what is a_0? do separately:

$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} dt\, \underbrace{e^{j0\omega_0 t}}_{1} \cdot 1 = \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{1}{T}\left[t\right]_{-T_1}^{T_1} = \frac{2T_1}{T}$$

Fourier Series Representation for a discrete-time periodic signal:

$x[n]$; periodic with period N: $x[n+N] = x[n]$

$\cos[\omega_0 n]$; (not all ω_0 will make this discrete-time signal a periodic signal)

$$\cos[\omega_0 (n+N)] = \cos[\omega_0 n] \quad \text{or} \quad \omega_0 N = m2\pi$$

$$\text{or} \quad \omega_0 = \frac{m2\pi}{N}$$

$$\rightarrow \omega_0 = \frac{2\pi}{N} \quad (m = 1 \text{ for lowest frequency})$$

$e^{jk\omega_0 n}$ or $e^{jk\frac{2\pi}{N}n}$

$\underbrace{e^{jk\omega_0 t}}$

There is no repeat here if k is increased by N

$\downarrow$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\underbrace{e^{jk\frac{2\pi}{N}n}}$

There is a repeat if k is increased by N

$$e^{j(k+N)\frac{2\pi}{N}n} = e^{jk\frac{2\pi}{N}n}\underbrace{e^{j2\pi n}}_{\text{always } 1}$$

$$\boxed{x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\frac{2\pi}{N}n} \quad \text{(only } N \text{ terms)}}$$

F.S. representation for a discrete-time periodic signal

How to obtain a_k from $x[n]$?

$$\boxed{a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jk\frac{2\pi}{N}n}}$$

Properties of Fourier Series for discrete-time signal:

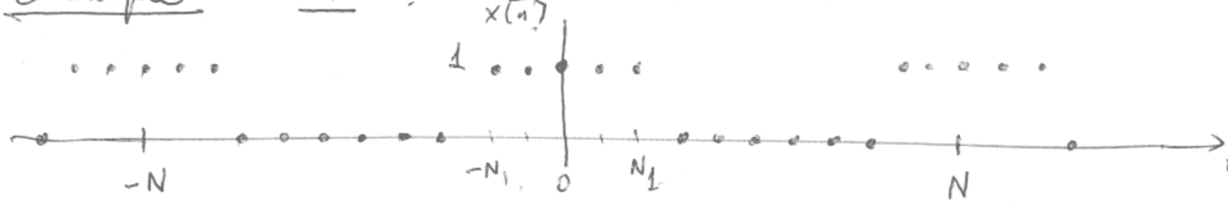
Table 3.2 pg 221

Linearity
Time shift
Frequency shift
Time scaling
Multiplication
(Differentiation) → First difference
(integration) → Running sum

Parseval's relation (for discrete-time signal).

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$

Example: 3.12 :



rectangular pulse of width $2N_1$; period N

$$a_k \stackrel{?}{=} \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jk\frac{2\pi}{N}n} = \frac{1}{N} \sum_{n=-N_1}^{N_1} x[n]\, e^{-jk\frac{2\pi}{N}n} = \frac{1}{N} \sum_{n=-N_1}^{N_1} e^{-jk\frac{2\pi}{N}n}$$

geometric sum.

pick any interval of width N
→ could be $\left(-\frac{N}{2}, \frac{N}{2}\right)$
since $\frac{N}{2} > N_1$

$$\left(\sum_{k=0}^{n} \alpha^{k} = \frac{1-\alpha^{n+1}}{1-\alpha} \right. \quad \text{from page 13} \left. \vphantom{\frac{1}{1}} \right)$$

$$\sum_{n=-N_1}^{N_1} \alpha^{n} \;;= \sum_{m=0}^{2N_1} \alpha^{m-N_1} = \alpha^{-N_1} \sum_{m=0}^{2N_1} \alpha^{m} = \alpha^{-N_1} \frac{1-\alpha^{2N_1+1}}{1-\alpha}$$

$m = n + N_1$ or $n = m - N_1$

$$\boxed{\alpha \equiv e^{-jk\frac{2\pi}{N}}}$$

$$\rightarrow \quad a_k = \frac{1}{N} e^{jk\frac{2\pi N_1}{N}} \boxed{\frac{1-e^{-jk\frac{2\pi}{N}(2N_1+1)}}{1-e^{-jk\frac{2\pi}{N}}}}$$

$$a_k = \frac{1}{N} e^{jk\frac{2\pi N_1}{N}} \cdot \frac{e^{-jk\frac{2\pi}{N}(N_1+\frac{1}{2})} \left(e^{jk\frac{2\pi}{N}(N_1+\frac{1}{2})} - e^{-jk\frac{2\pi}{N}(N_1+\frac{1}{2})} \right)}{e^{-jk\frac{2\pi}{2N}} \left(e^{jk\frac{2\pi}{2N}} - e^{-jk\frac{2\pi}{2N}} \right)}$$

$$= \frac{1}{N} \frac{\cancel{2j} \sin \frac{k2\pi}{N}(N_1+\frac{1}{2})}{\cancel{2j} \sin \frac{k2\pi}{2N}} \quad ; \quad k \neq 0, \pm N, \pm 2N, \pm 3N, \ldots$$

What is a_k when $k = 0, \pm N, \pm 2N, \ldots$?

$$a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} \underbrace{x[n]}_{1} \underbrace{e^{-jk\frac{2\pi}{N}n}}_{\substack{\text{always 1 when} \\ k=0;\, \pm N;\, \pm 2N}} = \frac{1}{N}(2N_1+1)$$

Plot is in figure 3.17 pg 219

Compare with plot for a_k of same but continuous-time signal in 3.7 pg 195: the effect of discretizing a signal is the ~~addition~~ of ~~attenuated~~ repetition of the original spectrum in frequency.
(→ filter before signal reconstruction)