Ch5: Discrete-Time Fourier Transform:

Continuous-time F.T.

Direct: $\hat{X}(j\omega) = \int_{-\infty}^{\infty} dt\, x(t)\, e^{-j\omega t}$

Inverse: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, \hat{X}(j\omega)\, e^{j\omega t}$

The continuous-time Fourier Transform is NOT periodic in frequency:

$$\hat{X}(j\omega) \neq \hat{X}(j(\omega + 2\pi))$$

b/c: $e^{j(\omega+2\pi)t} \neq e^{j\omega t}$

Discrete-time F.T.

$$\hat{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} d\omega\, \hat{X}(e^{j\omega})\, e^{j\omega n}$$

a) $e^{j(\omega+2\pi)n} = e^{j\omega n} \cdot \underbrace{e^{j2\pi n}}_{\text{For any integer } n \text{ is always } 1}$

$\boxed{\text{Only integrate over a range of } 2\pi \text{ for inverse discrete-time Fourier Transform}}$

Also: $\hat{X}(e^{j(\omega+2\pi)}) = \hat{X}(e^{j\omega})$

The discrete-time Fourier Transform is always periodic in frequency

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Properties: Table 5.1 (linearity, time-shift, convolution, etc)
Table 5-2 (pairs of DTFT)

5.8 Use Tables 5.1 & 5.2 to obtain the inverse DTFT

for $\hat{X}(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\sin\frac{3}{2}\omega}{\sin\frac{\omega}{2}}\right) + 5\pi\delta(\omega)$

$-\pi < \omega < \pi$

What property in Table 5.1 is relevant? → 5.3.5 Accumulation:

$$\sum_{k=-\infty}^{n} x[k] \xrightarrow{DTFT} \frac{1}{1-e^{-j\omega}}\hat{X}(e^{j\omega}) + \pi\hat{X}(e^{j0})\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$$

$$\hat{X}(e^{j\omega}) = \frac{\sin\frac{3}{2}\omega}{\sin\frac{1}{2}\omega} \longrightarrow x_1[n] = ?$$

Table 5.2: What pairs of DTFT can we use to obtain $x_1[n]$? Row #9:

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & |n| > N_1 \end{cases} \longrightarrow \frac{\sin\left(N_1+\frac{1}{2}\right)\omega}{\sin\frac{1}{2}\omega}$$

In our problem $N_1 = 1$

$$\to x_1[n] = \begin{cases} 1 & |n| \le 1 \\ 0 & |n| > 1 \end{cases} \xrightarrow{\sim} x_1[k] = \begin{cases} 1 & |k| \le 1 \\ 0 & |k| > 1 \end{cases}$$

Let's apply Accumulation property 5.3.5 with this result:

$$\sum_{k=-\infty}^{\infty} x_1[k] \xrightarrow{DTFT} \frac{1}{1-e^{-j\omega}}\frac{\sin\frac{3}{2}\omega}{\sin\frac{1}{2}\omega} + \pi\left[\frac{\sin\frac{3}{2}\omega}{\sin\frac{1}{2}\omega}\right]_{\omega\to 0}\underbrace{\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)}$$

L'Hopital Rule:

$$\left[\frac{\frac{3}{2}\cos\frac{3}{2}\omega}{\frac{1}{2}\cos\frac{1}{2}\omega}\right]_{\omega\to 0} = 3$$

$\boxed{-\pi < \omega < \pi}$ → $k=0$

If DTFT is applied to the Combined system :

$$\hat{X}(e^{j\omega}) \longrightarrow \boxed{\hat{H}(e^{j\omega})} \longrightarrow \hat{Y}(e^{j\omega})$$

$\hat{Y} = \hat{X} \cdot \hat{H}$ ① (Convolution property)

→ <u>Parallel connection</u> : both systems receive a same input

→ <u>Series connection</u> : input to 2^{nd} system is the output to the 1^{st} system.

How do we find $h_2[n]$?

$$y = y_1 + y_2 = x * h_1 + x * h_2 = x * (h_1 + h_2)$$

or $y = x * (h_1 + h_2)$ If we apply DTFT on both sides.

$$\hat{Y}(e^{j\omega}) = \hat{X}(e^{j\omega}) . (\hat{H}_1 + \hat{H}_2)$$

Table 5.1 (Prop # 5.4 : Convolution)

Because of ①: $\hat{Y}(e^{j\omega}) = \hat{X}(e^{j\omega}) . \hat{H}(e^{j\omega})$

$\hat{H} = \hat{H}_1 + \hat{H}_2$

given

can calculate from $h_1[n]$ given

$$\boxed{\text{First get } \hat{H}_2 \text{ , then } h_2[n] = DTFT^{-1}[\hat{H}_2]}$$

$$\hat{H}_2 = \hat{H} - \hat{H}_1$$

$$h_1[n] = \left(\frac{1}{3}\right)^n u[n] \longrightarrow \hat{H}_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}} = \frac{3}{3 - e^{-j\omega}}$$

Table 5.2 row #8

$$a^n u[n] \longrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$|a| < 1$

$$\hat{H}_2 = \hat{H} - \hat{H}_1 = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}} - \frac{3}{3 - e^{-j\omega}}$$

$$= \frac{(-12 + 5e^{-j\omega})(3 - e^{-j\omega}) - 3(12 - 7e^{-j\omega} + e^{-j2\omega})}{(12 - 7e^{-j\omega} + e^{-j2\omega})(3 - e^{-j\omega})}$$

$$= \frac{-36 + 27e^{-j\omega} - 5e^{-j2\omega} - 36 + 21e^{-j\omega} - 3e^{-j2\omega}}{36 - 33e^{-j\omega} + 10e^{-j2\omega} - e^{-j3\omega}}$$

$$= \frac{-72 + 48e^{-j\omega} - 8e^{-j2\omega}}{+36 - 33e^{-j\omega} + 10e^{-j2\omega} - e^{-j3\omega}}$$

$$= \frac{-2(36 - 24e^{-j\omega} + 4e^{-j2\omega})}{36 - 33e^{-j\omega} + 10e^{-j2\omega} - e^{-j3\omega}}$$

Division:

$$\begin{array}{rrrrr|l}
 & 36 & -33e^{-j\omega} & +10e^{-j2\omega} & -e^{-j3\omega} & 36 - 24e^{-j\omega} + 4e^{-j2\omega} \\
+ & & +9e^{-j\omega} & -6e^{-j2\omega} & +e^{-j3\omega} & \frac{1}{4}e^{-j\omega} - 1 \\
\hline
+ & 36 & -24e^{-j\omega} & +4e^{-j2\omega} & +0 & \\
 & -36 & +24e^{-j\omega} & -4e^{-j2\omega} & & \\
\hline
 & 0 & +0 & +0 & +0 &
\end{array}$$

$$36 - 33e^{-j\omega} + 10e^{-j2\omega} - e^{-j3\omega} = (36 - 24e^{-j\omega} + 4e^{-j2\omega})\left(\frac{1}{4}e^{-j\omega} - 1\right)$$

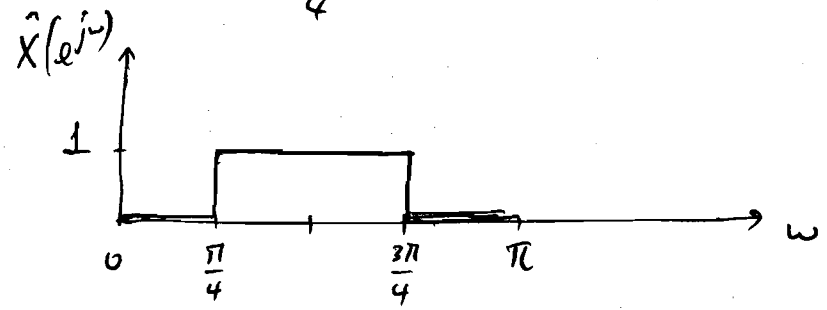
$$\rightarrow \hat{H}_2 = \frac{-2\,\cancel{(36 - 24e^{-j\omega} + 4e^{-j2\omega})}}{\cancel{(36 - 24e^{-j\omega} + 4e^{-j2\omega})}\left(\frac{1}{4}e^{-j\omega} - 1\right)} = \frac{2}{1 - \frac{1}{4}e^{-j\omega}}$$

$$\longrightarrow \boxed{h_2[n] = 2\left(\frac{1}{4}\right)^n u[n]}$$

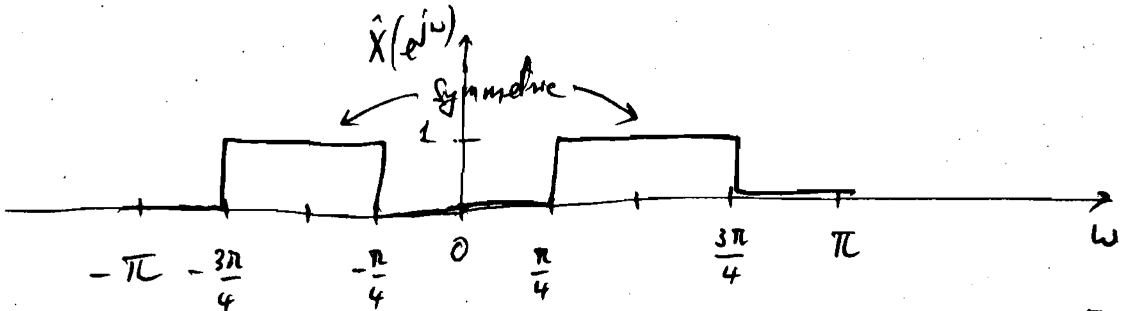
Table 5-2 row #8

5-22

a) $$\hat{X}(e^{j\omega}) = \begin{cases} 1 & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\omega| \le \pi \ ; \quad 0 \le |\omega| < \frac{\pi}{4} \end{cases}$$



Since $\hat{X}(e^{j(\omega+2\pi)}) = \hat{X}(e^{j\omega})$ <u>DTFT is always periodic!</u>



$$x[n] \underset{\text{DTFT}^{-1}}{=} \frac{1}{2\pi}\int_{2\pi} d\omega\, \hat{X}(e^{j\omega})\, e^{j\omega n} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\omega\, \underbrace{\hat{X}(e^{j\omega})}_{\text{symmetric wrt } 0} e^{j\omega n}$$

$$= \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} d\omega\, 1\, e^{j\omega n} + \frac{1}{2\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\omega\, 1\, e^{j\omega n}$$

$$= \frac{1}{2\pi j n}\left(e^{-j\frac{\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right) + \frac{1}{2\pi j n}\left(e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}\right)$$

① $$= \frac{1}{2\pi j n}\left(e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right) + \frac{-1}{2\pi j n}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$

$$= \frac{\sin\frac{3\pi}{4}n}{\pi n} - \frac{\sin\frac{\pi}{4}n}{\pi n} \quad \checkmark$$

② $$= \frac{e^{-jn\frac{2\pi}{4}}}{2\pi j n}\left(e^{+j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right) + \frac{e^{jn\frac{2\pi}{4}}}{2\pi j n}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$

$$= \frac{1}{\pi n} \underbrace{\left(e^{-jn\frac{2\pi}{4}} + e^{jn\frac{2\pi}{4}}\right)} \sin \frac{\pi}{4} n$$

$$= \frac{2 \cos \frac{2\pi}{4} n \ \sin \frac{\pi}{4} n}{\pi n} \quad \checkmark$$

HW5 → due next Thursday.

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ch 6: Time & Frequency Characterization of Signals & Systems

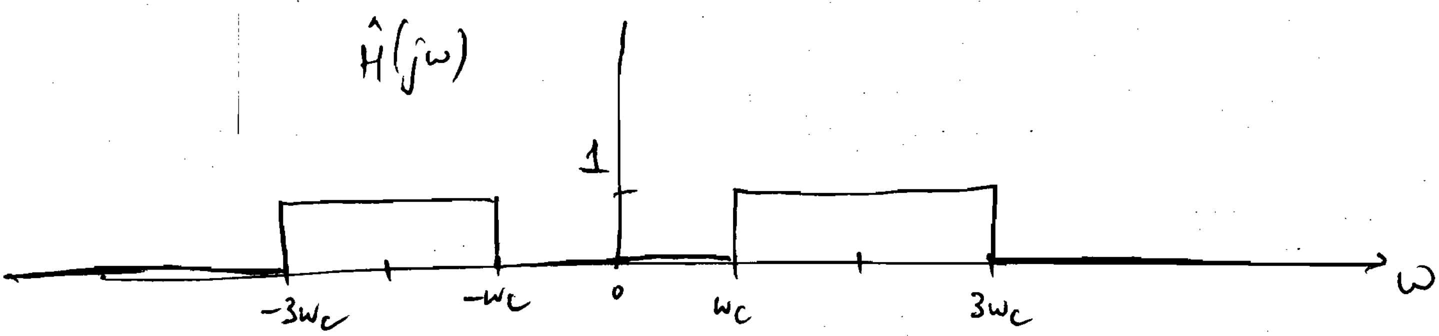
HW6: 6.5; 6.18; 6.19; 6.23; 6.27; 6.30; 6.31; 6.33; 6.40; 6.51

6.5 System: continuous-time bandpass filter

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$\hat{H}(j\omega) = \begin{cases} 1 & \omega_c \le |\omega| \le 3\omega_c \\ 0 & \text{elsewhere} \end{cases} \quad ; \quad \omega_c = \text{cut-off frequency}$$

a) Determine $g(t)$ such that $h(t) = g(t) \dfrac{\sin \omega_c t}{\pi t}$

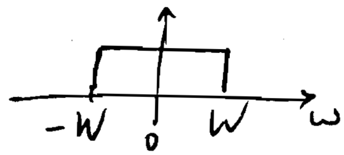


To find $g(t)$: $\quad g(t) = \dfrac{\mathcal{F}^{-1}[\hat{H}(j\omega)]}{\dfrac{\sin \omega_c t}{\pi t}}$

We need to find $h(t) = F^{-1}[\hat{H}]$

Since Ch6 is application of Ch 4 & 5 to Signals & Systems:

Use Table 4.2: $\hat{X}(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$



$\longleftrightarrow \dfrac{\sin Wt}{\pi t}$

$$\rightarrow \hat{H}(j\omega) = \underbrace{\hat{X}(j(\omega - 2\omega_c))} + \underbrace{\hat{X}(j(\omega + 2\omega_c))}\,; \quad W = \omega_c$$

frequency shift

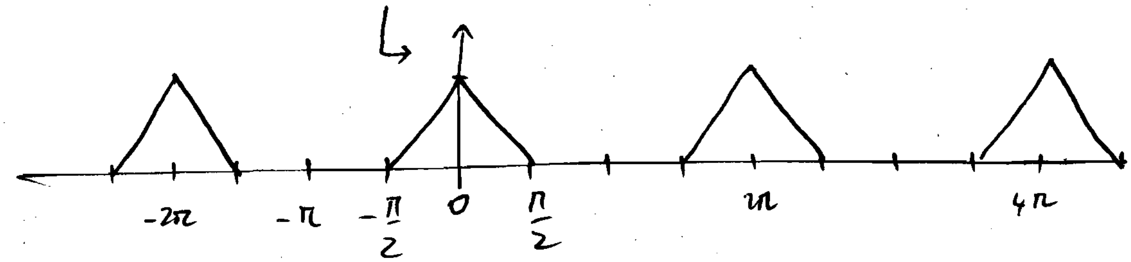
$$h(t) = e^{j2\omega_c t}\,\frac{\sin \omega_c t}{\pi t} + e^{-j2\omega_c t}\,\frac{\sin \omega_c t}{\pi t}$$

$$= \left(e^{j2\omega_c t} + e^{-j2\omega_c t}\right)\frac{\sin \omega_c t}{\pi t} = 2\cos 2\omega_c t \,\frac{\sin \omega_c t}{\pi t}$$

$$\boxed{g(t) = \frac{h(t)}{\frac{\sin \omega_c t}{\pi t}} = 2\cos 2\omega_c t}$$

b) $\omega_c \uparrow$ is $h(t)$ more or less concentrated about the origin? → <u>Matlab code</u> → extra credit $\frac{1}{2}$ HW 6

5.27/ a) $x[n] \rightarrow \hat{X}(e^{j\omega})$



Sketch DTFT of $w[n] = x[n]p[n]$: sketch $\hat{W}(e^{j\omega})$

(i) $p[n] = \cos \pi n$

Table 5.1 : sect 5.5 : $x[n]\, y[n] \rightarrow \frac{1}{2\pi}\int_{2\pi} d\theta\, X(e^{j\theta})Y(e^{j(\omega-\theta)})$

Sketch $\hat{W}(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} d\theta\, \hat{X}(e^{j\theta})\hat{P}(e^{j(\omega-\theta)})$

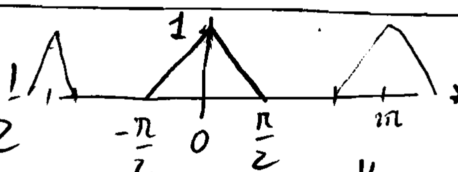
Table 5.2 : Find $\hat{P}(e^{j\theta})$

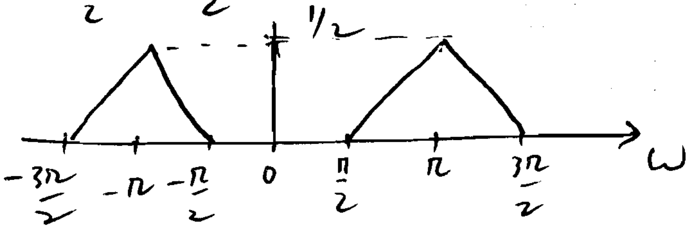
$\cos \omega_0 n \longrightarrow \pi \sum_{\ell} [\delta(\omega - \omega_0 - 2\pi\ell) + \delta(\omega + \omega_0 - 2\pi\ell)]$

$\cos \pi n \longrightarrow \pi [\delta(\omega - \pi) + \delta(\omega + \pi)]$ ($\ell = 0$ since we only need one interval $(-\pi, \pi)$)

$\hat{W}(e^{j\omega}) = \frac{1}{2\pi} \hat{X} * \pi[\delta(\omega - \pi) + \delta(\omega + \pi)]$

$\boxed{\text{Convolution with a delta function is simple: Notes p. 26 : } z(\omega) * \delta(\omega - \omega_0) = z(\omega - \omega_0)}$

$\hat{W}(e^{j\omega}) = \frac{1}{2}$  $* [\delta(\omega - \pi) + \delta(\omega + \pi)]$

$=$ 

(iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k]$

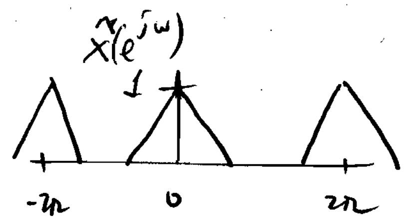
<u>Table 5.2</u>: $\hat{P}(e^{j\theta})$:

$$\sum_{k=-\infty}^{\infty} \delta[n-kN] \xrightarrow{\text{DTFT}} \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$$

$$\underset{(N=2)}{p[n]} \longrightarrow \hat{P} = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$$

we keep this summation since it is part of the signal.

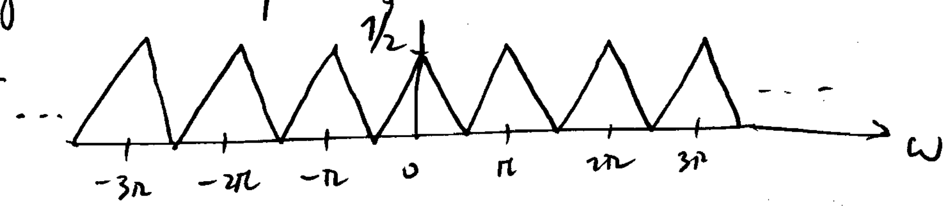
$\hat{W}(e^{j\omega}) = \frac{1}{2\pi}$



$* \; \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$

Convolution with our δ places the triangular pulses at positive & negative multiple of π's

$\hat{W}(e^{j\omega}) =$



(b)

$$w[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = ?$$

$$h[n] = \frac{\sin \pi n/2}{\pi n}$$

$w[n] = x[n] \cdot p[n]$, for different choices of $p[n]$

iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k] \rightarrow \hat{W}$: series of Δ pulses of amplitude $1/2$ centered at $\pm k\pi$

$$y[n] = w[n] * h[n]$$

Convolution property (Table 5.1)

$$\hat{Y}(e^{j\omega}) = \hat{W}(e^{j\omega}) \cdot \hat{H}(e^{j\omega})$$

✓ ?

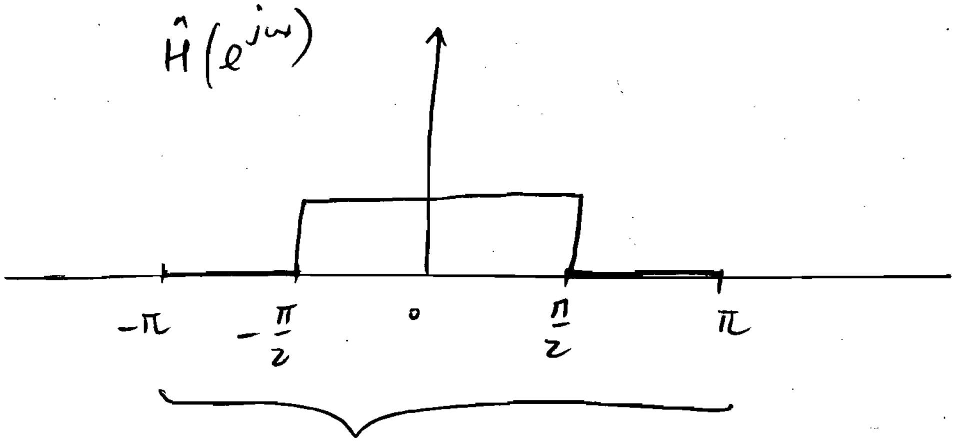
$$h[n] = \frac{\sin \pi n/2}{\pi n} \longrightarrow \hat{H}(e^{j\omega})$$

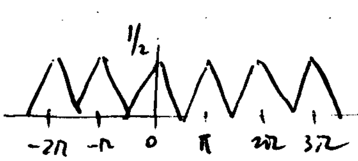
Table 5.2: $\frac{\sin Wn}{\pi n} \equiv \frac{W}{\pi}\,\text{sinc}\left(\frac{Wn}{\pi}\right) \longrightarrow \hat{X}(e^{j\omega}) = \begin{cases} 1 & 0 \le |\omega| \le W \\ 0 & W \le |\omega| \le \pi \end{cases}$

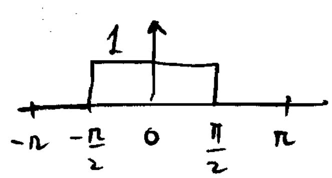
periodic w/ period of 2π

$h[n]$: $\longrightarrow$ $\hat{H}(e^{j\omega}) = \begin{cases} 1 & 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\omega| < \pi \end{cases}$

$W = \frac{\pi}{2}$



$\Rightarrow \hat{Y} = \hat{W} \cdot \hat{H} =$ 

$\cdot$ 

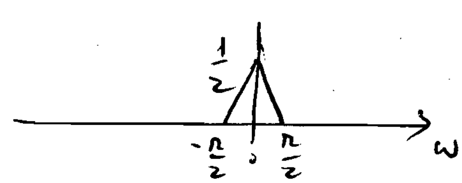
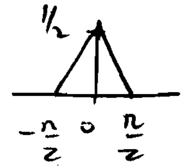
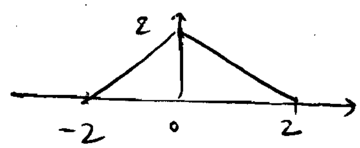
$\hat{Y} =$ 

Table 5.2 does not give direct answer for $\hat{Y} =$  $\to y[n]$

area $= \frac{\pi}{2}\times\frac{1}{2}\times\frac{1}{\cancel{2}}\times\cancel{2} = \frac{\pi}{4}$

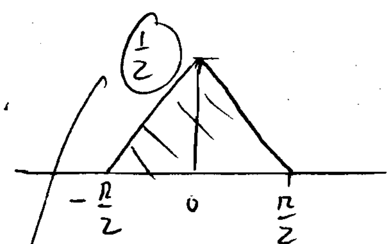
Problem 4.10 : we did.

$$\left(\frac{\sin t}{\pi t}\right)^2 \longrightarrow \frac{1}{2\pi}$$



area $\frac{2\times 2}{2}\times 2\times\frac{1}{2\pi} = \frac{2}{\pi}$

$$y[n] = \underbrace{\sim}_{\text{constant}}\left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)^2$$



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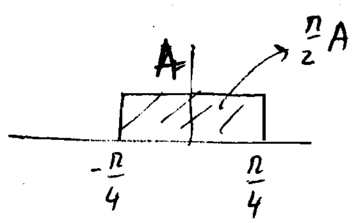

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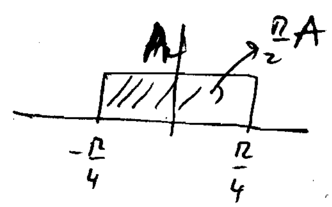


$$\frac{1}{2} = \frac{\pi}{\cancel{4}}A\times\cancel{2}$$

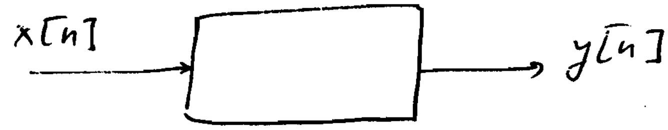
$$\boxed{A = \frac{1}{2\pi}}$$

$$\begin{cases} A & 0\le|w|\le W \\ 0 & W\le|w|\le\pi\end{cases} \xrightarrow{DTFT^{-1}} \frac{1}{2}\,\frac{\sin Wn}{\pi n}$$

$W = \frac{\pi}{4}$

$$= A\,\frac{\sin\frac{\pi}{4}n}{\pi n}$$

5.19



$$\text{DTFT}\left[y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]\right]$$

a) $\hat{H}(e^{jw})$?

$$\hat{Y}(e^{jw}) - \frac{1}{6}e^{-jw}\hat{Y}(e^{jw}) - \frac{1}{6}e^{-2jw}\hat{Y}(e^{jw}) = \hat{X}(e^{jw})$$

Since $\hat{H}(e^{jw}) = \dfrac{\hat{Y}(e^{jw})}{\hat{X}(e^{jw})} = \dfrac{1}{1-\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}}$

b) $h[n] = \text{DTFT}^{-1}\left[\hat{H}(e^{jw})\right]$

Partial Fraction Expansion:

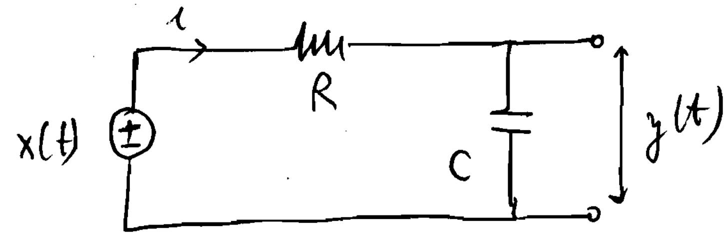
$$\hat{H}(e^{jw}) = \frac{1}{(1-\frac{1}{2}e^{-jw})(1+\frac{1}{3}e^{-jw})} = \frac{3/5}{1-\frac{1}{2}e^{-jw}} + \frac{2/5}{1+\frac{1}{3}e^{-jw}}$$

DTFT^{-1} Table 5-2 (row #8)

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$

Ch 6 (Cont.)

6.18



Can the <u>step response</u> be oscillatory?

$$u(t) \longrightarrow \boxed{RC} \longrightarrow$$ can this oscillate? No.

<u>Proof</u>: $y(t) = F^{-1}[\hat{Y}] = F^{-1}[\hat{X} \cdot \hat{H}]$

$$x(t) = Ri + \underbrace{\frac{1}{C}\int i\,dt}_{y(t)} \rightarrow i(t) = C\frac{dy}{dt}$$

$$\boxed{x(t) = RC\frac{dy}{dt} + y(t)}$$

input/output differential equation

Applying Fourier Transform on this equation:

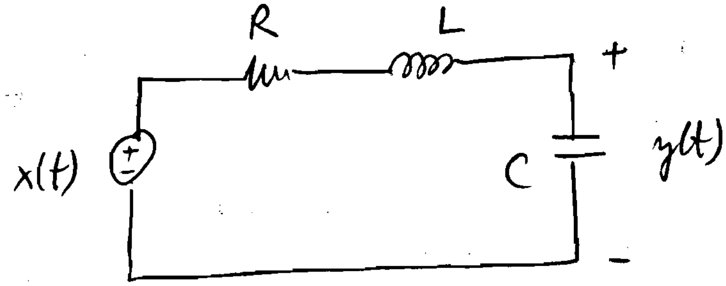
$$\hat{X} = RCj\omega\hat{Y} + \hat{Y} \rightarrow \hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{1}{RCj\omega + 1}$$

Also: $F[u(t)] = \frac{1}{j\omega} + \pi\delta(\omega)$

$$y(t) = F^{-1}[\hat{X} \cdot \hat{H}] = F^{-1}\left[\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)\frac{1}{RCj\omega + 1}\right]$$

<u>Table 4.2</u>: exponential function of time, no oscillation.

6.19



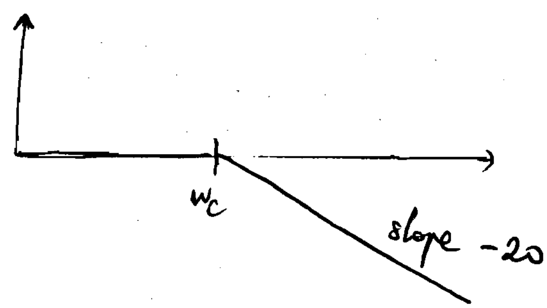
can this oscillate ?

$$\hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{\text{impedance at } C}{\text{total impedance}} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + (j\omega)^2 LC + 1}$$

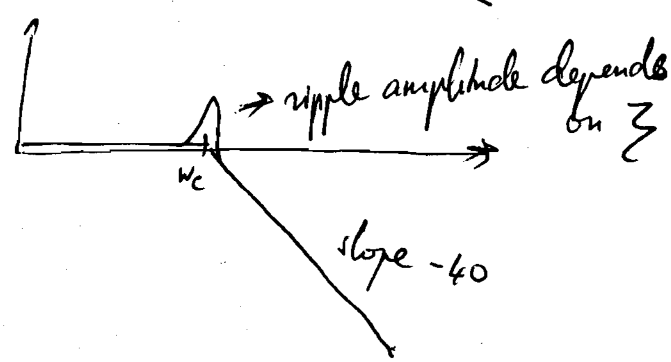
Voltage division using Phasors

2nd order pole.

Bode Plot: 1st order pole:



2nd order pole:



$$\frac{1}{1 + 2(j\omega\tau\xi) + (j\omega\tau)^2}$$

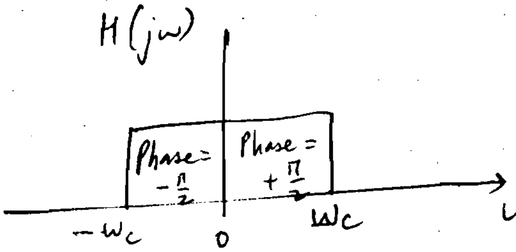
tau xi

$$\tau = \sqrt{LC} \rightarrow 2\tau\xi = RC$$

$$\xi = \frac{RC}{2\tau} = \frac{RC}{2\sqrt{LC}} = \boxed{\frac{R}{2}\sqrt{\frac{C}{L}}}$$

$$y(t) = F^{-1}\left[\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) \underbrace{\frac{1}{j\omega RC + (j\omega\sqrt{LC})^2 + 1}}_{\text{has a ripple depending on } \xi}\right] \rightarrow \boxed{\text{oscillations}}$$

c) $$\text{Phase}(\hat{H}(j\omega)) = \begin{cases} \frac{\pi}{2} & \omega > 0 \\ -\frac{\pi}{2} & \omega < 0 \end{cases}$$



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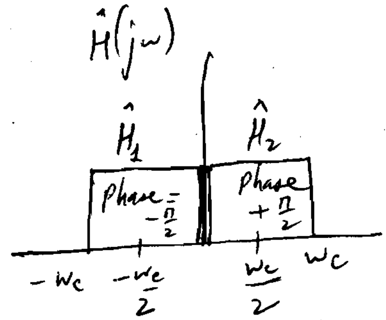
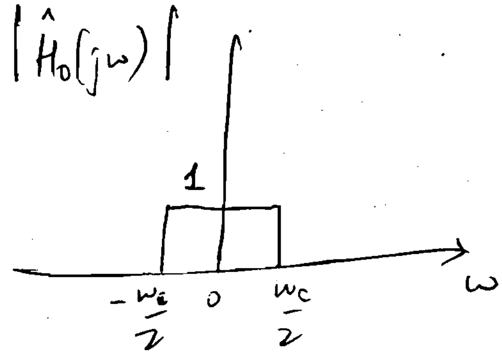


$$\hat{H}(j\omega) = \hat{H}_1(j\omega) + \hat{H}_2(j\omega) = -j\left|\hat{H}_0\left(j\left(w + \frac{w_c}{2}\right)\right)\right| + j\left|\hat{H}_0\left(j\left(\omega - \frac{w_c}{2}\right)\right)\right|$$

phase = $-\frac{\pi}{2}$ phase = $+\frac{\pi}{2}$



Shift in frequency: convolution with a delta function

$$\hat{H}(j\omega) = \underbrace{|\hat{H}_0(j\omega)|}_{\downarrow \text{ Table 4.2 row \#9}} * \underbrace{\left[-j\,\delta\left(\omega + \frac{w_c}{2}\right) + j\,\delta\left(\omega - \frac{w_c}{2}\right)\right]}_{\downarrow \text{ Table 4.2 row \#4}}$$

$$h(t) = \frac{\sin \frac{w_c}{2} t}{\pi t} \qquad \left(-\frac{1}{\pi} \sin \frac{w_c}{2} t\right)$$

$$h(t) = \left(-\frac{1}{\pi}\right) \frac{\sin^2 \frac{w_c}{2} t}{\pi t}$$

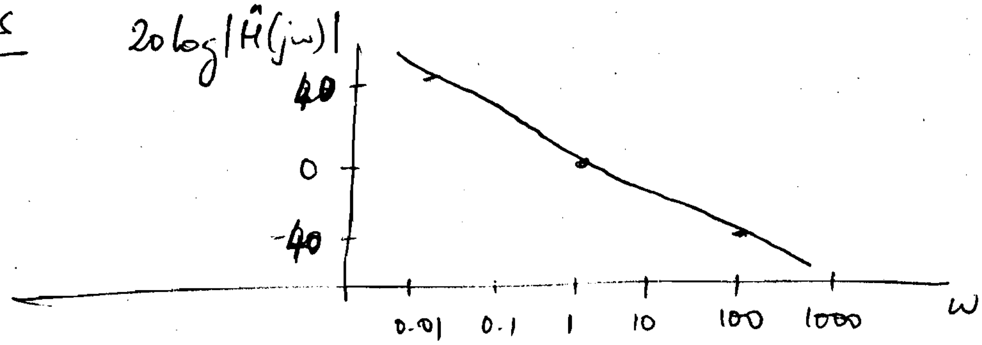
6.31 Integrator: $\hat{H}(j\omega) = \frac{1}{j\omega} + \underbrace{\pi\delta(\omega)}$

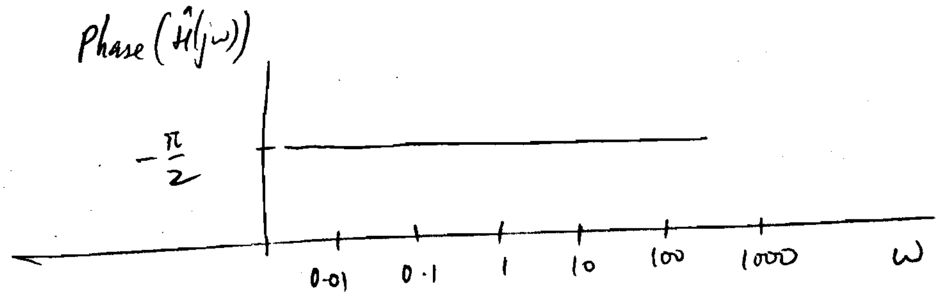
b/c integration of a constant from $t=-\infty$

at $\omega = 0$

If only examine $\omega > 0 \rightarrow \begin{cases} 20\log|\hat{H}(j\omega)| = 20\log\left|\frac{1}{j\omega}\right| = -20\log\omega \\ \text{Phase}(\hat{H}(j\omega)) = \text{Phase}(-j) = -\frac{\pi}{2} \end{cases}$

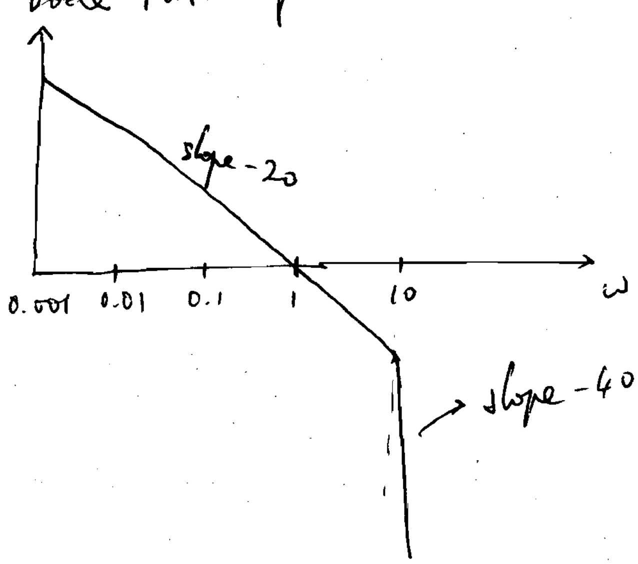
Bode Plots

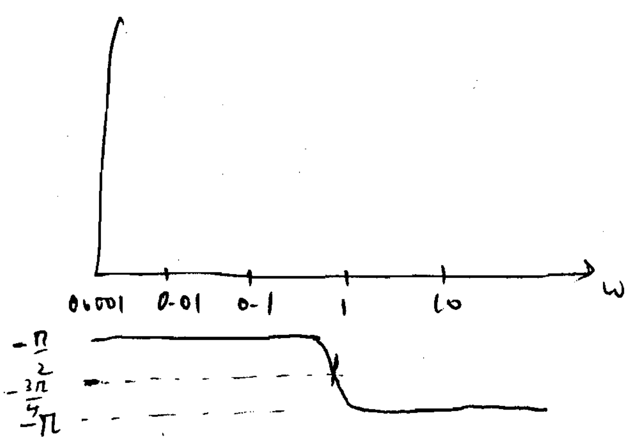




(a) $\hat{H}(j\omega) = \frac{1}{j\omega(1 + j\frac{\omega}{10})} + \underbrace{\pi\delta(\omega)}_{0}$ (Electric motor)

Bode Plots for $\omega > 0.001$





Ch7 Sampling

HW7 7.7 ; 7.9; 7.15; 7.24; 7.30; 7.31; 7.40; 7.45; 7.49; 7.50