3.28c

$x[n]$ periodic, $N=4$

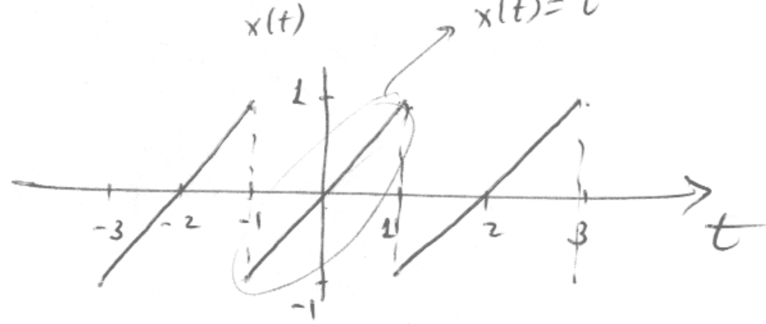
$x[n] = 1 - \sin\frac{\pi n}{4}$ for $0 \le n \le 3$

$$a_k = \frac{1}{4}\Big(\underset{n=0}{1} + \underset{n=1}{\left(1-\frac{1}{\sqrt{2}}\right) e^{-jk\frac{2\pi}{4}}} + \underset{n=2}{0} + \underset{n=3}{2e^{-jk\frac{3\pi}{2}}} \Big)$$

3.28d

(38)

3.22 a) P.3.22a)



$$a_k = \frac{1}{T}\int_T dt\, e^{-jk\omega_0 t}\, x(t)$$

choose one period.

$$\frac{2\pi}{T} = \pi$$

$$a_k = \frac{1}{2}\underbrace{\int_{-1}^{1} dt\, e^{-jk\pi t}\, t}_{-\frac{d}{d(\pi k)}\int_{-1}^{1} dt\, e^{-jk\pi t}}$$

Ch 4 Continuous-time Fourier Transform

HW 4: 4.10; 4.14; 4.19; 4.23; 4.26; 4.28;
→ 4.34; 4.44; 4.46; 4.53

$$x(t) \longrightarrow \hat{X}(j\omega) = \int_{-\infty}^{\infty} dt\, e^{-j\omega t} x(t)$$

Fourier Transform of $x(t)$

$$\hat{X}(j\omega) \longrightarrow x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega\, e^{j\omega t} \hat{X}(j\omega)$$

Inverse Fourier Transform of $\hat{X}(j\omega)$

Properties: Table 4.1

Time shifting:

$$x(t) \longrightarrow \hat{X}(j\omega)$$

$$x(t-t_0) \overset{?}{\longrightarrow} \int_{-\infty}^{\infty} dt\, e^{-j\omega t} x(t-t_0) =$$

write this in term of $\hat{X}(j\omega)$ by doing a change of variable $t' = t - t_0$

$$\rightarrow \begin{cases} dt' = dt \\ t = t' + t_0 \end{cases}$$

$$\int_{-\infty}^{\infty} dt'\, e^{-j\omega(t'+t_0)} x(t') = e^{-j\omega t_0} \underbrace{\int_{-\infty}^{\infty} dt'\, e^{-j\omega t'} x(t')}_{\hat{X}(j\omega)}$$

Fourier transform of a time shifted signal is $e^{-j\omega t_0}\hat{X}(j\omega)$ (4.3.2)

Convolution: $\quad x(t) \longrightarrow \hat{X}(j\omega)\ ; \quad y(t) \longrightarrow \hat{Y}(j\omega)$

What is the Fourier Transform of $x * y$ in terms of $\hat{X}$ & $\hat{Y}$?

$$\hookrightarrow \int_{-\infty}^{\infty} dt\, e^{-j\omega t}\left[x(t) * y(t)\right] = \int_{-\infty}^{\infty} dt\, e^{-j\omega t} \int_{-\infty}^{\infty} d\tau\, x(\tau)\, y(t-\tau)$$

$$= \int_{-\infty}^{\infty} d\tau\, x(\tau) \underbrace{\int_{-\infty}^{\infty} dt\, e^{-j\omega t}\, y(t-\tau)}$$

$$= e^{-j\omega\tau}\, \hat{Y}(j\omega)$$

from previous derivation of the time-shift property for Fourier transform

$$= \left[\int_{-\infty}^{\infty} d\tau\, x(\tau)\, e^{-j\omega\tau}\right] \hat{Y}(j\omega)$$

$$= \hat{X} \cdot \hat{Y}$$

$\Rightarrow$ Fourier Transform of a convolution is the product of the Fourier Transforms !! (4.4)

Very important for linear System Analysis!

$$x(t) \longrightarrow \boxed{\;h(t)\;} \longrightarrow y(t)$$

$$y(t) = x * h$$

Convolution property: $\quad \hat{Y} = \hat{X} \cdot \hat{H}$ a simple product.

$\rightarrow$ an alternative way to computing the convolution to get the output signal from the input signal and the impulse response of the system is

To do an inverse Fourier Transform on $\hat{X} \cdot \hat{H}$.

Pairs of Fourier Transform (Table 4.2)

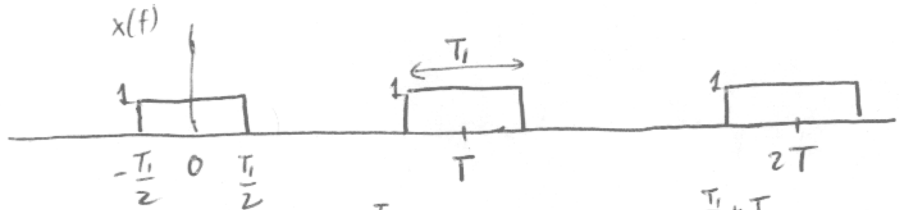
* $x(t) = \cos w_0 t \rightarrow \hat{X}(jw) = \int_{-\infty}^{\infty} dt\, e^{-jwt} \cos w_0 t = \frac{1}{2}\left[\int_{-\infty}^{\infty} dt \left(e^{-j(w-w_0)t} + e^{-j(w+w_0)t} \right) \right]$

$\cos w_0 t = \dfrac{e^{jw_0 t} + e^{-jw_0 t}}{2}$

$= \frac{2\pi}{2}\left[\delta(w-w_0) + \delta(w+w_0) \right]$

$$\boxed{\begin{aligned} \int e^{j(k-n)w_0 t}\, dt &= 2\pi\, \delta((k-n)w_0) \\ &= \frac{2\pi}{w_0}\delta[k-n] \\ \delta(at) = \frac{1}{|a|}\delta(t) \qquad &= T\,\delta[k-n] \text{ as in page \#24} \end{aligned}}$$

* $x(t) = 1 \rightarrow \hat{X}(jw) = \int_{-\infty}^{\infty} dt\, e^{-jwt} \cdot 1 = 2\pi\delta(w)$

* 

$$\hat{X}(jw) = \underbrace{\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} dt\, e^{-jwt}}_{(\text{at } 0)} + \underbrace{\int_{-\frac{T_1}{2}+T}^{\frac{T_1}{2}+T} dt\, e^{-jwt}}_{(\text{at } T)} + \cdots$$

$\dfrac{e^{-jw\frac{T_1}{2}} - e^{jw\frac{T_1}{2}}}{-jw} = \dfrac{2\sin w\frac{T_1}{2}}{w}$

$\dfrac{e^{-jw\left(\frac{T_1}{2}+T\right)} - e^{-jw\left(-\frac{T_1}{2}+T\right)}}{-jw}$

$e^{-jwT} \dfrac{2\sin w\frac{T_1}{2}}{w}$ ✓ $\qquad e^{-jw2T}\dfrac{2\sin w\frac{T_1}{2}}{w}$

$$= \boxed{\frac{2\sin w\frac{T_1}{2}}{w} \underbrace{\sum_{n=-\infty}^{\infty} e^{-jwnT}}_{\infty \text{ Geometric Sum.}}} \longrightarrow \text{One HW set of extra credit.}$$

$$e^{jA} = \cos A + j\sin A$$

$$+ \quad e^{-jA} = \cos A - j\sin A$$

$$e^{jA} + e^{-jA} = 2\cos A \longrightarrow \cos A = \frac{e^{jA} + e^{-jA}}{2}$$

substract →

$$e^{jA} - e^{-jA} = 2j\sin A \longrightarrow \sin A = \frac{e^{jA} - e^{-jA}}{2j}$$