HW#2

2-22)

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

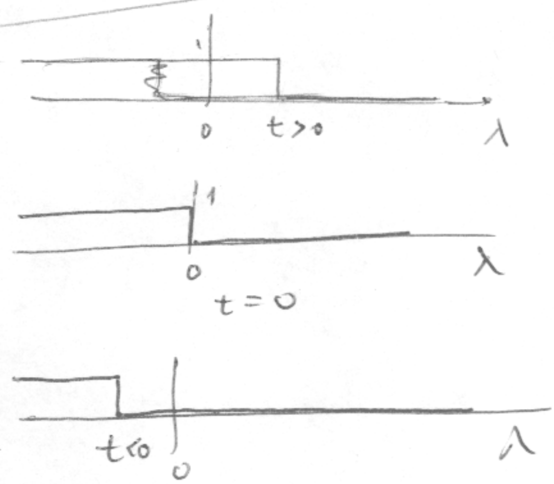
a) $x(t) = e^{-\alpha t} u(t)$; $h(t) = e^{-\beta t} u(t)$

initially at rest

$$y(t) = x * h = \int_{-\infty}^{\infty} d\lambda \; \underbrace{x(\lambda)}_{e^{-\alpha\lambda}\underbrace{u(\lambda)}} \; \underbrace{h(t-\lambda)}_{e^{-\beta(t-\lambda)} u(t-\lambda)}$$

$$= e^{-\beta t} \int_{0}^{\infty} d\lambda \; e^{-(\alpha-\beta)\lambda} \; \underbrace{u(t-\lambda)}$$

$=$



$t>0$ $\quad y(t) = e^{-\beta t} \int_0^t d\lambda \; e^{-(\alpha-\beta)\lambda} = e^{-\beta t} \left[\dfrac{e^{-(\alpha-\beta)\lambda}}{-(\alpha-\beta)} \right]_0^t$

$$= e^{-\beta t} \frac{\left(e^{-(\alpha-\beta)t} - 1\right)}{-(\alpha-\beta)} = \frac{e^{-\beta t} - e^{-\alpha t}}{\alpha - \beta}$$

$\boxed{\alpha \neq \beta}$

$$y(t) = \frac{e^{-\beta t} - e^{-\alpha t}}{\alpha - \beta} u(t)$$

$t \leq 0$ $\quad y(t) = 0 \longrightarrow \int_0^0 (\;\;) = 0$

$t>0$ $\quad y(t) = e^{-\beta t} \underbrace{\int_0^t d\lambda} = e^{-\beta t}\, t$

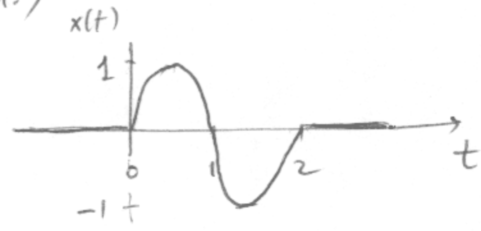
$\boxed{\alpha = \beta}$

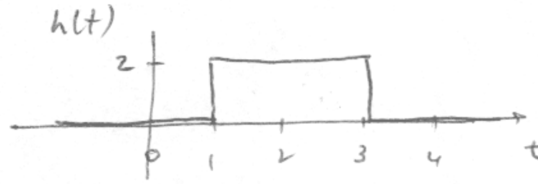
$$y(t) = t e^{-\beta t} u(t)$$

$t \leq 0$ $\quad y(t) = 0$

2.22 (cont.)

(c)





one period of $\sin \pi t$

$\omega = \pi \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2s$

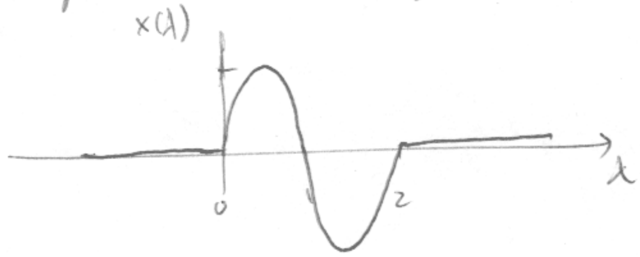
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} d\lambda \, \underbrace{x(\lambda)}_{\uparrow} \, \underbrace{h(t-\lambda)}_{\uparrow}$$

a) use math expression for whole signals $x(t)$ & $h(t)$

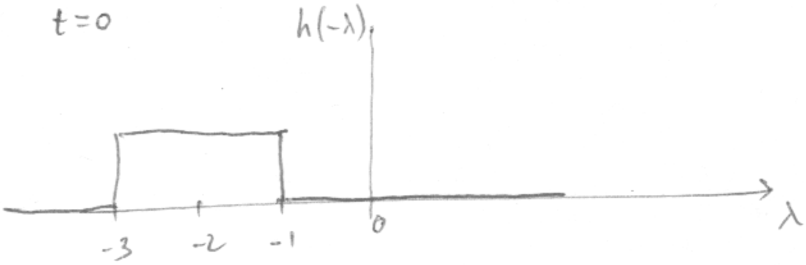
$$\begin{cases} x(t) = \sin \pi t \left[u(t) - u(t-2)\right] \\ h(t) = u(t-1) - u(t-3) \end{cases}$$

→ b) use visualization to do integral.

Approach b): drawing graphs and checking on overlaps b/w $x(\lambda)$ and $h(t-\lambda)$

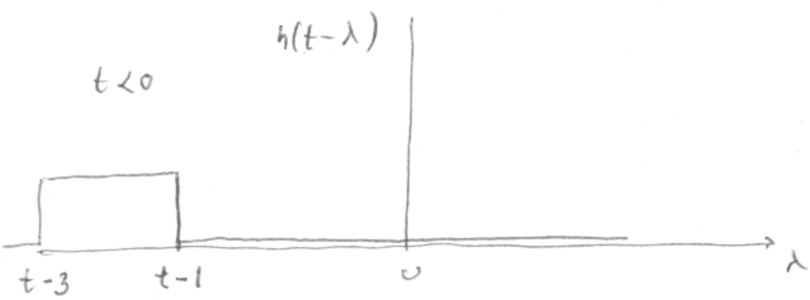


t=0



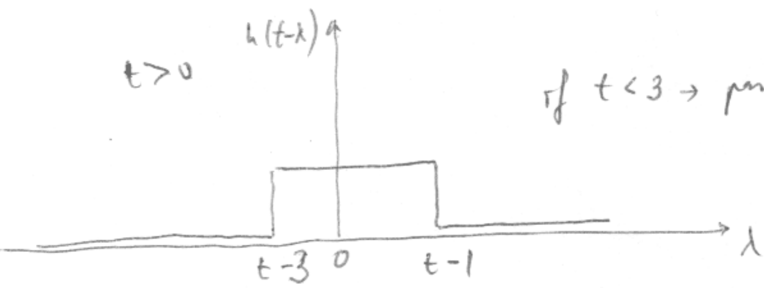
$y(t) = x * h = 0 \quad (t=0)$

t<0



$y(t) = x * h = 0 \quad (t<0)$

t>0



if $t<3 \rightarrow$ part of rectangle is negative

$$y(t) = x * h = \begin{cases} t-1 \le 2 : = 2\int_0^{t-1} d\lambda \, \sin(\pi\lambda) \\ t-1 > 2 : 2\int_{t-3}^{2} d\lambda \, \sin(\pi\lambda) \end{cases}$$

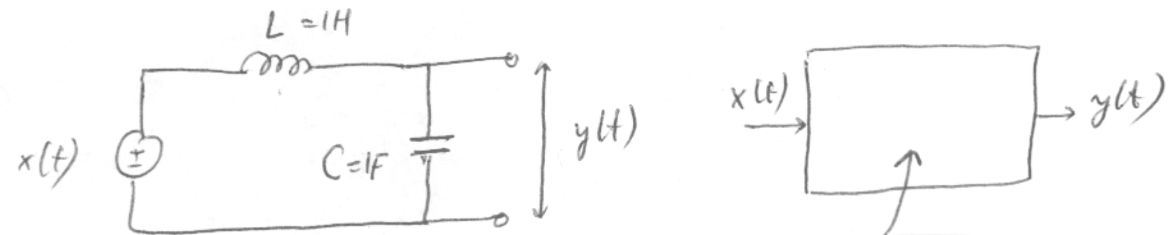
$$t>0 \quad y(t) = x(t) * h(t) = \begin{cases} t-1 \le 2 \text{ or } t<3 & = -2\left[\frac{\cos \pi\lambda}{\pi}\right]_0^{t-1} = \frac{2}{\pi}\left[1 - \cos \pi(t-1)\right] \\ t-1>2 \text{ or } t>3 & = -2\left(\frac{\cos \pi\lambda}{\pi}\right)_{t-3}^{2} = \frac{2}{\pi}\left[\cos \pi(t-3) - 1\right] \end{cases}$$

Observation: $t \le 1$ ($t-1 \le 0$): right border of rectangle $h(t-\lambda)$ is still negative (or zero) → no overlap.

$$\boxed{\text{Summary } y(t) = \begin{cases} 0 & t \le 1 \ ; \ t>5 \\ \frac{2}{\pi}\left[1-\cos \pi(t-1)\right] & 1 < t \le 3 \\ \frac{2}{\pi}\left[\cos \pi(t-3) - 1\right] & 3 < t \le 5 \end{cases}}$$

Observation: $t-3>2$ or $t>5$ → left border of rectangle $h(t-\lambda)$ has passed right border of $x(\lambda)$ → no overlap → 0 output.

2.61



(i) D.E relating $x(t)$ to $y(t)$: $\boxed{x(t) = LC\frac{d^2y}{dt^2} + y(t)}$

$$v_L(t) = L\frac{di_L}{dt}$$

$$i_L = i_C \rightarrow y(t) = v_C(t) = \frac{1}{C}\int i_C \, dt \rightarrow i_C(t) = i_L(t) = C\frac{dy}{dt}.$$

$$\rightarrow v_L(t) = LC\frac{d^2y}{dt^2}$$