%This code will generate a signal, add noise, show the Fourier transform, %then reconstruct the signal.

% Sinusoid generation
t=-3:0.1:3; %time series: -3,-2.9,...,0,...,2.9,3
time_freq=100; %period is 0.01s
f=sin(2*pi*freq*t); %frequency shift
figure(1), plot(t,f)
t1=-0.02:0.001:0.02; %there were not enough points for each period
fl=sin(2*pi*freq*t1); %frequency shift
figure(2), plot(t1,fl)
t2=-0.02:0.0001:0.02; %there were not yet enough points for each period, we get flat peaks
f2=sin(2*pi*freq*t2); %frequency shift
figure(3), plot(t2,f2)

% add 1000% noise or SNR=1:10
f2n=f2+10*randn(1,length(t2));
figure(4), plot(t2,f2n)

%show frequency spectrum
ff2n=fftshift(f2n);
figure(5), plot(abs(ff2n))

%do lowpass filter
band=floor(length(t2)/4)+10;
filt_one=ones(1,band);
filt_zero=zeros(1,length(t2)-band);
filt=[filt_one filt_zero];
f2n=f2n.*filt;
figure(6), plot(abs(f2n))
%show inverse Fourier Transform
iff2n=ifft(f2n);
figure(7), plot(real(iff2n))
Figure 1: 2 points for every 10 periods. Increment in time was 0.1s while the period was $\frac{1}{f_{\text{freq}}} = \frac{1}{100 \text{Hz}} = 0.01 \text{s}$.
Figure 2: 100 points for every 10 periods (10 points per period): decreased movement in time from 0.1s to 0.001s. Not enough to represent the peaks!

Also needed to narrow time interval from (3, 3) to (-0.02, 0.02) otherwise we would have such a dense graph that oscillations are not seen.
Figure 3: 100 points in every period by decreasing the time increment to 0.0001s.
Figure 4: added 1000 Hz noise to sinusoid
Figure 5: Fourier Transform of the signal plus 10% noise
Figure 6: A simple low-pass filter on Figure 5.
Figure 7: Inverse Fourier Transform of Figure 6.

Difference w.r.t. original signal plus noise (Figure 4):

1) Lower amplitude
2) Less dense: 6/1c of the elimination of higher frequencies by applying the low-pass in frequency-domain.
Figure 2: Simulation of 4 current traces

Second Matlab example
Figure 5: Magnetic field by the 4 current traces in Figure (2) at $z = 10 \mu m$.

These look blurred because of "noise" by deflecting the field at separation $z$. 
Figure 9: Remained current traces from the magnetic field at z = 10 μm. (Edges are much sharper than in Figure 4.)

"Filter" that convert the field back to the current that produces it. This filter is applied in frequency domain to the Fourier Transform of the magnetic field.
Signal: a function of time

\[ f(t) = \sin(2\pi ft); \quad \exp(-at) \]

\[ f[n] = \sin [2\pi fn]; \quad \exp[-an] \]

Discrete-time: time is given in increments of \( \Delta t \)

Properties:

1) Transformation of the time variable:

\[ t \rightarrow t' = at + b \]

scale factor \( \rightarrow \) shift

2) Periodicity (sinusoids are periodic); non-periodicity (exponentials are non-periodic)

Periodic signal: \( x(t + T) = x(t) \) (period is \( T \))

\[ x[n + N] = x[n] \] (period is \( N \))

What is the period for \( e^{j\omega t} \) ?

\[ e^{j\omega (t + T)} = e^{j\omega t} \]

or \( e^{j\omega T} = 1 \)

in general \( e^{j\theta} = \cos \theta + j \sin \theta \)

\[ \text{Real} \] \hspace{1cm} \text{Imag}

\[ \cos \theta = \text{Real} \]
\[ \sin \theta = \text{Imag} \]

\[ e^{j\omega T} = \cos(\omega T) + j \sin(\omega T) = 1 \rightarrow \sin \omega T = 0 \]

\[ \cos(\omega T) = 1 \rightarrow \omega T = 2\pi \rightarrow T = \frac{2\pi}{\omega} \]

\( \omega \) is the angular frequency.
\[
\begin{align*}
    y_1[n] &= 2x_1[n] + 4x_1[n-1] \\
    y_2[n] &= x_2[n-2] + \frac{1}{2} x_2[n-3] \quad \text{Input/Output difference equation}
\end{align*}
\]

Combine \( S_1 \) with \( S_2 \) in series: (Output to \( S_1 \) is input to \( S_2 \))

\[
\begin{align*}
    x &= x_1 \\
    y &= y_2
\end{align*}
\]

What is the input/output difference equation relating \( y[n] \) to \( x[n] \)?

- Substitution & manipulation:
  \[
  y_2[n] = x_2[n-2] + \frac{1}{2} x_2[n-3] = 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2} 2x_1[n-3] + \frac{1}{2} 4x_1[n-4] \\
  = 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]
  \]
Periodic signals:
\[ x(t+T) = x(t) \]
\[ x[n+N] = x[n] \]

Complex representation:
\[ \sin(\omega t) = \text{Im} [e^{j\omega t}] \]
\[ \cos(\omega t) = \text{Re} [e^{j\omega t}] \]

What is the period \( T \) for \( e^{j\frac{\pi}{3}t} \)?
\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{3}} = \frac{3}{2} \text{ s} \]

What is \( N \) for \( e^{j\frac{3\pi}{2}n} \)?

- Discrete-time:
  \[ t = nT = n \frac{3}{2} \text{ s} \]
  \[ N \text{ must be an integer} \]
  \[ e^{j\frac{3\pi(n+N)}{2}} = e^{j\frac{3\pi n}{2}} \cdot e^{j\frac{3\pi N}{2}} = 1 \]
  \[ \Rightarrow \frac{3\pi N}{2} = m \frac{2\pi}{3}, \ m \text{ integer} \]
  \[ N = \frac{m \frac{2\pi}{3}}{3} = \frac{m \pi}{3} \]

The first time \( N \) is an integer is when \( m = 3 \) \( \Rightarrow N = 2 \)

What is \( T \) for \( e^{j\frac{\pi}{3}t} \)?
\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{3}} = \frac{3}{2} \text{ s} \]

What is \( N \) for \( e^{j\frac{3\pi}{2}n} \)?

\[ 3N = m \frac{2\pi}{3}, \ m \text{ integer} \]
\[ N = m \frac{2\pi}{9} \]
No period b/c \( \pi \)

Conclusion:
- Different from continuous and discrete-time signals:
  \( e^{j\omega t} \) is always periodic (for all \( \omega \)) but \( e^{j\omega n} \) is not periodic if \( \omega n \) contains no \( \pi \) in it.
Functions that are useful to represent signals:

- Unit impulse: \( \delta(t) \)
  \[
  \delta(t) = \lim_{\epsilon \to 0} \frac{\epsilon}{t}
  \]

- Unit step function: \( u(t) \)

Integrals involving \( \delta(t) \):

\[
\int_{-\infty}^{\infty} f(t) \delta(t) \, dt = f(0)
\]

(centre of \( \delta(t) \))

\[
\int_{-\infty}^{\infty} f(t) \delta(t-a) \, dt = f(a)
\]

(centre of \( \delta(t) \))

Questions:

\[
\int_{-3}^{3} f(t) \delta(t) \, dt = f(0)
\]

\[
\int_{-3}^{3} f(t) \delta(t-4) \, dt = 0
\]
Systems: represented as a box whose content we don't know.

\[ x(t) \xrightarrow{S} y(t) \]

It is identified by the equation \( y(t) \) with which it relates \( y(t) \) to \( x(t) \)

e.g. \( y(t) = 2x(t) - x^2(t+1) \)

Properties:
1) Memory (current outputs depend on past inputs)
   (e.g. a table of food)
2) Invertibility \( S \) and \( S^{-1} \)

\[ x(t) \xrightarrow{S} y(t) \xrightarrow{S^{-1}} x(t) \]

3) Causality (if output just depends on current and/or past inputs)
4) Stability (if small inputs would not produce extremely large outputs)
5) Time-invariance: same input gets same output regardless of the time it happens.
   (e.g. gravity)
6) Linearity
   \[ x \xrightarrow{\text{Linear}} y \]
   \[ ax \xrightarrow{\text{Linear}} ay \]
   \[ x_1 + x_2 \xrightarrow{\text{Linear}} y_1 + y_2 \]

Non-linear systems:
\[ x \xrightarrow{S} y(t) = x^2(t) \]
\[ x_1 + x_2 \xrightarrow{S} y_1 + y_2 \]
\[ ax \xrightarrow{\text{Non-linear}} a^2y \]
\[ x_1 + x_2 \xrightarrow{\text{Non-linear}} x_1^2 + x_2^2 + 2x_1x_2 \]
Another non-linear system:

\[ y = \sin(x(t)) \]

\[ \sin(ax) \neq a \sin(x) \]

(1.10) Fundamental period of: 

\[ x(t) = 2 \cos \left( \frac{10(t+1)}{4} - \frac{\sin(4t-1)}{4} \right) \]

\[ 10T_1 = n_1 \pi \]

\[ T_1 = n_1 \frac{2\pi}{10} = \left\{ \frac{2\pi}{10}, \frac{4\pi}{10}, \frac{6\pi}{10}, \frac{8\pi}{10}, \pi, \frac{12\pi}{10}, \ldots \right\} \]

\[ 4T_2 = n_2 \frac{2\pi}{4} \]

\[ T_2 = n_2 \frac{2\pi}{4} = \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots \right\} \]

The first time both signals repeat at the same time is when 

\[ T_1 = T_2 = \frac{\pi}{2} \rightarrow \text{fundamental period of } x(t) \]

(1.11) Fundamental period of: 

\[ x(u) = 1 + e^{\frac{j2\pi u}{7}} - e^{\frac{j2\pi u}{5}} \]

\[ N_1 = \frac{7}{2} n_1 = \left\{ 7, 14, 21, 28, 35, 42, \ldots \right\} \]

\[ N_2 = 5 n_2 = \left\{ 5, 10, 15, 20, 25, 30, 35, 40, \ldots \right\} \]

\[ N_1 = N_2 = 35 \]

Both signals #1 & #2 need to come back to same value
for the combined signal to have its first period!

HW due next Tuesday: 9/19.
original signal

signal with 1000% noise added

Fourier Transform of signal with 1000% noise added

Low-pass filtered used to eliminate noise (bandwidth=12)
% This code will generate a signal, add noise, show the Fourier transform, 
% then reconstruct the signal.

% Sinusoid generation
% t=-3:0.1:3; % time series: -3,-2.9,...,0,...,2.9,3
freq=100; % period is 0.01s
f=sin(2*pi*freq*t);
figure(1),plot(t,f)
tl=-0.02:0.001:0.02;%there were not enough points for each period
fl=sin(2*pi*freq*tl);
figure (2), plot(tl,fl)
t2=-0.02:0.0001:0.02;%there were not yet enough points for each period, we get flat peaks
f2=sin(2*pi*freq*t2);
figure(3), plot(t2,f2)

% add 100% noise or SNR=1:10
f2n=f2+10*randn(1,length(t2));
figure(4), plot(t2,f2n)

% show frequency spectrum
ff2n=fftshift(f2n);
figure(5), plot(abs(ff2n))

% do lowpass filter
band=floor(length(t2)/128)+9;%floor(length(t2)/4)+10;
filt_one=ones(1,band);
filt_zero=zeros(1,length(t2)-band);
filt=[filt_one filt_zero];
ff2n=ff2n.*filt;
figure(6), plot(abs(ff2n))
% show inverse Fourier Transform
iff2n=ifft(ff2n);
figure(7),plot(real(iff2n))

reconstructed signal
a) \( y(t) = x(t-2) + x(2-t) \)  

1) Memory : Yes  
2) Time-invariance : 
   \[ x(t) \rightarrow x(t+\alpha) \]
   \[ x((t+\alpha)-2) + x(2-(t+\alpha)) = y(t+\alpha) \]
3) Linear : Yes  
   Some input/output relationship for all time shifts.
4) Causal : \( t = 0 \) 
   \[ y(0) = x(-2) + x(2) \]
   Causal only for \( t \geq 2 \)!
5) Stable ? \( \frac{2}{4} \)
   \[ y(2) = x(0) + x(0) \]
   Noncausal for \( t < 2 \)
6) \( y(t) = x\left(\frac{t}{3}\right) \)  

1) Memory : \( y(6) = x(2) \) Yes!  
2) Time-invariance : 
   \[ t \rightarrow t+3 \]
   \[ x\left(\frac{t}{3}\right) \rightarrow x\left(\frac{t+3}{3}\right) = x\left(\frac{t}{3} + 1\right) \]
3) Linear : Yes
4) Causal : Yes \( \left(\frac{t}{3} < t\right) \)
5) Stable : Yes

\[ y(30s) = x(10s) \]  
Output of 30s is  
Input at 10s  

\[ +60s \]  
Output of 90s is  
Input at 30s  

Shift of 60s in output corresponds with a shift of 20s in input (not time invariant).