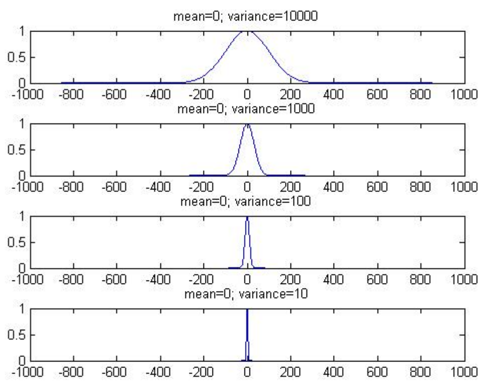
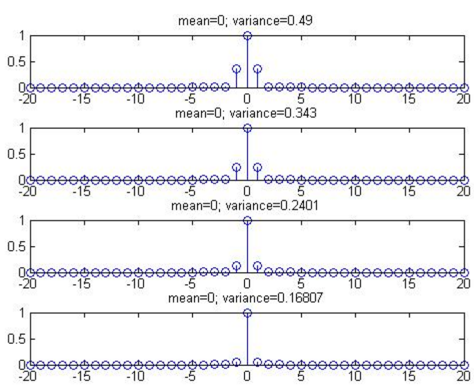
More on signals:

<u>Unit impulse</u>: δ(t)



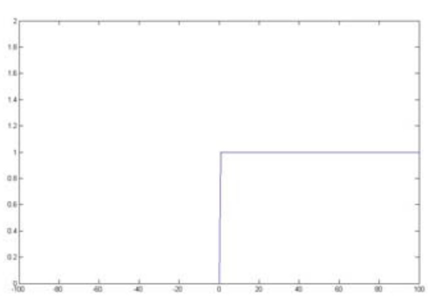

```
%Gaussian distribution
clear all
close all
for i=1:4
var=10^(5-i);%1000;%variance
sigma=sqrt(var);%standard deviation
xbar=0;%%mean
x=-1000:1000;
fx=1/(sqrt(2*pi)*sigma)*exp(-(x-
xbar).^2/(2*var));
fx=fx/max(fx);
figure(1), subplot(4,1,i),plot(x,fx),
title(strcat('mean=',num2str(xbar),';
variance=',num2str(var)))
end
```

A unit impulse is the limit of gaussian function when the width (variance; standard deviation) tends to zero. “Unit” when the maximum value is 1. The discrete-time version of it is δ[n], as shown in the figure below.

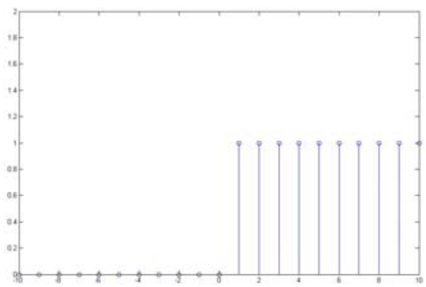



```
%Gaussian distribution: discrete-time
clear all
%close all
for i=1:4
var=.7^(i+1);%1000;%variance
sigma=sqrt(var);%standard deviation
xbar=0;%%mean
x=-20:20;
fx=1/(sqrt(2*pi)*sigma)*exp(-(x-
xbar).^2/(2*var));
fx=fx/max(fx);
figure(2), subplot(4,1,i),stem(x,fx),
title(strcat('mean=',num2str(xbar),';
variance=',num2str(var)))
end
```

<u>Unit step function:</u> u(t) or u[n]



```
close all
clear all
xstep=-100:1:100;
step=[zeros(1, 101) ones(1,100)];
figure(1),plot(xstep,step)
```

	`xstepdt=-10:1:10;`
	`stepdt=[zeros(1, 11) ones(1,10)];`
	`figure(2), stem(xstepdt, stepdt)`

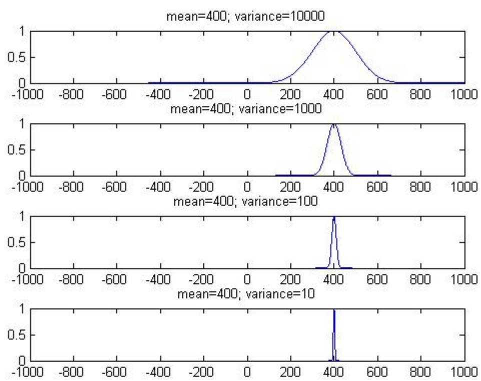
Relationship:

$$\partial(t) = \frac{du}{dt}$$

Integrals involving impulse functions δ(t):

$$\int_{-\infty}^{+\infty} dt \quad f(t)\delta(t) \quad = \quad f(0)$$

Below is a impulse centered at 400



$$\int_{-\infty}^{+\infty} dt \quad f(t)\delta(t-400) \quad = \quad f(400) \quad \text{(impulse centered at 400)}$$

$$\int_{-\infty}^{+\infty} dt \quad f(t)\delta(t+400) \quad = \quad f(-400) \quad \text{(impulse centered at -400)}$$

$$\int_{-3}^{+3} dt \quad f(t)\delta(t) \quad = \quad f(0)$$

$$\int_{-3}^{+3} dt \quad f(t)\delta(t-400) \quad = \quad 0$$

1.39 From the relationship between impulse and the step functions:

$$u_\Delta(t) = \int_{-\infty}^{t} d\tau\ \delta_\Delta(\tau); \quad u(t) = \lim_{\Delta\to 0} u_\Delta(t)$$

u_Δ is a step function with a finite slope during Δ at the transition from 0 to 1; when $\Delta \to 0$ the transition has an infinite slope, we get the actual step function u(t). Similarly δ_Δ is an impulse with a finite width Δ starting from 0, when $\Delta \to 0$ we get back the actual impulse $\delta(t)$.

The problem asks for this proof:

$$\lim_{\Delta\to 0}[u_\Delta(t)\delta(t)] = 0$$

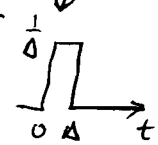
This is obvious if we replace the definition of u_Δ:

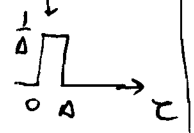
$\lim_{\Delta\to 0}[u_\Delta(t)\delta(t)] = \delta(t)\lim_{\Delta\to 0}\int_{-\infty}^{t} d\tau\ \delta_\Delta(\tau);$ since when t=0 the lim is 0; same when t<0; when t>0 the limit is nonzero however $\delta(t)=0$, so the result is still 0.

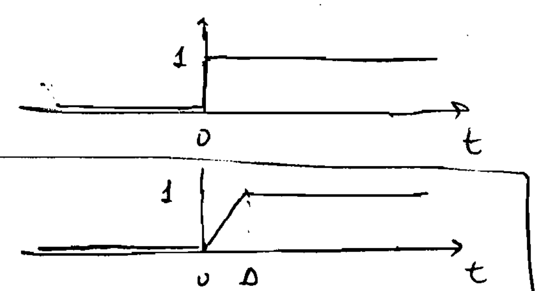
The problem also asks for this proof:

$$\lim_{\Delta\to 0}[u_\Delta(t)\delta_\Delta(t)] = \frac{1}{2}\delta(t)$$

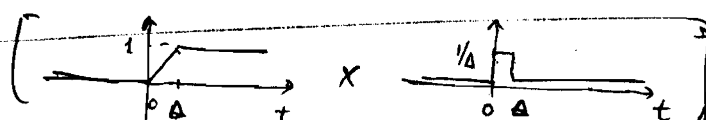
$$\lim_{\Delta \to 0} \left[u_\Delta(t)\, \delta_\Delta(t) \right] = \lim_{\Delta \to 0} \left[\quad \times \int_{-\infty}^{t} \delta_\Delta(\tau)\, d\tau \right]$$

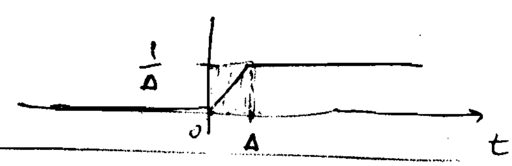






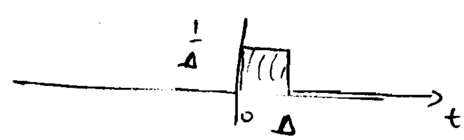
$$\lim_{\Delta \to 0} \left[u_\Delta(t)\, \delta_\Delta(t) \right] = \lim_{\Delta \to 0} \left[\quad \times \quad \right]$$





→ Recall:

$$\lim_{\Delta \to 0} \quad = \delta(t)$$



So → $\frac{1}{2}\delta(t)$.

$\frac{1}{2}$ → half area.