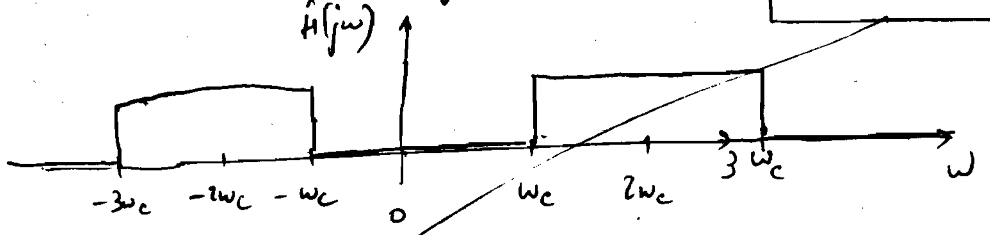
Ch 6: Time & Frequency Characterization of Signals & Systems.

HW6: 6.5 ; 6.18 ; 6.19 ; 6.23 ; 6.27 ; 6.30 ; 6.31 ; 6.33 ;
6.40 ; 6.51

6.5| Continuous-time band pass filter:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \qquad \hat{H}(j\omega) = \begin{cases} 1 & \omega_c \le |\omega| \le 3\omega_c \\ 0 & \text{elsewhere} \end{cases}$$

a) determine $g(t)$ such that $\underbrace{h(t) = \frac{\sin \omega_c t}{\pi t} \cdot \underbrace{g(t)}_{}}_{?}$:

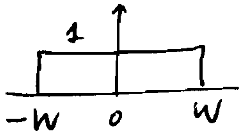


Method a:) $\hat{H}(j\omega) = F\left[\frac{\sin \omega_c t}{\pi t}\right] * \hat{G}(j\omega) \to \hat{G}(j\omega)$.

Method b?) $g(t) = \frac{F^{-1}[\hat{H}]}{\frac{\sin \omega_c t}{\pi t}}$ ✓

$F^{-1}[\hat{H}] =$

Table 4.2: $\hat{X}(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| < W \end{cases}$



$\longrightarrow \frac{\sin Wt}{\pi t}$

$$\hat{H}(j\omega) = \underbrace{\hat{X}(j(\omega - 2\omega_c))}_{\downarrow} + \underbrace{\hat{X}(j(\omega + 2\omega_c))}_{\downarrow} \qquad (W = \omega_c)$$

$$\to h(t) = e^{j2\omega_c t} \cdot \frac{\sin \omega_c t}{\pi t} + e^{-j2\omega_c t} \cdot \frac{\sin \omega_c t}{\pi t} \longrightarrow g(t) = e^{j2\omega_c t} + e^{-j2\omega_c t}$$

$$= 2 \cos 2\omega_c t$$

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$; $\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$, $\boxed{e^{j\alpha} = \cos\alpha + j\sin\alpha}$

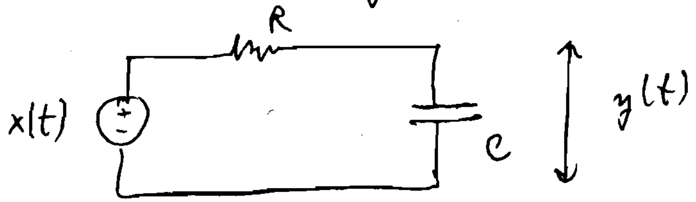
de Moivre relation

b) $w_c \uparrow$ $h(t)$ more/less concentrated about the origin?

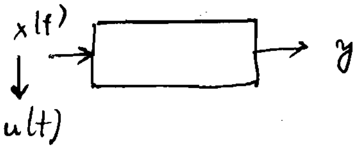
$$h(t) = \frac{\sin(w_c t)}{\pi t} \underbrace{2\cos(2w_c t)}_{g(t)}$$

→ Matlab plot → x credit (half a HW set).

$\underline{6.18|}$ RC circuit LTI system.



Step response:



$\to y(t)$: can this be oscillatory?

→ No. Proof: $y(t) = F^{-1}[\hat{Y}] = F^{-1}\Big[\underbrace{\left(\frac{1}{jw} + \pi\delta(w)\right)}_{\hat{X}} . \hat{H}\Big]$

(Four. Transf of step function).

$\hat{H}$? : Circuit loop equation:

$$x(t) = Ri + \underbrace{\frac{1}{C}\int i\,dt}_{y(t)} \Rightarrow i(t) = C\frac{dy}{dt}$$

$$F\left[\,x(t) = RC\frac{dy}{dt} + y(t)\right] \to \hat{X} = RCjw\hat{Y} + \hat{Y}$$

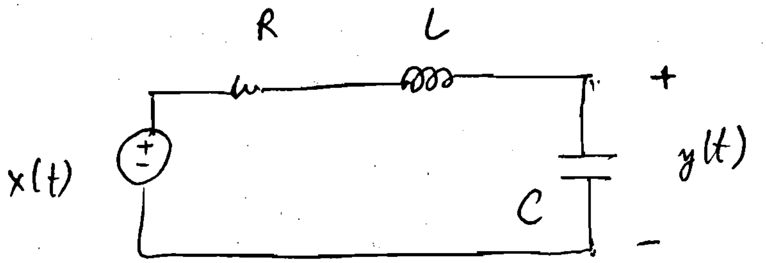
$$\to \hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{1}{RCjw + 1}$$

$$y(t) = F^{-1}\left[\frac{\frac{1}{jw} + \pi\delta(w)}{\boxed{1 + jwRC}}\right] = \to$$ exponential decay in time !

Table 4.2 ↑

→ No oscillations.

6.19



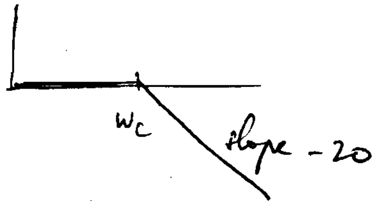
Can a step response have osc. behavior ?

$$\hat{H} = \frac{\hat{Y}}{\hat{X}} \underset{\downarrow}{=} \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + (j\omega)^2 LC + 1}$$

From Circuit Analysis II (Voltage Division in frequency)

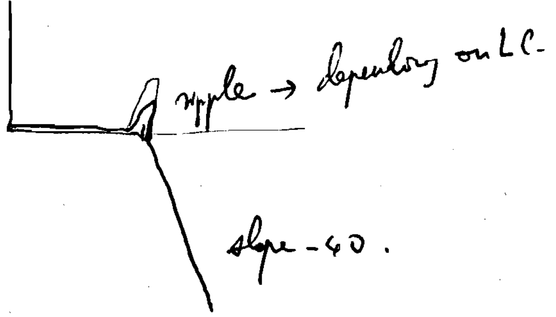
2nd-order denominator → Bode Plot =

Bode Plot : 1st order pole :



2nd order pole.

$$\boxed{\frac{1}{1+2(j\omega\underset{\downarrow\,\text{tau}}{\tau}\underset{\downarrow\,\text{xi}}{\xi})+(j\omega\tau)^2}}$$



No ripple if $\underset{xi}{\xi} \geq 1$

$\tau = \sqrt{LC}$; $\rightarrow \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

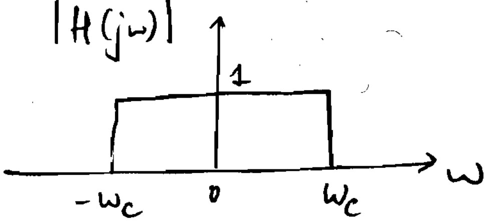
No ripple if $\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$ or $\boxed{R \geq 2\sqrt{\frac{L}{C}}}$

No osc.

Plot inverse F with Matlab.
(half HW set extra credit); → different ξ to see osc.

6.23

Low-pass filter:



Sketch impulse response $h(t) = F^{-1}[\hat{H}(j\omega)]$

a) $\text{Phase}(\hat{H}(j\omega)) = 0 \rightarrow \hat{H}(j\omega) = |\hat{H}(j\omega)|\underbrace{e^{j0}}_{1}$

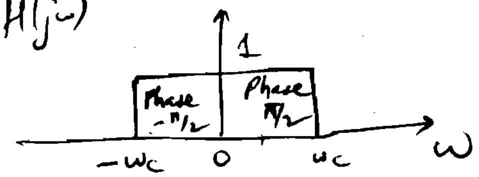
$\rightarrow h(t) = \frac{\sin \omega_c t}{\pi t}$

$\hat{z} = |z|e^{j\,\text{Phase}(\hat{z})}$

b) $\text{Phase}(\hat{H}(j\omega)) = \omega T \rightarrow \hat{H}(j\omega) = |\hat{H}(j\omega)|e^{j\omega T} \rightarrow h(t) \overset{?}{=} \frac{\sin \omega_c(t+T)}{\pi(t+T)}$

c) $\text{Phase}(\hat{H}(j\omega)) = \begin{cases} \frac{\pi}{2} & \omega > 0 \\ -\frac{\pi}{2} & \omega < 0 \end{cases}$

$\hat{H}(j\omega)$



$\hat{H}(j\omega) = |\hat{H}(j\omega)|e^{-j\frac{\pi}{2}}$ $\quad$ $\hat{H}(j\omega) = |\hat{H}(j\omega)|e^{j\frac{\pi}{2}}$ $\Big\}\ \hat{H}(j\omega) =$

$$\hat{H}(j\omega) = |\hat{H}(j\omega)|\{e^{-j\frac{\pi}{2}}u(-\omega) + e^{j\frac{\pi}{2}}\underbrace{u(\omega)}\}$$

$\downarrow$

$\frac{1}{jt} + \pi\delta(t)$?

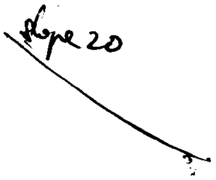
(6.31) Integrator: $\hat{H}(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$

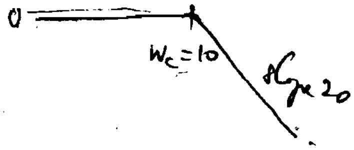
$(h(t) = u(t))$

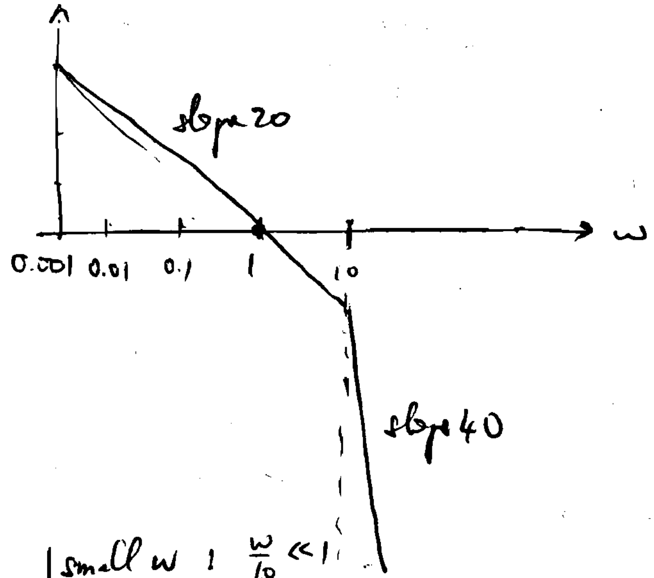
$\underline{\omega > 0} \rightarrow \begin{cases} 20\log|\hat{H}(j\omega)| = 20\log\left|\frac{1}{j\omega}\right| = -20\log\omega \\ Phase(\hat{H}(j\omega)) = Phase\left(\frac{1}{j\omega}\right) = -\frac{\pi}{2}. \end{cases}$

Bode Plot for $\hat{H}(j\omega) = \frac{1}{j\omega(1+j\frac{\omega}{10})} + \underbrace{\pi\delta(\omega)}_{0}$, $\omega > 0.001$

Magnitude: $20\log|\hat{H}(j\omega)| = -20\log_{10}|j\omega| - 20\log_{10}\left|1+j\frac{\omega}{10}\right|$

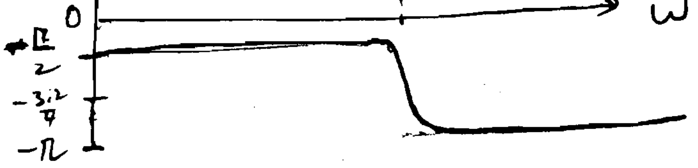






Phase: $\frac{1}{j\omega(1+j\frac{\omega}{10})}$ $\begin{cases} \text{small } \omega : \frac{\omega}{10} \ll 1 \\ \hookrightarrow -\frac{\pi}{2} \longrightarrow \text{intermediate: } \omega \simeq 10 \quad j\omega(1+j) \\ \text{large } \omega : \frac{\omega}{10} \gg 1 \rightarrow -\pi \end{cases}$

Phase.



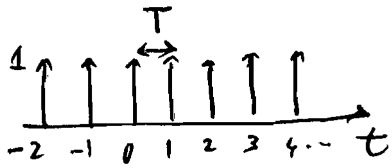
Ch.7 : Sampling

HW7: 7.7, 7.9; 7.15, 7.24; 7.30; 7.31; 7.40; 7.45; 7.49; 7.50

Sampled signal: $x_p(t) = x(t) \cdot p(t)$

$\uparrow$ original signal

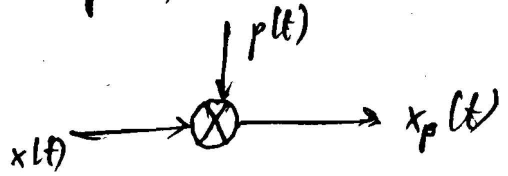
$p(t) \rightarrow$



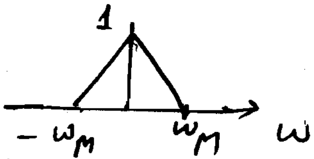
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

By multiplying the pulse signal p(t) to any signal x(t) we get the sampled signal $x_p(t)$.

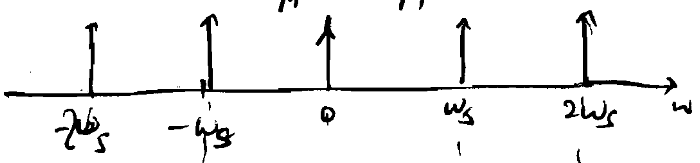
Sampling process:



$x(t) \rightarrow \hat{X}(j\omega)$ is triangular:

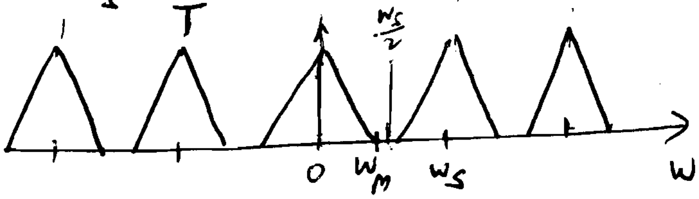


$p(t) \rightarrow \hat{P}(j\omega)$:



$\omega_s = \frac{2\pi}{T}$

$\hat{X}_p(j\omega) = \hat{X} * \hat{P}$:



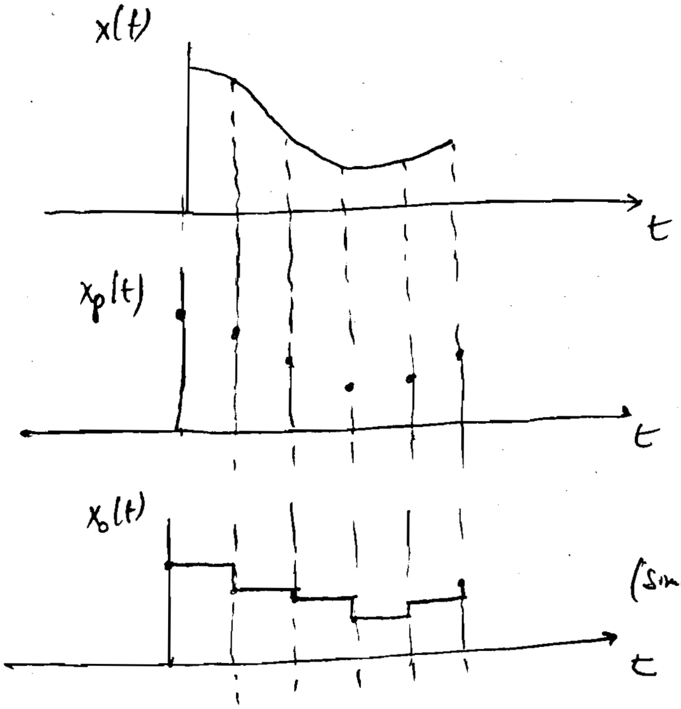
$\omega_s \geq 2\omega_M$: there is no overlap of the repetition in frequency $\rightarrow$ perfect reconstruction of original signal.

Sampling or Nyquist Theorem.

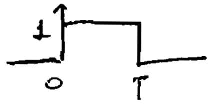
(Sampling freq has to be at least 2 times the max. freq.

The chapter defines :

1) zero-order hold: hold value until next sample, or doing a convolution with a rectangular pulse



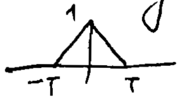
convolute with

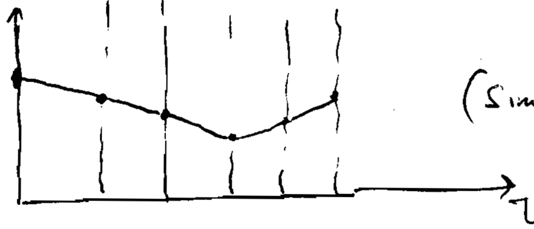


(simulate a continuous signal by holding values until the next sample)

2) First-order hold: use linear interpolation to project value until next sample, or doing a convolution w/ a triangular pulse.

convolution with





(Simulating a continuous signal by linear interpolation until the next sample)

7.7

$x_0(t) \longrightarrow \boxed{\hat{H}\,?} \longrightarrow x_1(t)$ → $x_0 * h = x_1$ or $\hat{X}_0 \cdot \hat{H} = \hat{X}_1$

$x_0(t) = x_p(t) * h_0(t)$
↳ rectangular pulse (we know)

↓

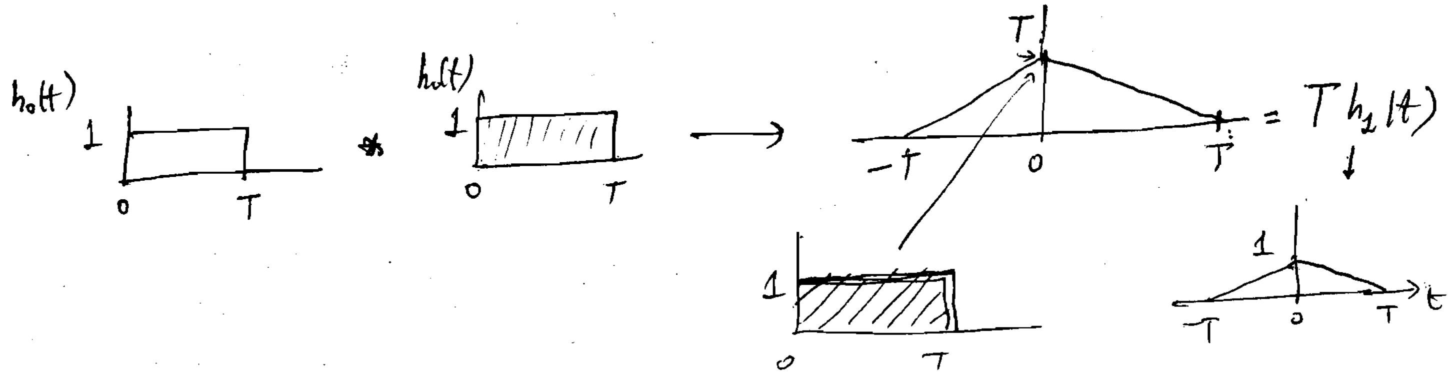
$\hat{X}_0 = \hat{X}_p \cdot \hat{H}_0$

$x_1(t) = x_p(t) * h_1(t)$
↳ triangular pulse (we know)

$\hat{X}_1 = \hat{X}_p \cdot \hat{H}_1$

so by eliminating $\hat{X}_p$ = $\hat{X}_1 = \frac{\hat{X}_0}{\hat{H}_0} \cdot \hat{H}_1$ → $\boxed{\hat{H} = \frac{\hat{H}_1}{\hat{H}_0}}$

Any connection b/w $\hat{H}_1$ & $\hat{H}_0$? or any connection b/w rectangular & triangular pulses?



that is

$$\begin{array}{cccccc} h_0(t) & * & h_0(t) & = & T & h_1(t) \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ \hat{H}_0 & \cdot & \hat{H}_0 & = & T & \hat{H}_1 \end{array} \rightarrow \boxed{\hat{H}_1 = \frac{1}{T}\hat{H}_0 \cdot \hat{H}_0}$$

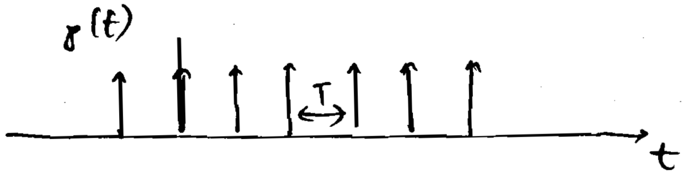
$$\rightarrow \quad \hat{H} = \frac{\hat{H}_1}{\hat{H}_0} = \frac{\frac{1}{T}\hat{H}_0 \cdot \cancel{\hat{H}_0}}{\cancel{\hat{H}_0}} = \underset{\downarrow}{\frac{1}{T}\hat{H}_0} = \frac{2\sin\frac{\omega T}{2}}{\omega T} e^{-j\omega\frac{T}{2}}$$

Fourier Transform of a rectangular pulse

Table 4.2 :

2.9/ $x(t) = \left(\frac{\sin(50\pi t)}{\pi t}\right)^2$ sampled at $w_s = 150\pi$

$\rightarrow g(t) = x(t)\,p(t)$

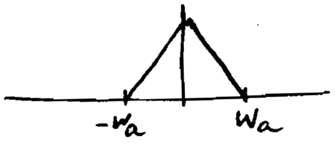


$T = \frac{2\pi}{w_s} = \frac{2\pi}{150\pi} = \frac{1}{75}\,s$

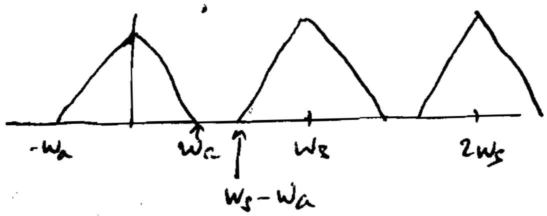
$$\hat{G}(jw) = 75\,\hat{X}(jw) \quad \text{for } |w| \le \overset{\downarrow ?}{\boxed{w_0}}$$

Recall: sampling theory:

$x(t) \xrightarrow{\qquad} \hat{X}(jw) =$



$x(t) \longrightarrow \overset{p(t)}{\otimes} \longrightarrow x_p(t) \rightarrow \hat{X}_p(jw) =$



No aliasing if $w_s - w_a \geq w_a$

or $w_s \geq 2w_a$ (Nyquist Theorem).

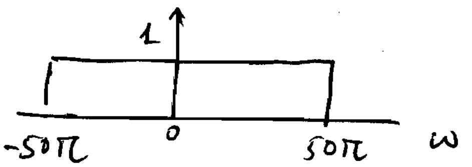
"If the left edge of the second group is larger or equal than the right edge of the first group, there is no aliasing".

This problem: is asking what is the right edge of the first group so the transform of the sampled signal $\hat{G}$ is proportional to the transform of the original signal $\hat{X}$.

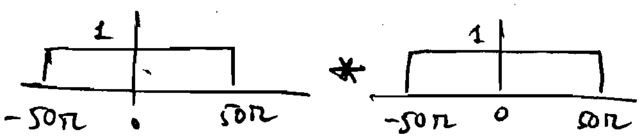
Max wa is when the right edge of 1st group falls on the left edge of the 2nd group.

We need $\hat{X}(j\omega)$:

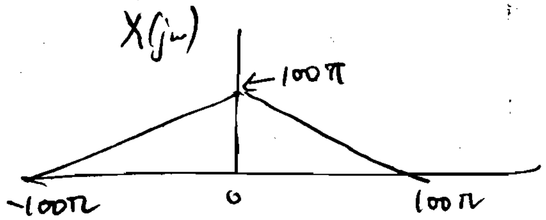
Table 4.2: $\frac{\sin 50\pi t}{\pi t} \longrightarrow$



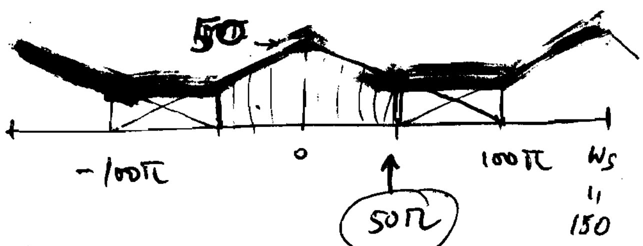
$\left(\frac{\sin(50\pi t)}{\pi t}\right)^2 \longrightarrow$



$\hat{X}(j\omega)$



$\hat{G}(j\omega)$



$w_s = 150\pi$: what is max w_0 for $\hat{G}(j\omega) \propto \hat{X}(j\omega)$

$\left(\hat{G} = \hat{X} * \hat{P}, \text{ this carries } \frac{1}{2\pi}\right)$

7.40

Real-life application of sampling.

$v(t) = \underset{\overset{\downarrow}{1}}{A} \cos(w_0 t + \phi)$; $w_0 = 2\pi f = 120\pi$

(f = 60Hz or 15 rev/s ×4 cycles)

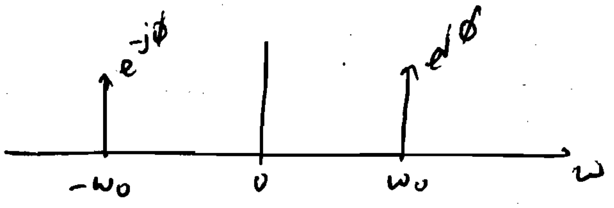
eye can't see at 60Hz → $-\boxed{\text{Low-pass at 20Hz}}-$

illumination: $i(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT)$; $\frac{1}{T}$ strobe freq in Hz.

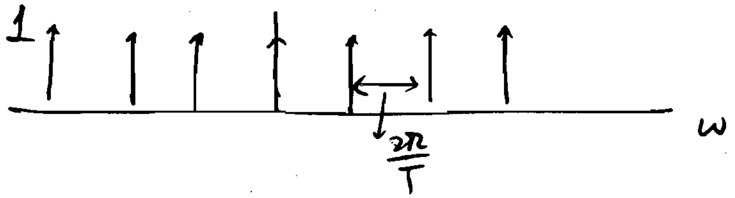
Sampled signal: $r(t) = v(t) \cdot i(t)$

$\hat{R} = \hat{V} * \hat{I}$

a) Sketch $\hat{V}$:



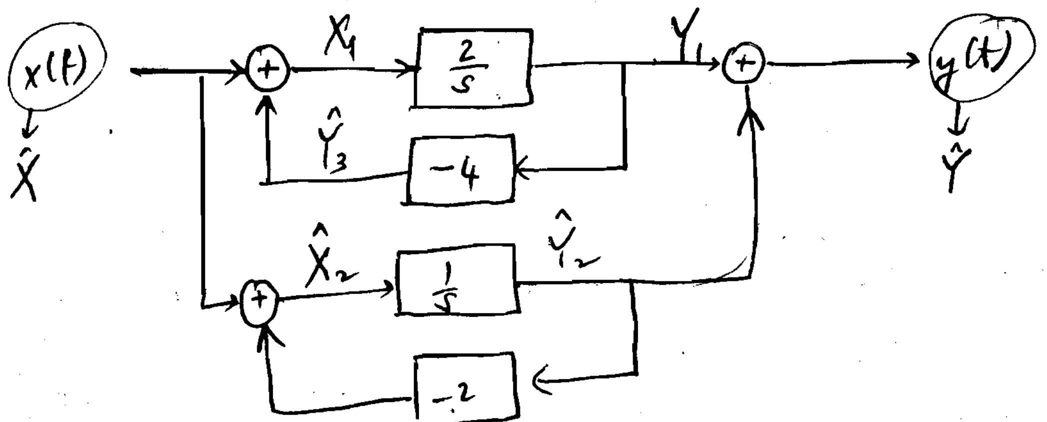
b) Sketch $\hat{I}$



Ch 9: Laplace & Signals & Systems.

HW9: 9.17, 9.24; 9.31; 9.34; 9.38; 9.42; 9.51
(optional)

9.17 .



D.E: relating x & y → find equation relating $\hat Y$ & $\hat X$, go back by doing inverse Laplace Transform using its properties

Equations: $\hat X_1 = \hat X + \hat Y_3 = \hat X - 4\hat Y_1$ ① → $\hat X_1 = \hat X - \frac{8}{s}\hat X_1$

$\hat X_2 = \hat X - 2\hat Y_2$ ② → $\hat X_2 = \hat X - \frac{2}{s}\hat X_2$

$$\left.\begin{aligned}\hat Y_1 &= \hat X_1 \frac{2}{s}\\ \hat Y_2 &= \hat X_2 \frac{1}{s}\end{aligned}\right\} \boxed{\hat Y = \hat Y_1 + \hat Y_2 = \hat X_1 \frac{2}{s} + \hat X_2 \frac{1}{s} = \left(\frac{\frac{2}{s}}{1+\frac{8}{s}} + \frac{\frac{1}{s}}{1+\frac{2}{s}}\right)\hat X}$$

① $\hat X_1\left(1 + \frac{8}{s}\right) = \hat X$ or $\hat X_1 = \frac{1}{1+\frac{8}{s}}\hat X$

② $\hat X_2\left(1 + \frac{2}{s}\right) = \hat X$ or $\hat X_2 = \frac{1}{1+\frac{2}{s}}\hat X$

$$\hat Y = \hat X\left(\frac{2}{s+8} + \frac{1}{s+2}\right) = \hat X\left(\frac{3s+12}{s^2+10s+16}\right)$$

→ $s^2\hat Y + 10s\hat Y + 16\hat Y = 3s\hat X + 12\hat X$

↓ $$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 16y = 3\frac{dx}{dt} + 12x .$$