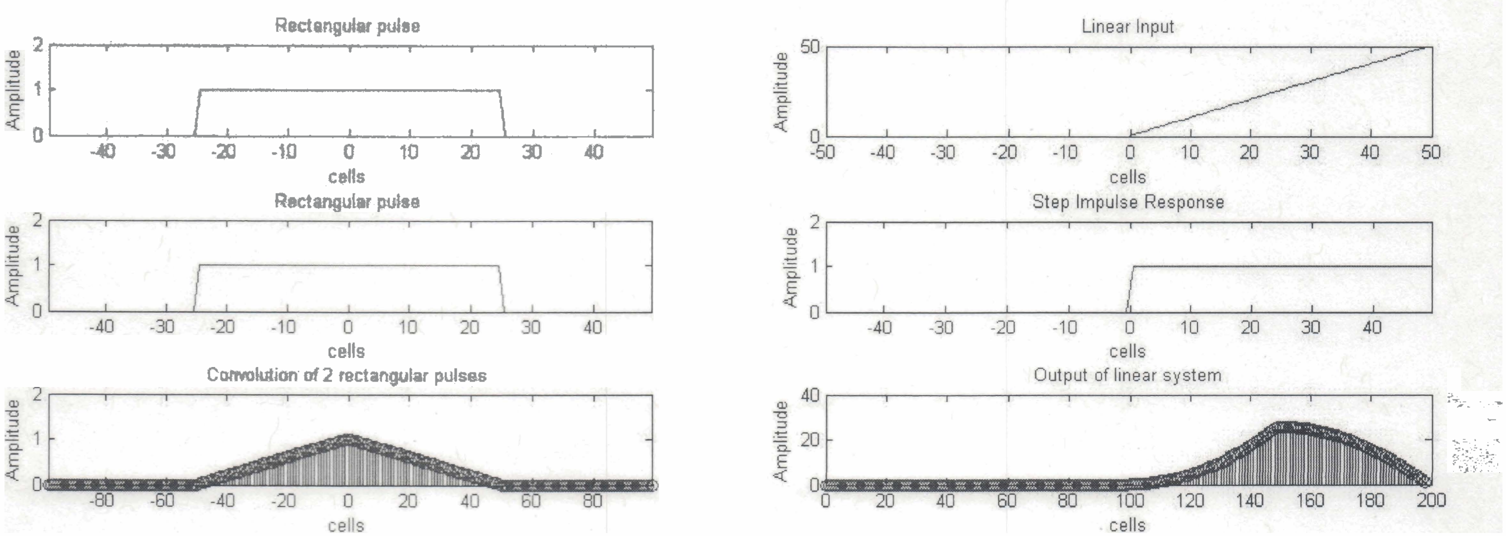
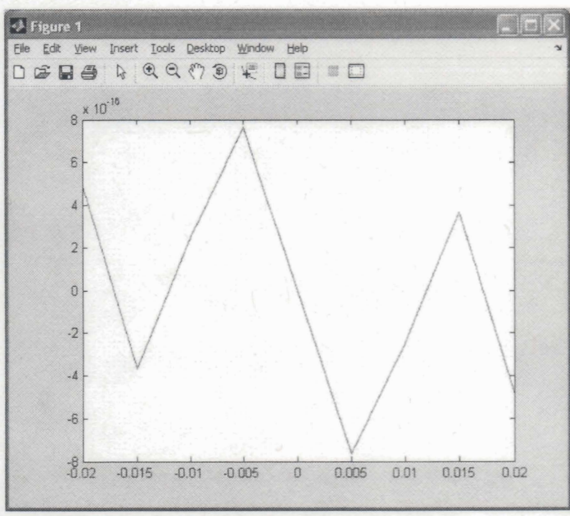


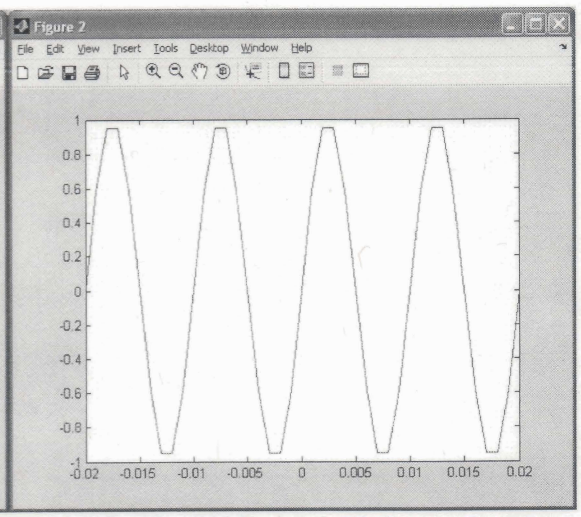
Applications of the convolution to obtaining output signals of linear systems of known impulse response. Graphs generated using Matlab

Introduction to signals and noise elimination using Matlab

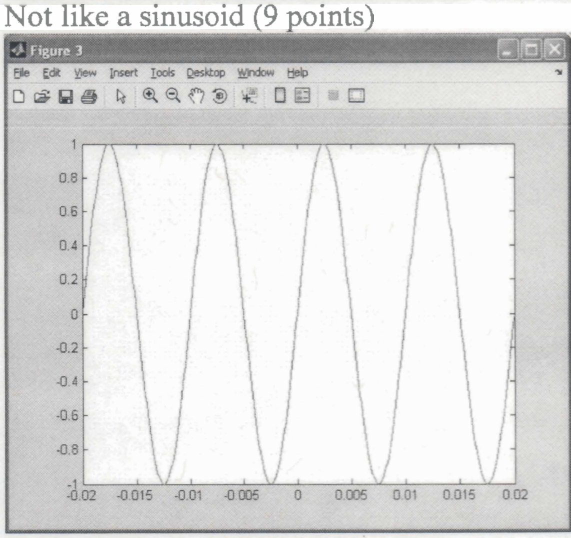
```
%This code will generate a signal, add noise, show the Fourier transform,
%then reconstruct the signal.

% Sinusoid generation
t=-0.02:0.005:0.02; %time series t (sec): we use 9  points
freq=100; %period is 0.01s
f=sin(2*pi*freq*t);
figure(1),plot(t,f)
t1=-0.02:0.001:0.02;%time series t1 (sec): we use 41 points
f1=sin(2*pi*freq*t1);
figure(2), plot(t1,f1)
t2=-0.02:0.0001:0.02;%time series t2 (sec): we use 401 points
f2=sin(2*pi*freq*t2);
figure(3), plot(t2,f2)
```



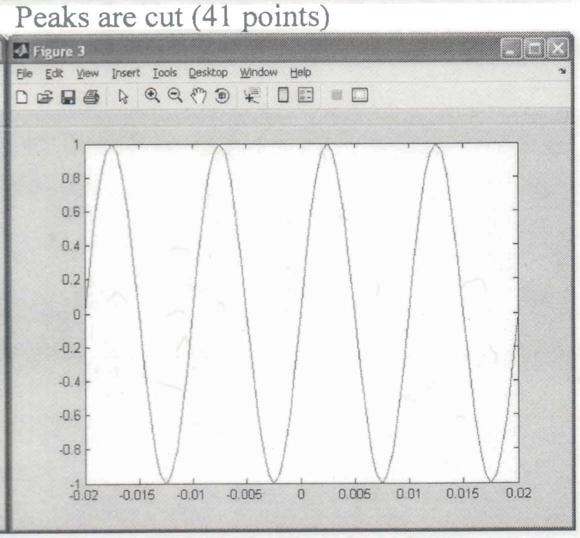
Not like a sinusoid (9 points)



Peaks are cut (41 points)



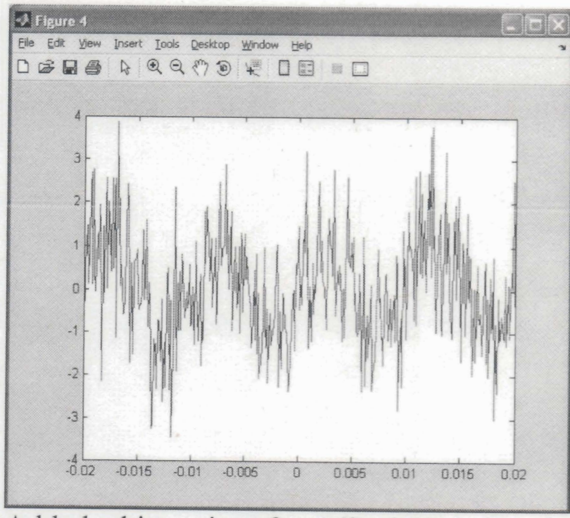
Peaks are included (401 points)



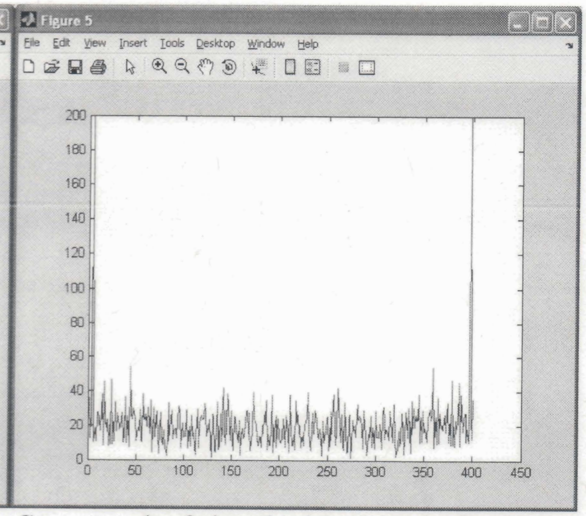
Peaks are included (81 points)*

*Although 81 points shows a good sinusoid; 401 points is needed for the noise elimination shown below.

```
%add noise to the sinusoid in figure 3
f2n=f2+1*randn(1,length(t2)); %randn gives a gaussian noise (white noise)
figure(4), plot(t2,f2n)

%show frequency spectrum for the signal in figure 4 (with noise) using fft
ff2n=fft(f2n);
figure(5), plot(abs(ff2n))
```



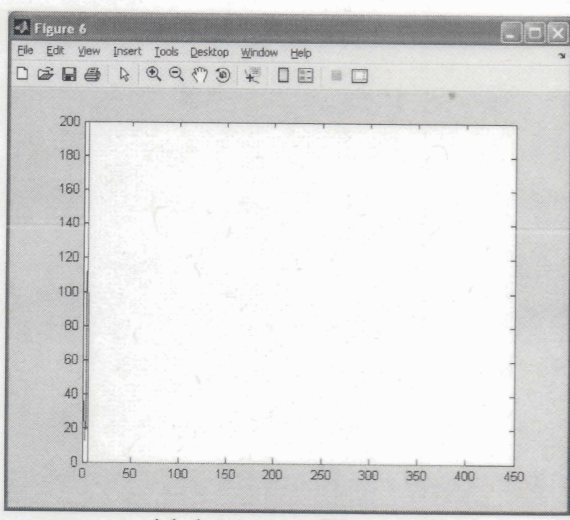
Added white noise of amplitude 1



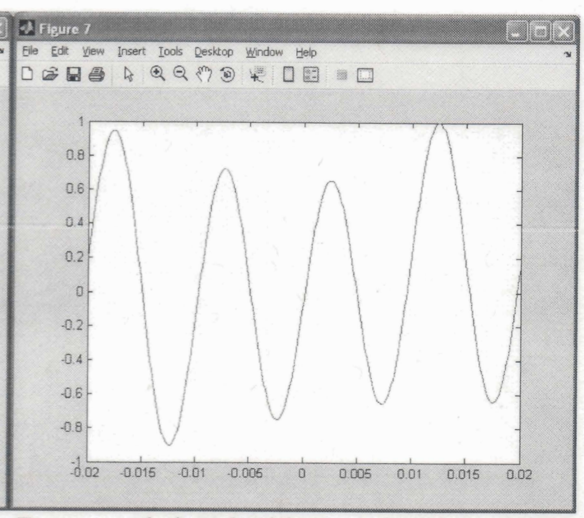
Spectrum* of signal with noise in figure 4

*Spectrum: graph with different frequencies with their amplitudes
Noise oscillatates a lot, we cut out the high-frequency portion of the spectrum before doing inverse Fourier Transform to recover the signal without noise (we apply a low-pass filter)

```
%do lowpass filter
band=floor(length(t2)/100)+1;%10;
filt_one=ones(1,band);
filt_zero=zeros(1,length(t2)-band);
filt=[filt_one filt_zero];
ff2n=ff2n.*filt;
figure(6), plot(abs(ff2n))

%show inverse Fourier Transform
iff2n=ifft(ff2n);
figure(7) ,plot(t2,real(iff2n)/max(real(iff2n)))
```



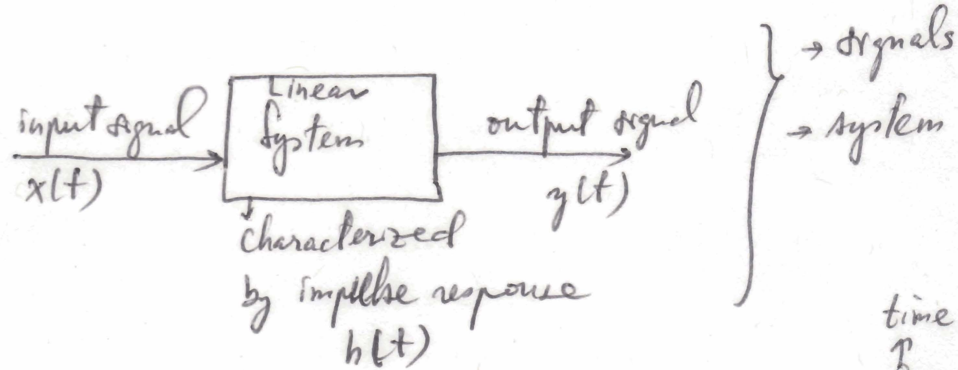
spectrum with low-pass filter applied



Recovered signal without noise (Inverse FFT)

Linear System Theory I (Fa'08)

Introduction to Signals & Systems:



Signal: a function of time

- continuous-time: $x(t) = \sin(\underbrace{2\pi f}_{\omega} t)$ (t: time variable), $x(t) = \exp(-at)$, etc.
- discrete-time: $x[n] = \sin[2\pi f n]$ (n: time index), $x[n] = \exp[-an]$, etc.

$t = n\Delta t$

- t → time variable
- n → integer, time index
- Δt → time increment

Properties:

1) linear transformation of time variable:

$$t \longrightarrow t' = at + b$$

(a: scaling factor, b: time shift)

2) Periodicity:

A signal is periodic if

$$\begin{cases} x(t+T) = x(t) & (T: \text{period of signal}) \\ x[n+N] = x[n] & (N: \text{period of discrete-time signal}) \end{cases}$$

E.g.: sinusoids $\sin(2\pi f(t+T)) = \sin(2\pi f t)$

$T = \frac{1}{f}$ is the period.

Is $x(t) = e^{j\omega t}$ periodic? (angular frequency: ω) → In general: $e^{j\theta} = \cos\theta + j\sin\theta$

→ $x(t) = \cos\omega t + j\sin\omega t$ → It is periodic.

What is the period? if T is the period:

$$e^{j\omega(t+T)} = e^{j\omega t}$$

$$e^{j\omega t} \cdot e^{j\omega T}$$

$$e^{j\omega T} = 1 \rightarrow \begin{cases} \sin\omega T = 0 \\ \cos\omega T = 1 \end{cases}$$

$\omega T = 2m\pi$, $(m = 0, 1, 2, \ldots)$

→ $T_m = \dfrac{m\,2\pi}{\omega}$; $(m = 1, 2, 3, 4$ etc...$)$

Fundamental period : $m = 1$ → $\boxed{T_1 = \dfrac{2\pi}{\omega}}$

→ What is the period of $x(t) = e^{j\pi t}$?

$$T = \frac{2\pi}{\pi} = 2\ s$$

→ What is the period of $x(t) = e^{j5\pi t}$?

$$T = \frac{2}{5}\ s$$

→ What is period of $x(t) = e^{j5\pi n}$? Here for a discrete-time signal if the period is N (an integer)

$$e^{j5\pi(n+N)} = e^{j5\pi n}$$

$$e^{j5\pi n} \cdot e^{j5\pi N} = e^{j5\pi n} \rightarrow e^{j5\pi N} = 1 \begin{cases} \cos 5\pi N = 1 \\ \sin 5\pi N = 0 \end{cases}$$

→ $5\pi N = m\,2\pi$ $(m = 1, 2, 3,$ etc.$)$

→ $N = \frac{m 2\pi}{5\pi}$ (m = 1, 2, 3, etc.)

↓ First integer! (N has to be an integer)

$N = \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, (2)$

m=1   m=2   m=3   m=4   m=5

→ Summary → $\begin{cases} e^{j5\pi t} \rightarrow T = \frac{2}{5} s \\ e^{j5\pi n} \rightarrow N = 2 \end{cases}$ One difference b/w continuous-time & discrete-time signals.

→ What is the period N of $x[n] = e^{j5n}$?

→ $5N = m2\pi$ (m = 1, 2, 3, ...)

$N = m\frac{2\pi}{5}$

not an integer → N will never exist → this is not an integer

↓

Signal is not periodic !!

→ Conclusion: not all discrete-time sinusoid is periodic!

→ only those whose angular frequency carries π

e.g. $\begin{cases} e^{j5\pi n} \rightarrow \text{periodic } N = 2 \\ e^{j5n} \rightarrow \text{not periodic.} \end{cases}$

→ Another observation:

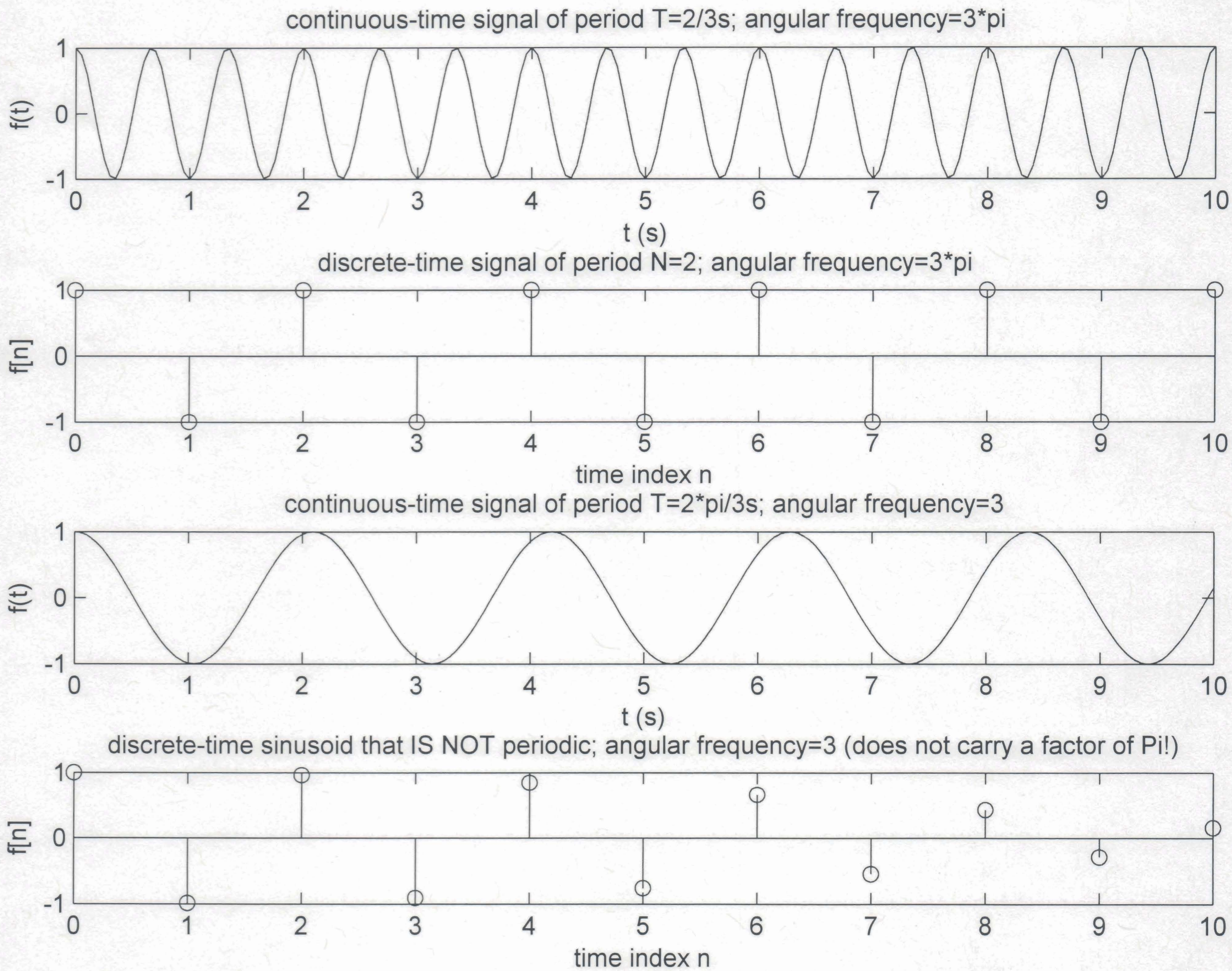
$$\begin{cases} e^{j2t} \neq e^{j(2+2\pi)t} \\ \quad \downarrow \qquad\qquad \downarrow \\ w=2 \qquad w=2+2\pi \\ e^{j2n} = e^{j(2+2\pi)n} = e^{j2n} \cdot \underbrace{e^{j2\pi n}}_{1\ \forall n} \end{cases}$$

↳ if you add 2π to the angular frequency, you get the same signal if it is discrete-time but a different signal if it is continuous time.

Visualize these 2 differences b/w continuous & discrete-time signals using Matlab.

HW #1: 1.10; 1.11; 1.36; 1.39 → signals
1.15; 1.16; 1.27; 1.28; 1.31; 1.46 → Systems
} due 9/16

```
%Engin 321
%TM
%Sept 4, 2008
%To visualize a difference between continuous-time and discrete-time
%signals: a continuous-time sinusoid is always periodic when a
%discrete-time sinusoid is only periodic if the angular frequency carries a
%factor of Pi

%Continuous and discrete-time periodic sinusoids
tc=0:0.05:10;%continuous-time vector
td=0:1:10;%discrete-time vector
omega=3*pi;
figure(1), subplot(4,1,1), plot(tc,cos(omega*tc))
title('continuous-time signal of period T=2/3s; angular frequency=3*pi')
xlabel('t (s)');
ylabel('f(t)');
subplot(4,1,2), stem(td,cos(omega*td))
title('discrete-time signal of period N=2; angular frequency=3*pi')
xlabel('time index n');
ylabel('f[n]');


%Continuous periodic and discrete-time non-periodic sinusoids
omega=3; %here the angular frequency does not carry a factor of Pi
subplot(4,1,3), plot(tc, cos(omega*tc))
title('continuous-time signal of period T=2*pi/3s; angular frequency=3')
xlabel('t (s)');
ylabel('f(t)');
subplot(4,1,4), stem(td,cos(omega*td))
title('discrete-time sinusoid that IS NOT periodic; angular frequency=3 (does not carry a↙
factor of Pi!)')
xlabel('time index n');
ylabel('f[n]');
```


continuous-time signal of period T=2/3s; angular frequency=3*pi
f(t)
t (s)
discrete-time signal of period N=2; angular frequency=3*pi
f[n]
time index n
continuous-time signal of period T=2*pi/3s; angular frequency=3
f(t)
t (s)
discrete-time sinusoid that IS NOT periodic; angular frequency=3 (does not carry a factor of Pi!)
f[n]
time index n