Ch 1: Intro to Signals & Systems

Matlab example:


```
%This code will generate a signal, add noise, show the Fourier transform,
%then reconstruct the signal.

% Sinusoid generation
t=-3:0.1:3; %time series: -3,-2.9,....0,....2.9,3
freq=100; %period is 0.01s
f=sin(2*pi*freq*t);
figure(1),plot(t,f)
t1=-0.02:0.001:0.02;%there were not enough points for each period
f1=sin(2*pi*freq*t1);
figure(2), plot(t1,f1)
t2=-0.02:0.0001:0.02;%there were not yet enough points for each period, we get flat peaks
f2=sin(2*pi*freq*t2);
figure(3), plot(t2,f2)

%add 1000% noise or SNR=1:10
f2n=f2+10*randn(1,length(t2));
figure(4), plot(t2,f2n)

%show frequency spectrum
ff2n=fftshift(f2n);
figure(5), plot(abs(ff2n))

%do lowpass filter
band=floor(length(t2)/4)+10;
filt_one=ones(1,band);
filt_zero=zeros(1,length(t2)-band);
filt=[filt_one filt_zero];
ff2n=ff2n.*filt;
figure(6), plot(abs(ff2n))
%show inverse Fourier Transform
iff2n=ifft(ff2n);
figure(7) ,plot(real(iff2n))
```

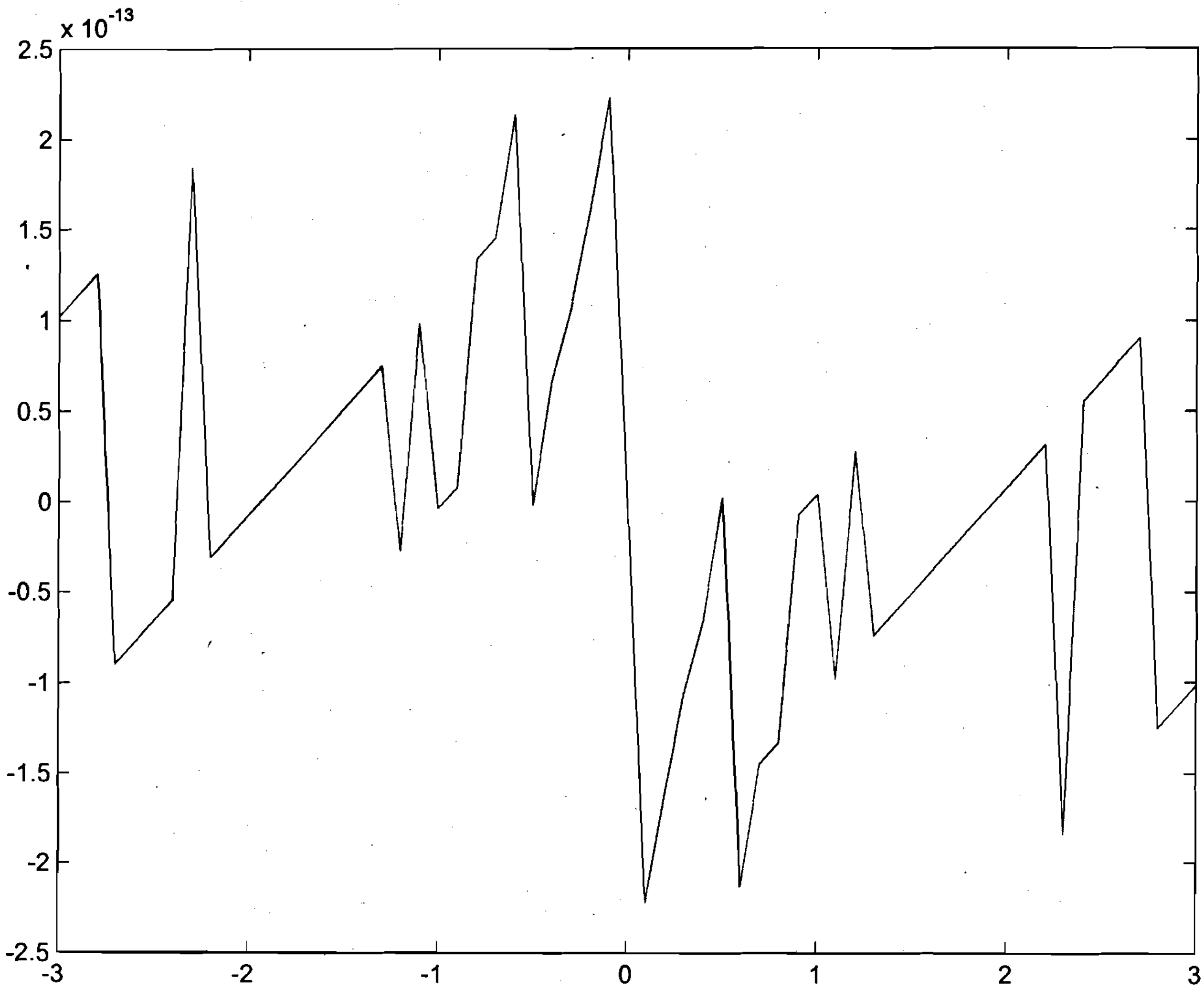


Figure 1: 2 points for every 10 periods: increment in time was 0.1s while the period was $\frac{1}{freq} = \frac{1}{100 Hz} = 0.01s$

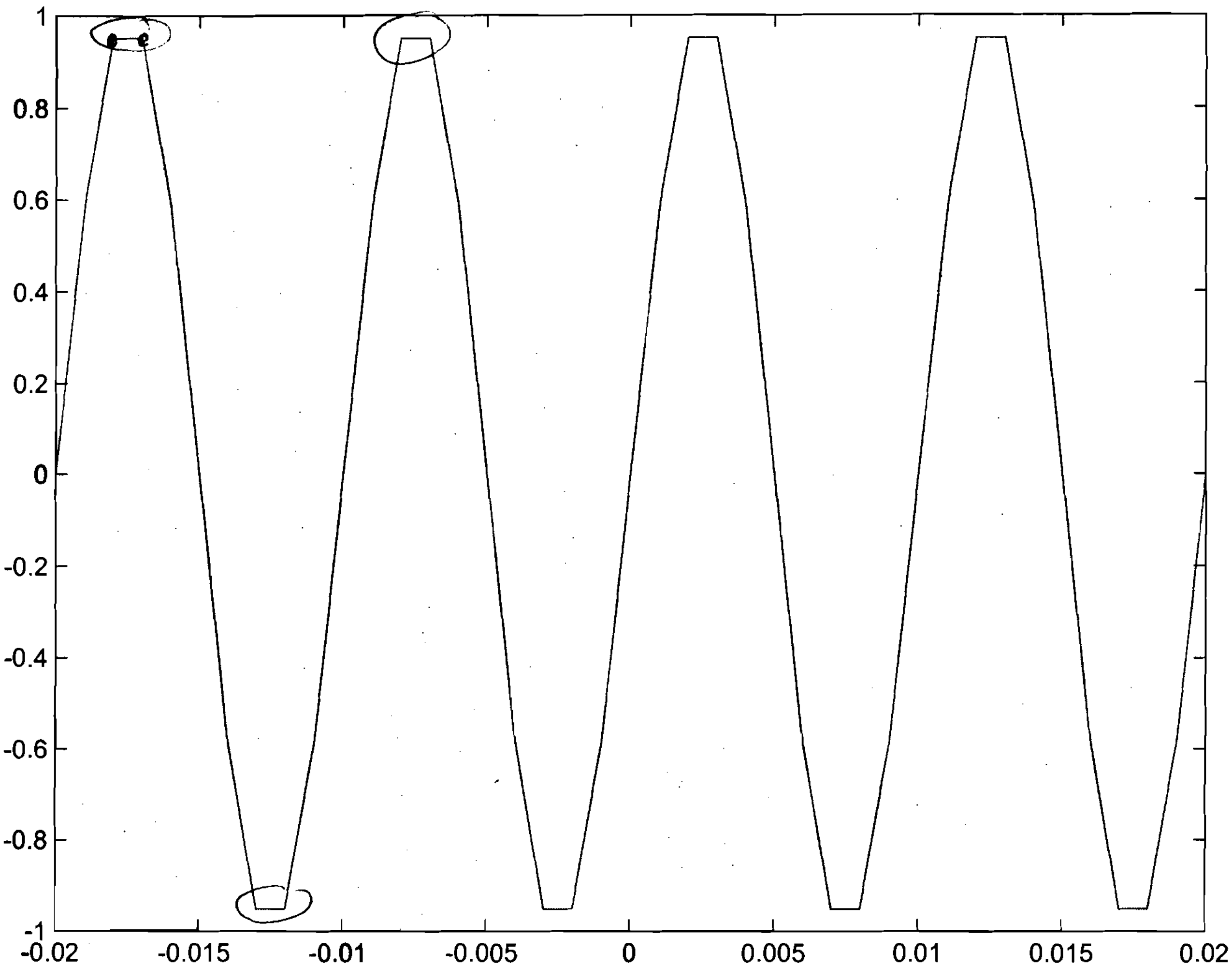


Figure 2: 100 points for every 10 periods (10 points per period):
decreased increment in time from 0.1s to 0.001s
Not enough to represent the peaks!
Also needed to narrow time interval from (-3,3) to (-0.02, 0.02)
otherwise we would have such a dense graph that oscillations are not seen.

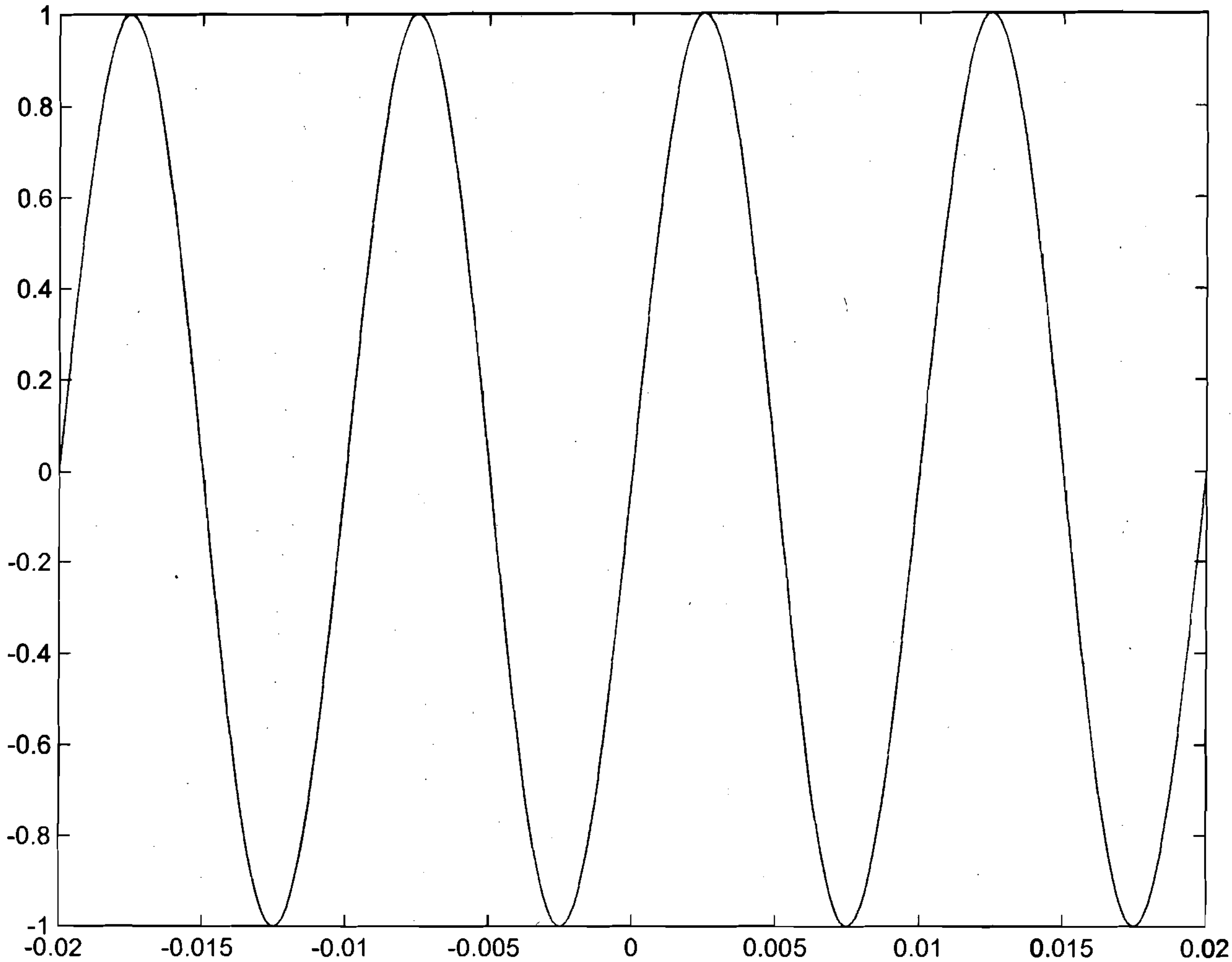


Figure 3: 100 points in every period by decreasing the time increment to 0.0001s.

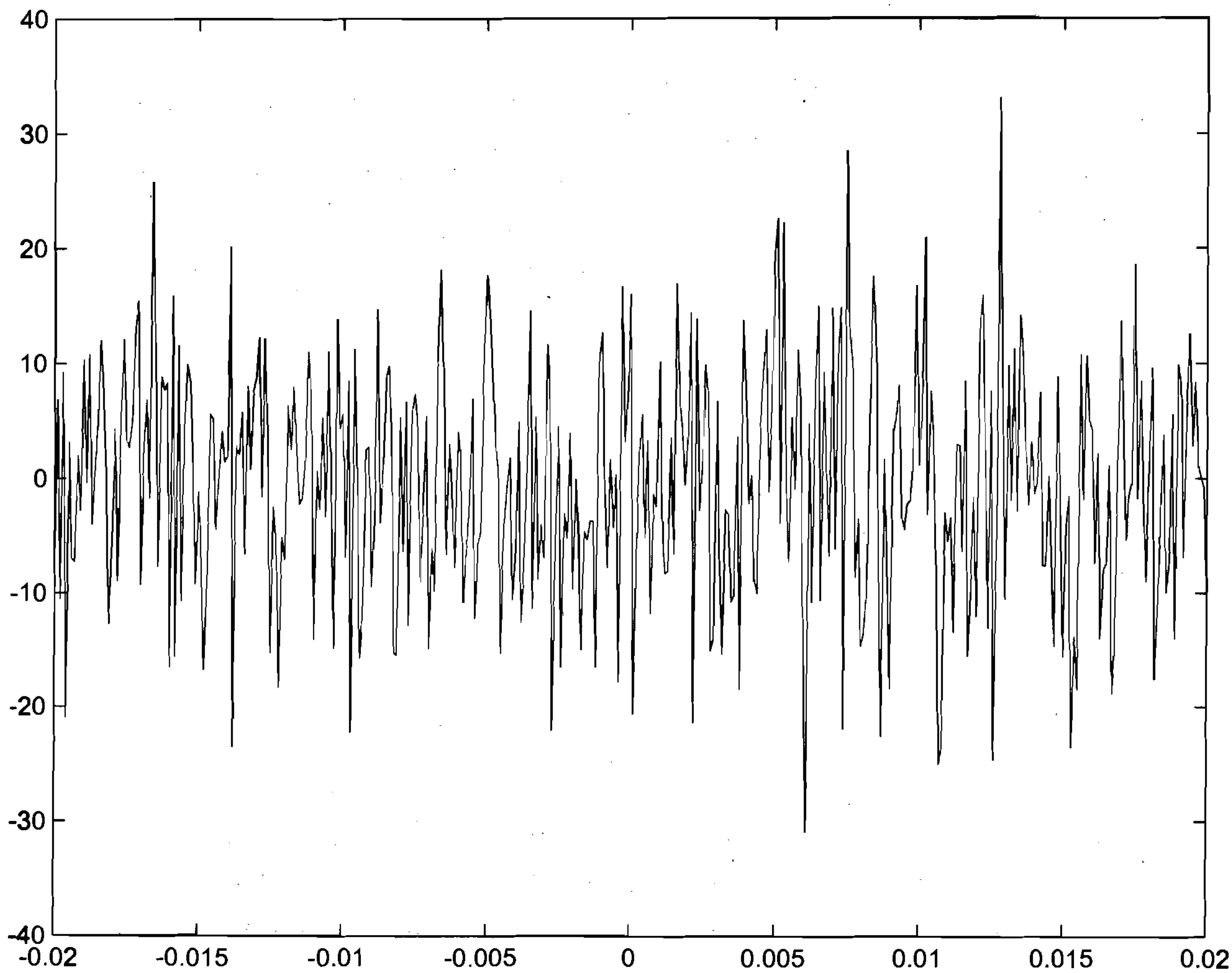


Figure 4: added 1000% noise to sinusoid

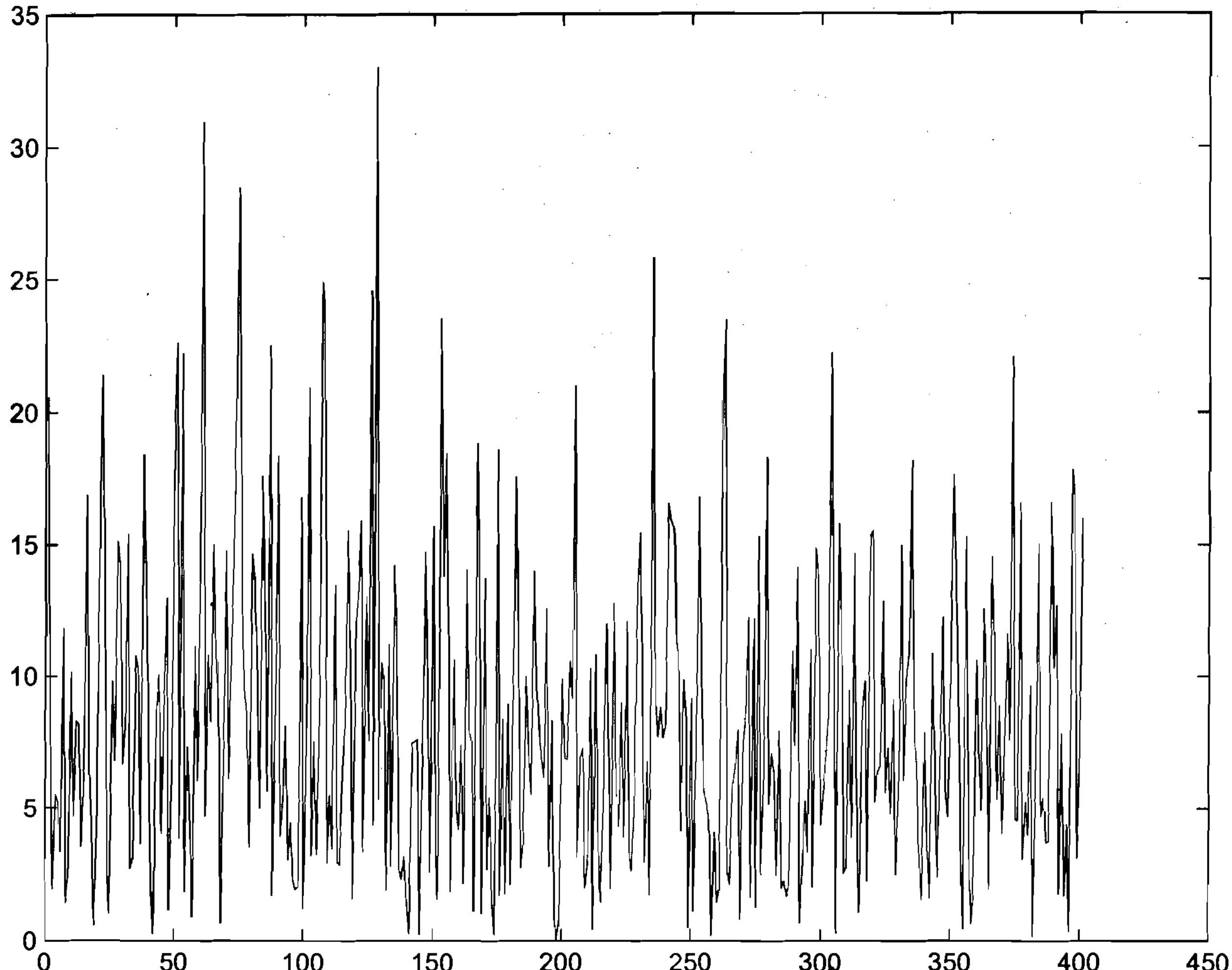


Figure 5: Fourier Transform of the signal plus 1000% noise

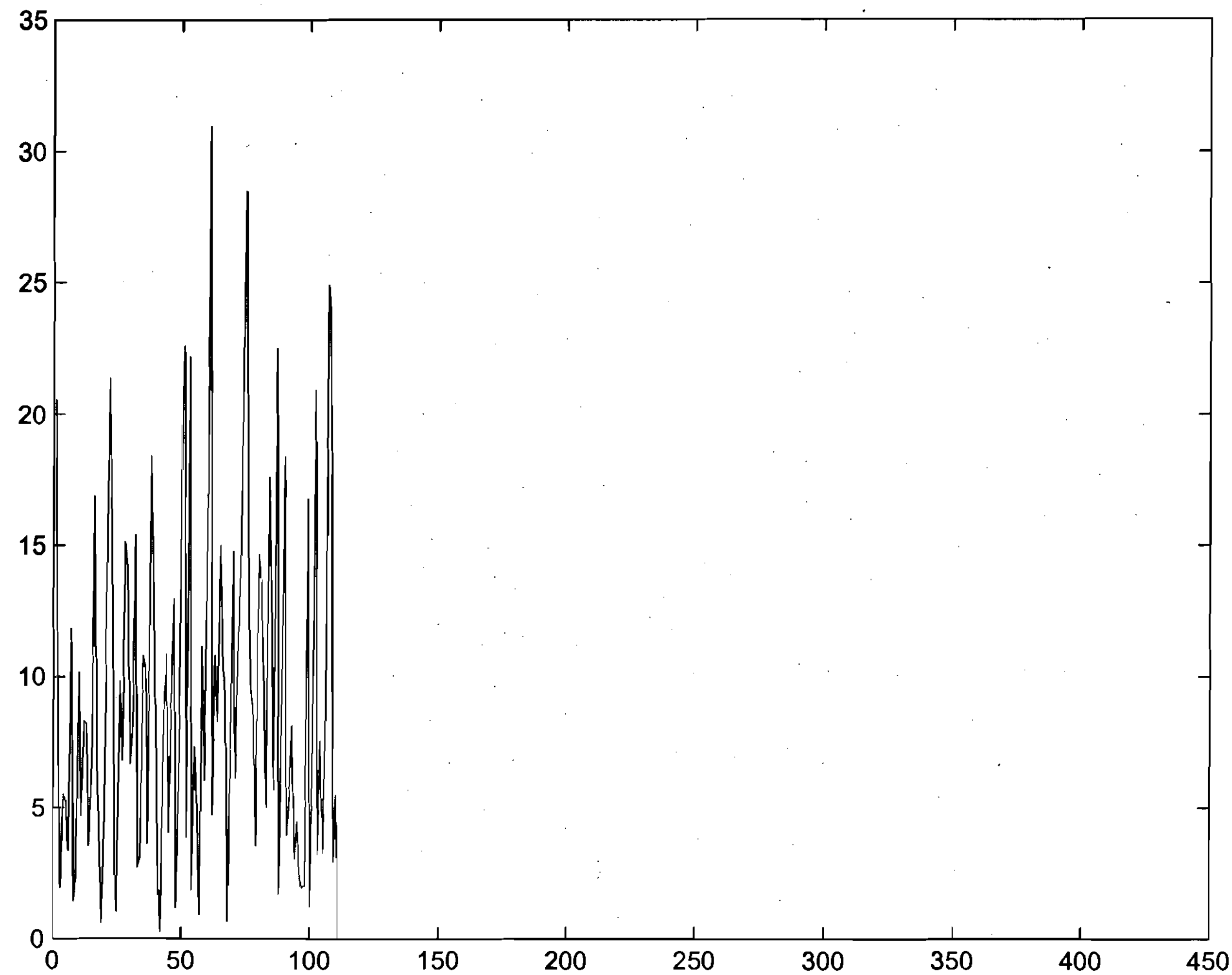


Figure 6: A simple Low-pass filter on Figure 5.

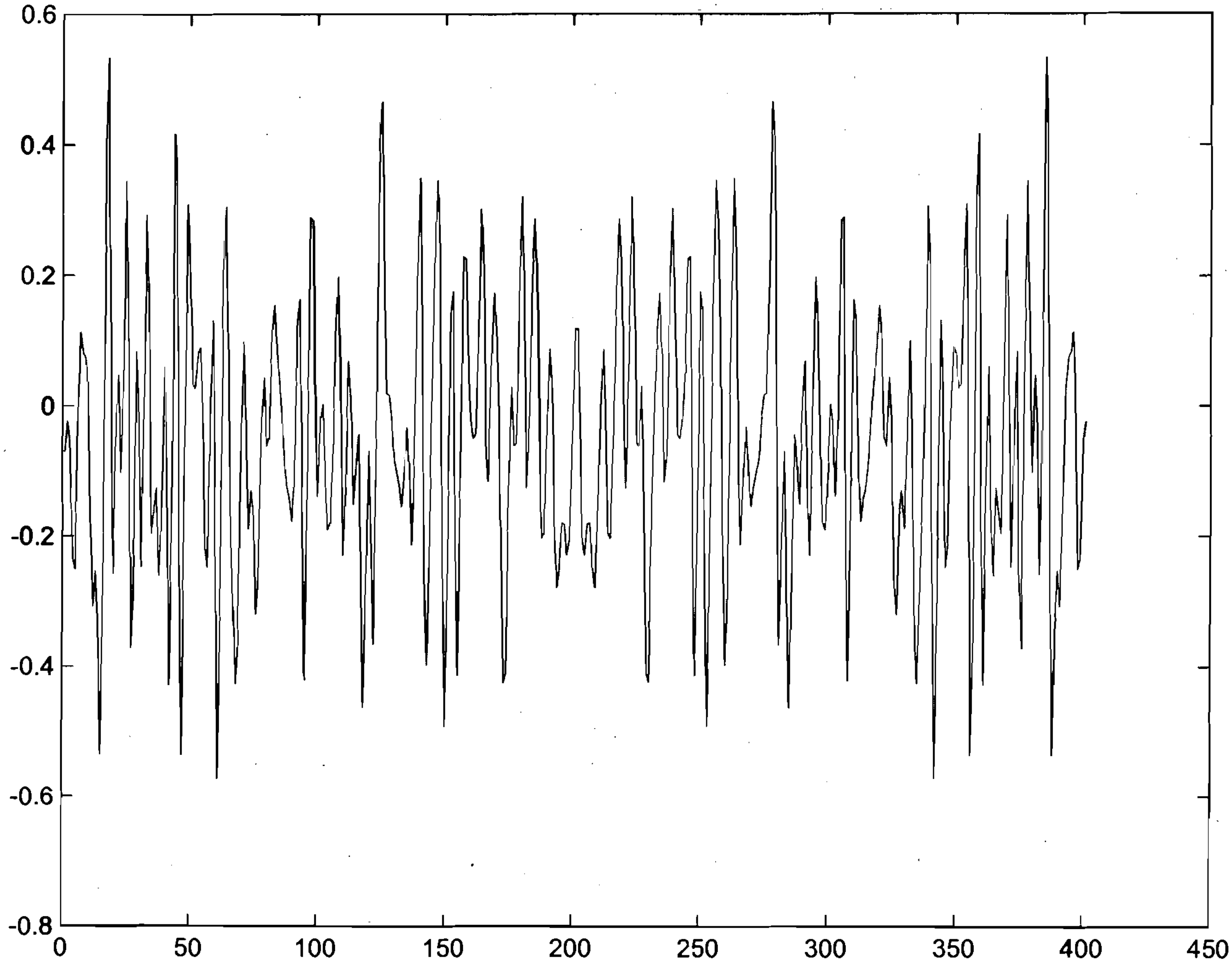


Figure 7: inverse Fourier Transform of Figure 6.

Differences w.r.t. original signal plus noise (Figure 4):

1) Lower amplitude
2) Less dense: b/c of the elimination of higher frequencies by applying the Low-pass in frequency-domain.

Second Matlab example:

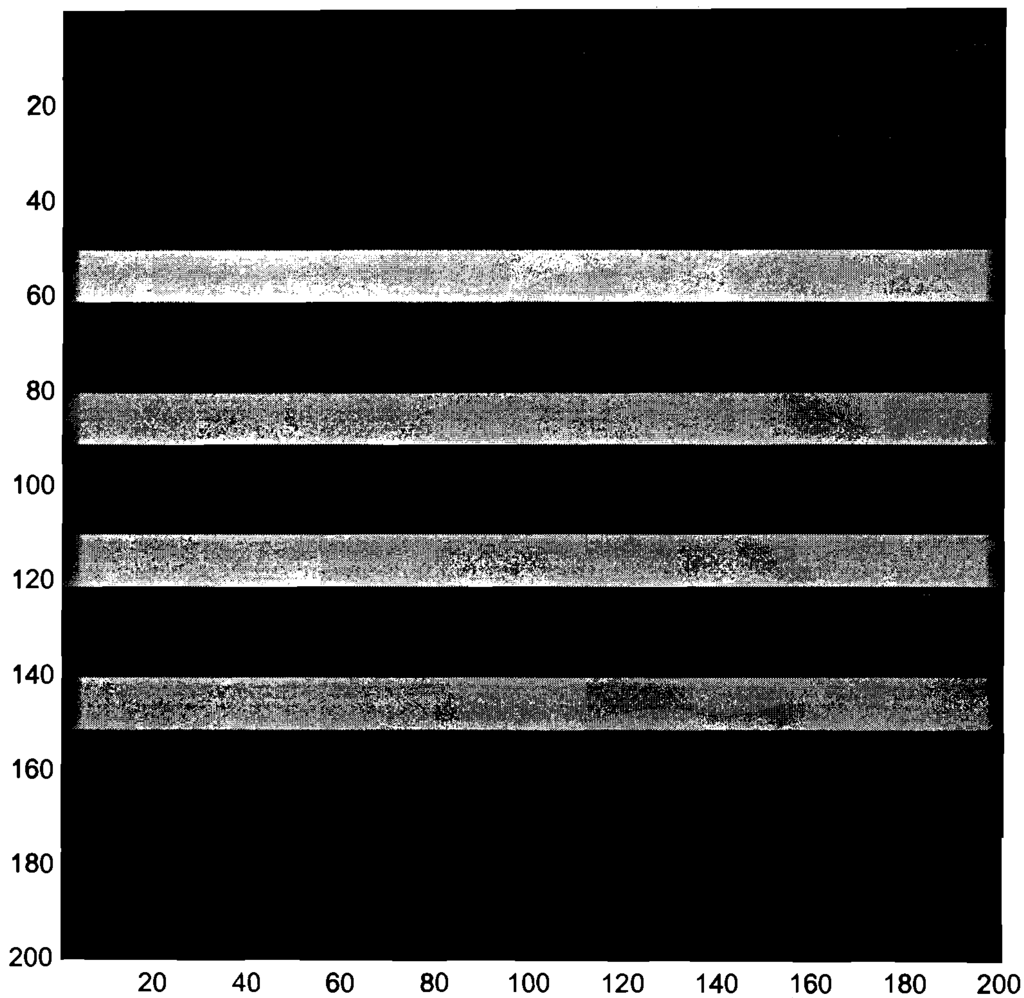


Figure 2: Simulation of 4 current traces

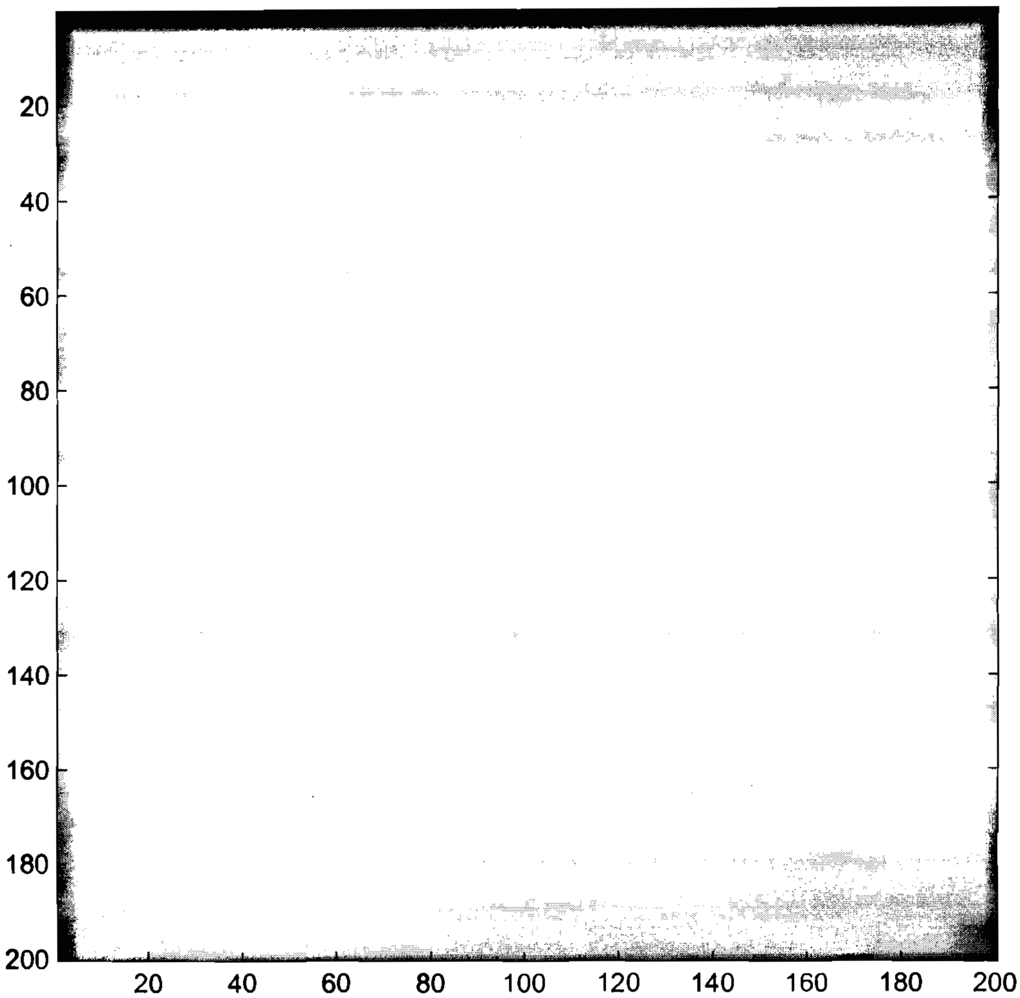


Figure 5: Magnetic field by the 4 current traces in Figure (2) at $z = 10\,\mu m$.

These look blurred because of "noise" by detecting the field at separation z.

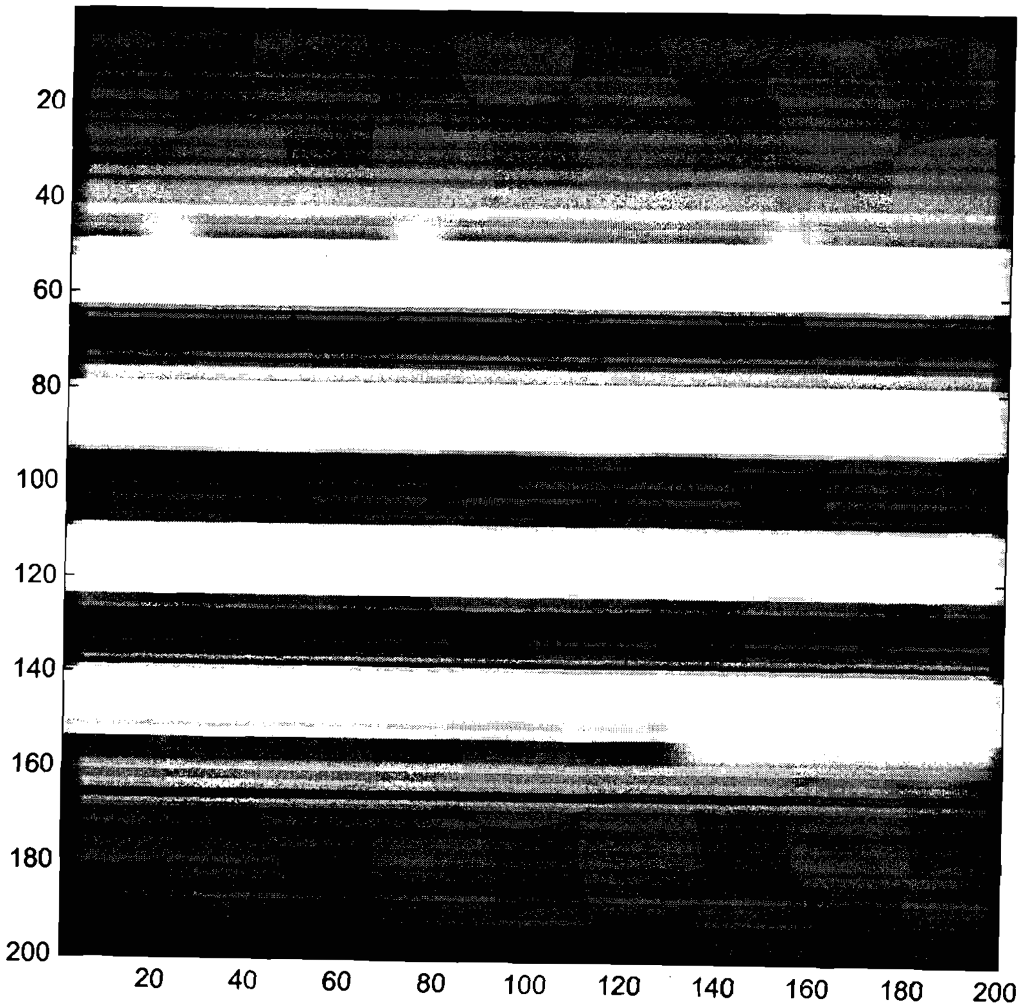


Figure 9: Recovered current traces from the magnetic field at $z = 10\ \mu m$. (Edges are much sharper than in Figure 4)

"Filter" that connect the field back to the current that produces it. This filter is applied in frequency-domain to the Fourier Transform of the magnetic field!

continuous-time

Signal: a function of time
$$\begin{cases} f(t) = \sin(2\pi f t);\ \exp(-at) \\ f[n] = \sin[2\pi f n];\ \exp[-an] \end{cases}$$

discrete-time : time is given in increments of Δt

$t = n \underbrace{\Delta t}_{\text{increment}}$

properties:

1) Transformation of the time variable:

$$t \rightarrow t' = \underbrace{a}_{\text{scaling factor}} t + \underbrace{b}_{\text{shift}}$$

2) Periodicity (sinusoids are periodic); non-periodicity (exponentials are non-periodic)

Periodic signal: $x(t+T) = x(t)$ (period is T)

$x[n+N] = x[n]$ (period is N)

→ What is the period for $e^{j\omega t}$?

$$e^{j\omega(t+T)} = e^{j\omega t}$$

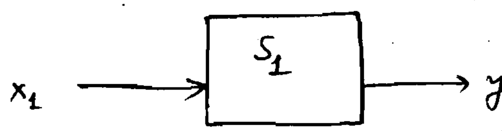
or $e^{j\omega T} = 1$

In general $e^{j\theta} = \underbrace{\cos\theta}_{\text{Real}} + j\underbrace{\sin\theta}_{\text{Imaginary}}$ $\begin{cases} \text{Real}(e^{j\theta}) = \cos\theta \\ \text{Imag}(e^{j\theta}) = \sin\theta \end{cases}$

$$e^{j\omega T} = \cos(\omega T) + j\sin(\omega T) = 1 \rightarrow \begin{cases} \sin\omega T = 0 \\ \& \cos\omega T = 1 \end{cases}$$

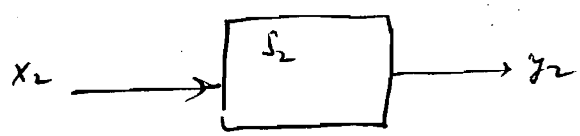
$\cos(\omega T) = 1 \rightarrow \omega T = 2\pi \rightarrow \boxed{T = \frac{2\pi}{\omega}}$

1.15



System (time-discrete)

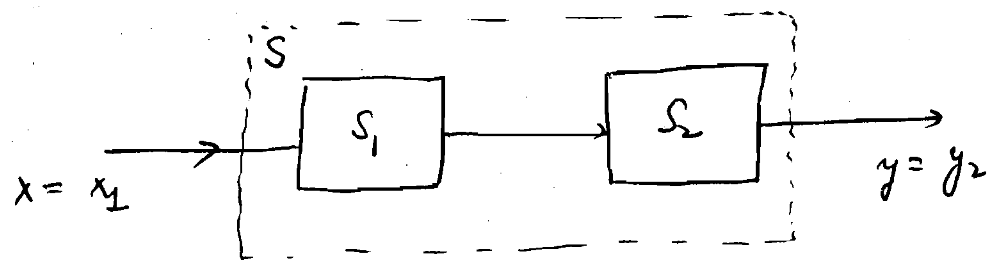
$$y_1[n] = 2x_1[n] + 4x_1[n-1]$$



$$y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$$

Input/Output difference equation

Combine S_1 with S_2 in series: (output to S_1 is input to S_2)



What is the input/output difference equation relating $y[n]$ to $x[n]$?

– Substitution & manipulation: $x_2[n] \to y_1[n]$

$$y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3] = 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}2x_1[n-3] + \frac{1}{2}4x_1[n-4]$$

$$= \cancel{2x\,n\,f}\ 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4]$$

Periodic signals : $\begin{cases} x(t+T) = x(t) \\ x[n+N] = x[n] \end{cases}$

Complex representation : $\sin(\omega t) = \underset{\substack{\uparrow \\ \text{imaginary part}}}{Im}[e^{j\omega t}]$

$$\cos(\omega t) = Re[e^{j\omega t}]$$

→ What is the period T for $e^{j3\pi t}$? $T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3}$ s.

→ " " " N for $e^{j3\pi n}$?

- Discrete-time : $t = n\Delta t$; n are integers → N has to be an integer.
- $e^{j3\pi(n+N)} = e^{j3\pi n} \cdot e^{j3\pi N} = 1$

$$\Rightarrow 3\pi N = m\,2\pi \text{ ; m integer}$$

$$\rightarrow N = \frac{m\,2\cancel{\pi}}{3\cancel{\pi}} = m\frac{2}{3}$$

The first time N is an integer is when $m = 3 \rightarrow \boxed{N = 2}$

→ e^{j3t} , what is T ? $T = \frac{2\pi}{3}$ s

→ e^{j3n} , what is N ?

$3N = m\,2\pi$, m integer

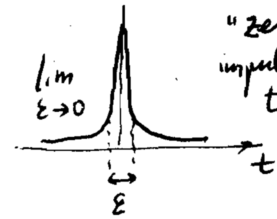
$N = m\,\frac{2\pi}{3}$ No period b/c π

Conclusion: 1st difference b/w continuous and discrete-time signals: $e^{j\omega t}$ is always periodic (for all ω) but $e^{j\omega n}$ is not periodic if ω ~~too~~ contains no π in it.

Functions that are useful to represent signals:

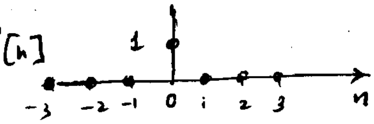
→ Unit impulse: δ "delta"

$\delta(t)$



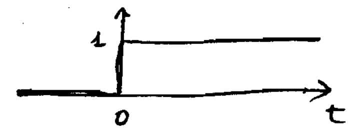
"zero-width" impulse centered at $t=0$

$\delta[n]$



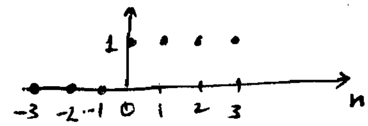
$$\delta(t) = \frac{du}{dt}$$

$u(t)$:



→ Unit step function: u

$u[n]$



Integrals involving $\delta(t)$:

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

center of $\delta(t)$

$$\int_{-\infty}^{\infty} f(t)\,\underbrace{\delta(t-a)}_{\text{center at } a}\,dt = f(a)$$

Questions:

$$\int_{-3}^{3} f(t)\,\delta(t)\,dt = f(0)$$

$$\int_{-3}^{3} f(t)\,\delta(t-4)\,dt = 0$$

Systems: represented as a box whose content we don't know

$$\underset{\text{input}}{x(t)} \longrightarrow \boxed{\quad S \quad} \longrightarrow \underset{\text{output}}{y(t)}$$

It is identified by the equation with which it relates $y(t)$ to $x(t)$

e.g. $\quad y(t) = 2x(t) - x^2(t+1)$

Properties:

1) Memory (current outputs depend on past inputs) (eg. a table of food)

2) Invertibility S and S^{-1}

$$x(t) \longrightarrow \underset{\text{system}}{\boxed{S}} \longrightarrow \underset{\text{inverse system.}}{\boxed{S^{-1}}} \longrightarrow x(t)$$

3) Causality (if output just depends on current and/or past inputs)

4) Stability (if small inputs would not produce extremely large outputs)

5) Time-invariance: same input gets same output regardless of the time it happens!

e.g. gravity

6) Linearity.

$$x \longrightarrow \boxed{\text{Linear } S} \longrightarrow y$$

$$ax \qquad\qquad ay$$

$$x_1 + x_2 \longrightarrow y + y_2$$

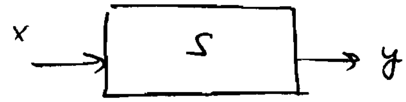
Non-linear systems: $\quad x \longrightarrow \boxed{S} \longrightarrow y$

$$y(t) = x^2(t)$$

$$ax \qquad\qquad a^2 \cdot y$$

$$x_1 + x_2 \qquad\qquad x_1^2 + x_2^2 + \underbrace{2x_1x_2}$$

Another non-linear system:



$$y = \sin(x(t))$$

$$ax \qquad\qquad \sin(ax) \neq a \sin x$$

(1.10) Fundamental period of: $x(t) = 2\cos \underbrace{(10t+1)}_{\#1} - \underbrace{\sin(4t-1)}_{\#2}$

$$10T_1 = n_1 2\pi$$

$$T_1 = n_1 \frac{2\pi}{10} = \left\{ \frac{2\pi}{10}, \frac{4\pi}{10}, \frac{6\pi}{10}, \frac{8\pi}{10}, \pi, \frac{12\pi}{10}, \dots \right\}$$

$$4T_2 = n_2 2\pi$$

$$T_2 = n_2 \frac{2\pi}{4} = \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots \right\}$$

The first time both signals repeat at the same time is when $T_1 = T_2 \doteq \pi \rightarrow$ fundamental period of $x(t)$.

(1.11) Fundamental period of: $x[n] = 1 + \underbrace{e^{j\frac{4\pi n}{7}}}_{\#1} - \underbrace{e^{j\frac{2\pi n}{5}}}_{\#2}$

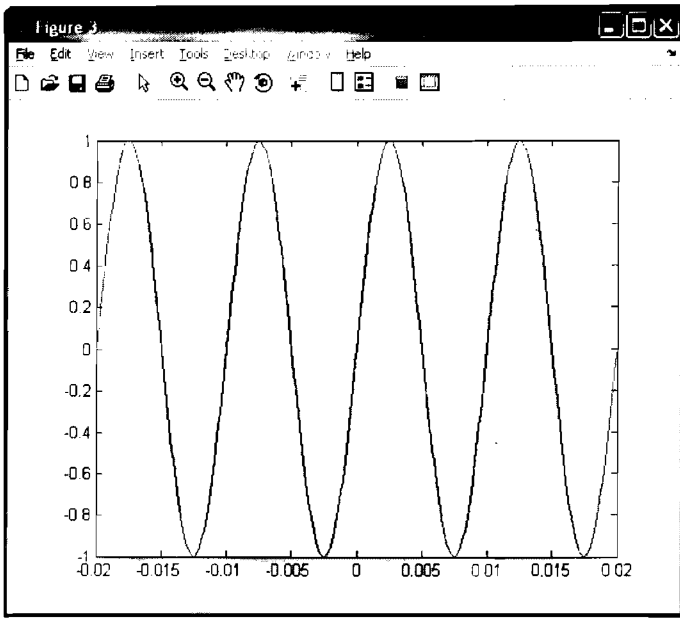
$$N_1 = \frac{7}{2} n_1 = \{7, 14, 21, 28, 35, 42 \dots\}$$

$$N_2 = 5 n_2 = \{5, 10, 15, 20, 25, 30, 35, 40 \dots\}$$

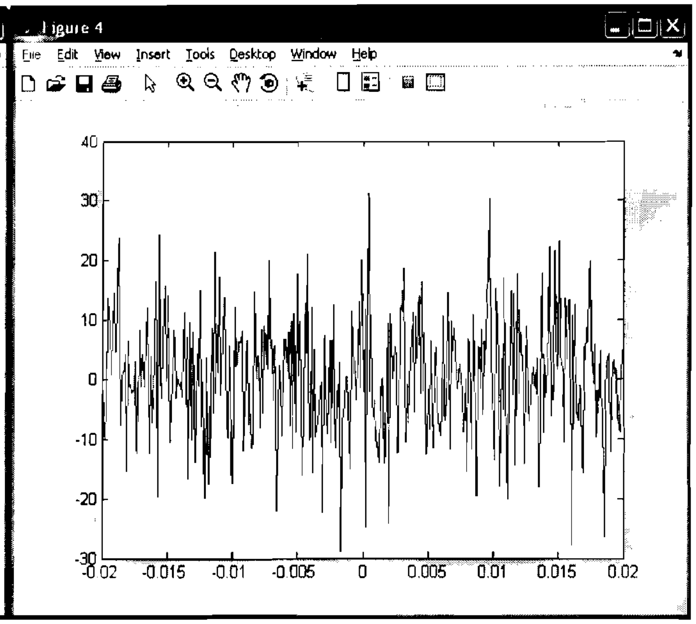
$$N_1 = N_2 = 35.$$

→ Both signals #1 & #2 need to come back to same value for the combined signal to have its first period!

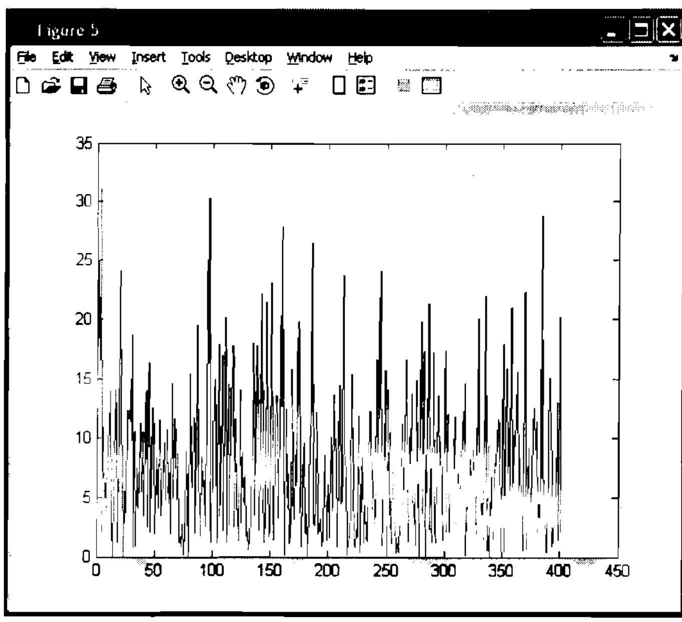
→ HW1 due next Tuesday: 9/19.



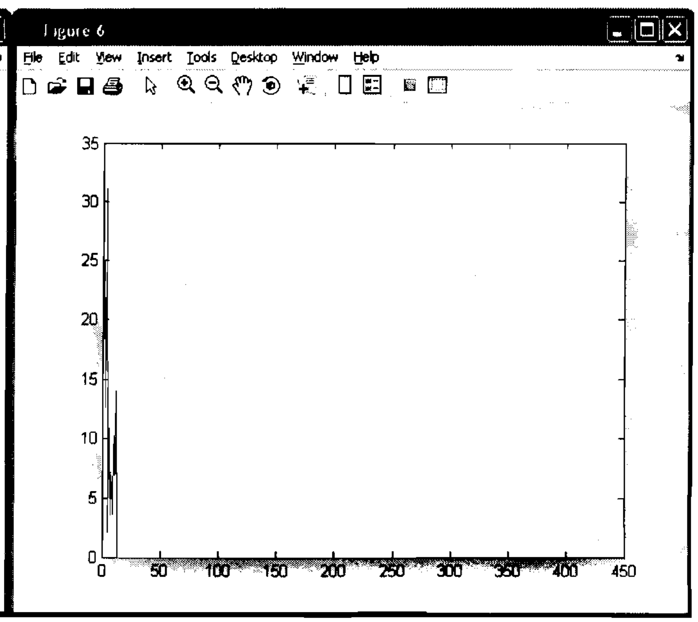
original signal



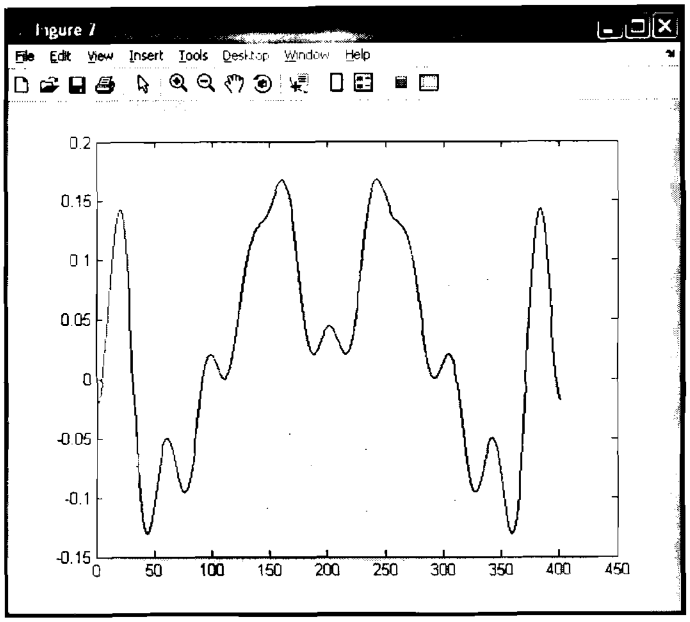
signal with 1000% noise added



Fourier Transform of signal with 1000% noise added



Low-pass filtered used to eliminate noise (bandwidth=12)



reconstructed signal

```
%This code will generate a signal, add noise, show the Fourier transform,
%then reconstruct the signal.

% Sinusoid generation
t=-3:0.1:3; %time series: -3,-2.9,....0,....2.9,3
freq=100; %period is 0.01s
f=sin(2*pi*freq*t);
figure(1),plot(t,f)
t1=-0.02:0.001:0.02;%there were not enough points for each period
f1=sin(2*pi*freq*t1);
figure(2), plot(t1,f1)
t2=-0.02:0.0001:0.02;%there were not yet enough points for each period, we get flat peaks
f2=sin(2*pi*freq*t2);
figure(3), plot(t2,f2)

%add 1000% noise or SNR=1:10
f2n=f2+10*randn(1,length(t2));
figure(4), plot(t2,f2n)

%show frequency spectrum
ff2n=fftshift(f2n);
figure(5), plot(abs(ff2n))

%do lowpass filter
band=floor(length(t2)/128)+9;%floor(length(t2)/4)+10;
filt_one=ones(1,band);
filt_zero=zeros(1,length(t2)-band);
filt=[filt_one filt_zero];
ff2n=ff2n.*filt;
figure(6), plot(abs(ff2n))
%show inverse Fourier Transform
iff2n=ifft(ff2n);
figure(7) ,plot(real(iff2n))
```

1.27 a) $y(t) = x(t-2) + x(2-t)$ System

1) Memory ? Yes.

2) Time-invariance ? Yes
$$\left\{ \begin{array}{l} x(t) \longrightarrow x(t+a) \\ x(t+a-2) + x(\underbrace{2-a-t}_{2-(t+a)}) = y(t+a) \\ \text{same input/output relationship for all time shifts.} \end{array} \right.$$

3) linear ? Yes

4) Causal ? $t=0$
$$\left\{ \begin{array}{l} y(0) = x(-2) + x(2) \\ y(1) = x(-1) + x(1) \\ y(2) = x(0) + x(0) \\ y(3) = x(1) + x(-1) \end{array} \right\}$$
Causal only for $t \geq 2$! Noncausal for $t<2$

5) Stable ? Yes.

f) $y(t) = x\left(\frac{t}{3}\right)$: System.

1) Memory - $y(6) = x(2)$ Yes!

2) Time-inv. $t \rightarrow t+3$

$$x\left(\frac{t}{3}\right) \rightarrow x\left(\frac{t+3}{3}\right) = x\left(\frac{t}{3}+1\right)$$

$$y(t+3)$$

$t = 30s$: $y(30s) = x(10s)$ (output at 30s is input at 10s)

+60s +20s

$t \rightarrow t + 60s = 90s$. $y(90s) = x(30s)$ (output at 90s is input at 30s)

Shift of 60s in output corresponds with a shift of 20s in input (not time invariant).

3) linear : Yes.

4) Causal . Yes $\left(\frac{t}{3} < t\right)$

5) Stable : Yes.