In Summary: in this example:

$$\boxed{y(t) = \left(\frac{1-e^{-at}}{a}\right)u(t)}$$

HW#2:

27/ $$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$$

$$g[n] \equiv u[n] - u[n-4]$$

a) $y[n]$? when $x[n] = \delta[n-1]$

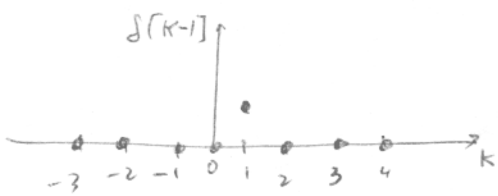
b) $y[n]$? when $x[n] = \delta[n-2]$

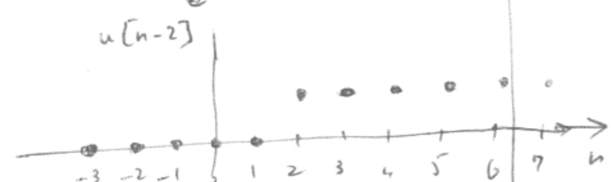
c) Is System LTI?

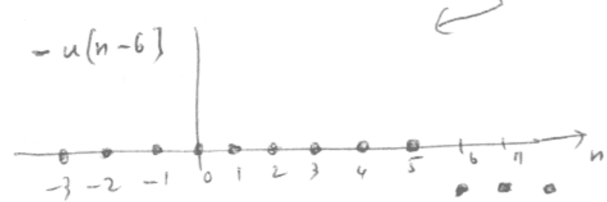
Linear & Time invariant?

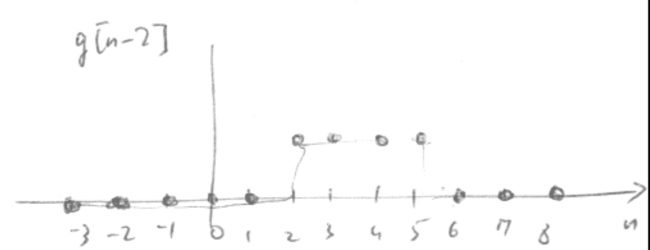
d) $y[n]$? when $x[n] = u[n]$

a) $$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]\, g[n-2k] = g[n-2] = u[n-2] - u[n-6]$$

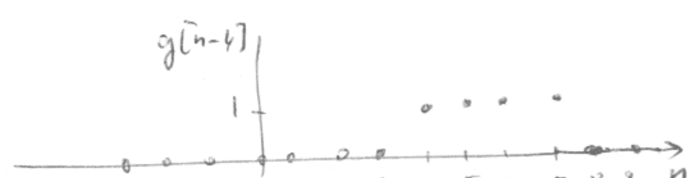








b) $$y[n] = u[n-4] - u[n-8]$$

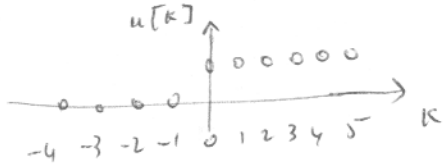


c) Relative time-shift for input b/w a) & b) is 1

but relative time-shift for output b/w a) & b) is 2

$\rightarrow$ Not Time-invariant.

d) $$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, g[n-2k] = \sum_{k=0}^{\infty} 1 \cdot g[n-2k] = \sum_{k=0}^{\infty} g[n-2k]$$



$$= g[n] + g[n-2] + g[n-4] + g[n-8] + \cdots$$

2.8 / $x * h$?

$$x(t) = \begin{cases} t+1 & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(t) = \delta(t+2) + 2\delta(t+1)$$

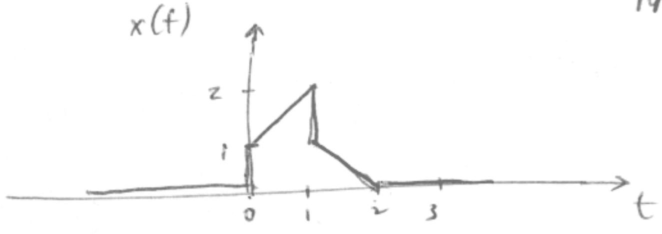
$$x * h = \underbrace{\int_{-\infty}^{\infty} d\lambda\; x(\lambda)\, h(t-\lambda)}_{\uparrow \text{ use this}} = \int_{-\infty}^{\infty} d\lambda\; x(t-\lambda)\, h(\lambda)$$

& Recall $$\int_{-\infty}^{\infty} d\alpha\; f(\alpha)\, \delta(\alpha+1) = f(-1)$$

2-17
2-24

2-8

$$x(t) = \begin{cases} t+1 & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & \text{elsewhere} \end{cases}$$



$$h(t) = \underset{\text{at } t=-2}{\delta(t+2)} + 2\,\underset{\text{at } t=-1}{\delta(\underbrace{t+1})}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} d\lambda \; x(\lambda)\; \underbrace{h(t-\lambda)}_{\text{reversed and shifted by } t}$$

$$= \int_{0}^{2} d\lambda \; x(\lambda) \left[\underbrace{\delta(t-\lambda+2)}_{\text{center at } t+2} + 2\, \underbrace{\delta(t-\lambda+1)}_{\text{center at } t+1} \right]$$

$$\int_{-\infty}^{\infty} d\alpha \; f(\alpha)\; \underbrace{\delta(b-\alpha)}_{\text{center at } b} = f(b)$$

center within limits of integration

if b is outside limits of integration $\rightarrow$ the integral is 0

We need to distinguish different values for t, to see if these centers are b/w limits of integration 0 & 2

a) $0 < t+2 < 2$ or $-2 < t < 0$

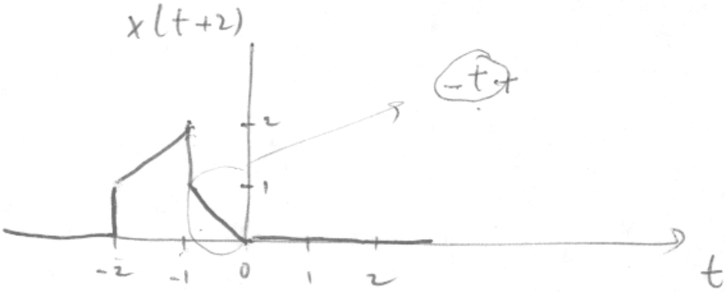
$0 < t+1 < 2$ or $-1 < t < 1$

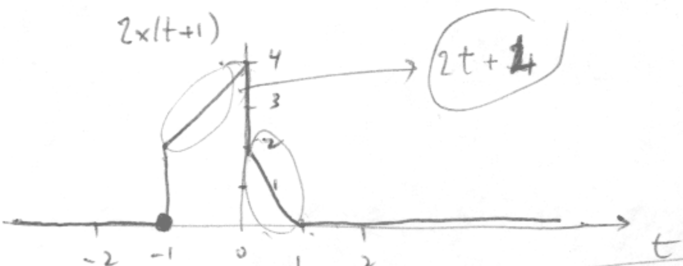
$$y(t) = \begin{cases} t < -2 & \longrightarrow 0 \\ -2 < t \le -1 & \longrightarrow x(t+2) \\ -1 < t \le 0 & \longrightarrow x(t+2) + 2x(t+1) \\ 0 < t \le 1 & \longrightarrow 2x(t+1) \\ 1 < t & \longrightarrow 0 \end{cases}$$

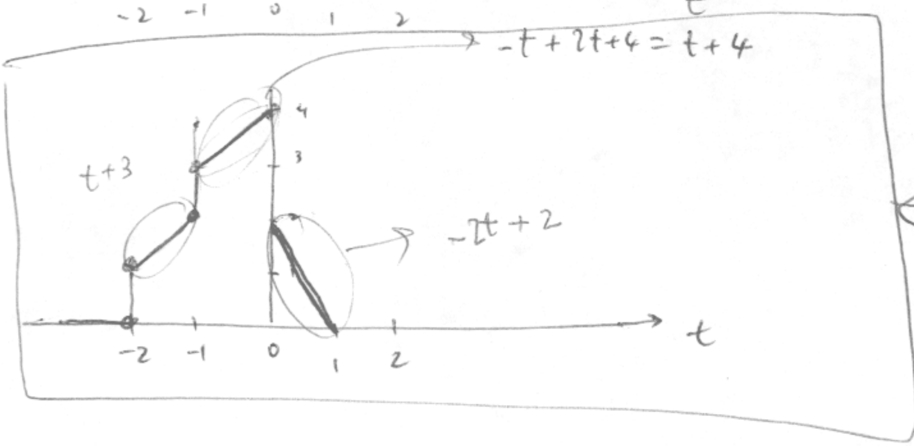
(second δ has center off interval defined by limits of integration 0 & 2)

$$
y(t) = \begin{cases}
-2 < t \le -1 \rightarrow x(t+2) = t+3 \checkmark \\
-1 < t \le 0 \rightarrow x(t+2) + 2x(t+1) = \cancel{-t+2(t+2) = 2t+3} \; t+4 \checkmark \\
0 < t \le 1 \rightarrow 2x(t+1) = -2t+2 \checkmark \\
\text{elsewhere} \rightarrow 0
\end{cases}
$$

expression.







graph.

2.17

$$\frac{dy}{dt} + 4y = x$$

L.T.I.
initial rest
$(t<0 \rightarrow x(t)=0;\ y(t)=0)$

a) if $x(t) = e^{(-1+3j)t} u(t) \rightarrow y(t) = ?$

Background (Math): $\frac{dy}{dt} + ay = bx$ (a, b are constants)

integrating factor: a) be^{at} : multiply to both sides of equation:

$$\underbrace{\frac{dy}{dt} be^{at} + ayb e^{at}}_{\cancel{b}\frac{d}{dt}\left(ye^{at}\right)} = \cancel{b}\, x e^{at}$$

b) integrate both sides $\int_{t_0}^{t}$

$$\left[y e^{at}\right]_{t_0}^{t} = b \int_{t_0}^{t} x e^{at}\, dt$$

$$y(t) \boxed{e^{at}} - y(t_0) e^{at_0} = b \int_{t_0}^{t} x(\lambda) e^{a\lambda}\, d\lambda$$

Solution of D.E.: $\boxed{y(t) = y(t_0)\, e^{-a(t-t_0)} + b \int_{t_0}^{t} x(\lambda)\, e^{-a(t-\lambda)}\, d\lambda}$

Back to 2.17: $t_0 = 0$; $y(t_0) = 0$

$$y(t) = \int_0^t x(\lambda)\, e^{-4(t-\lambda)}\, d\lambda = \int_0^t e^{(-1+3j)\lambda}\, e^{-4t}\, e^{4\lambda}\, d\lambda$$

$$x(\lambda) = e^{(-1+3j)\lambda} \underbrace{u(\lambda)}_{\substack{1 \\ 0<\lambda<t}}$$

$$y(t) = e^{-4t} \underbrace{\int_0^t e^{3(1+j)\lambda}\, d\lambda}$$

$$\left[\frac{e^{3(1+j)\lambda}}{3(1+j)} \right]_0^t = \frac{e^{3(1+j)t} - 1}{3(1+j)}$$

$$\rightarrow \quad y(t) = \frac{e^{-4t}\left(e^{(3+3j)t} - 1\right)}{3(1+j)} = \frac{1-j}{6}\left(e^{(-1+3j)t} - e^{-4t}\right)$$

add sign f: $u(t)$
to indicate
initial rest