1.10 Determine fundamental period T of

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

$$\#1 \qquad\qquad \#2$$

$$10\,T_1 = n_1\, 2\pi$$

$$T_1 = n_1 \frac{2\pi}{10} = \left\{ \frac{2\pi}{10}, \frac{4\pi}{10}, \frac{6\pi}{10}, \frac{8\pi}{10}, \pi, \frac{6\pi}{5}, \dots \right\}$$

$$n_1 = 1 \quad n_1 = 2 \ \dots \ n_1 = 5$$

$$4\,T_2 = n_2\, 2\pi$$

$$T_2 = n_2 \frac{2\pi}{4} = n_2 \frac{\pi}{2} = \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots \right\}$$

$$T = \pi$$

1.11 Fundamental period N of

$$x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$$

$$\#1 \qquad\qquad \#2$$

periodic — periodic

please check → $N_1 = n_1 \frac{7}{2}$ $= \{7, 14, 21, 28, 35, 42\}$

$\cancel{N_2 = n_2 \frac{1}{5}}$ $\{\cancel{\tfrac{1}{5}}, \cancel{\tfrac{2}{5}}, \dots \tfrac{5}{5}, \tfrac{35}{5} \dots\}$

$1, 2, 3, 4, 5, 6, 7$

$\boxed{N_2 = n_2 5} = \{5, 10, 15, 20, 25, 30, 35, \text{etc.}\}$

$\cancel{N = 7}$ $\boxed{N = 35}$

Both signals need to come back to same value for total signal to have its first repetition (or one period)

$$x[n] = 1 + \underbrace{e^{j\frac{4\pi n}{7}}}_{\text{Period } N_1} - \underbrace{e^{j\frac{2\pi n}{5}}}_{\text{Period } N_2}$$

$$e^{j\frac{4\pi(n+N_1)}{7}}$$

$$\cancel{e^{j\frac{4\pi n}{7}}} \cdot e^{j\frac{4\pi N_1}{7}} = \cancel{e^{j\frac{4\pi n}{7}}}$$

$$\Rightarrow e^{j\frac{4\pi N_1}{7}} = 1 \longrightarrow e^{j\frac{2\pi N_2}{5}} = 1 \quad (\text{if } N_2 \text{ is the period})$$

$$\frac{\cancel{4}^{\,2}\pi N_1}{7} = n_1 \cancel{2\pi} \qquad \frac{\cancel{2}\pi N_2}{5} = n_2 \cancel{2\pi}$$

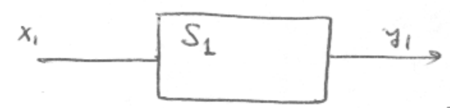
$$\rightarrow \boxed{N_1 = n_1 \frac{7}{2}} = \{7, 14, 28, 35, 42, \dots\} \qquad \boxed{N_2 = n_2 5} = \{5, 10, 15, 20, 25, 30, 35, 40, 45, \dots\}$$

$$\boxed{e^{j\alpha} = \cos\alpha + j\sin\alpha}$$

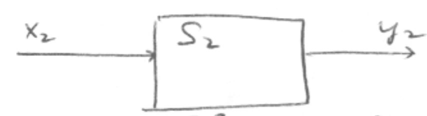
if $\alpha = n2\pi \rightarrow \begin{cases} \cos\alpha = 1 \\ \sin\alpha = 0 \end{cases}$

$\rightarrow e^{j2\pi n} = 1$

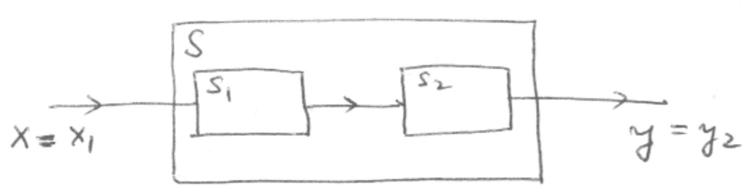
$\boxed{1.15}$



$y_1[n] = 2x_1[n] + 4x_1[n-1]$



$y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$



We are asked for an equation relating y to x : $y[n] = \dots$

· $\underbrace{\text{Output of } S_1}_{y_1}$ is $\underbrace{\text{input of } S_2}_{x_2}$ $\}$ $x_2[n] = y_1[n]$

$$y[n] = y_2[n] = \underbrace{y_1[n-2]}_{\uparrow} + \underbrace{\frac{1}{2}y_1[n-3]}_{\uparrow} = 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}2x_1[n-3] + \frac{1}{2}4x_1[n-4]$$

$$\Rightarrow y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

1-16 $\boxed{y[n] = x[n]\,x[n-2]}$ $\quad x \longrightarrow \boxed{S} \longrightarrow y$

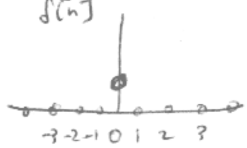
a) Memoryless?   b) Find output y when input $x[n] = A\,\delta[n]$

c) Is the system invertible?

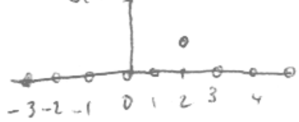
a) No, $y[n]$ depends on $x[n-2]$ (past input)

b) $y[n] = \underbrace{A\,\delta[n]}_{x[n]} \cdot \underbrace{A\delta[n-2]}_{x[n-2]} = A^2\,\delta[n]\,\delta[n-2] = 0$

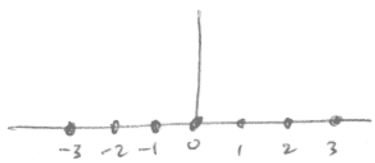
$\delta[n]$



$\delta[n-2]$



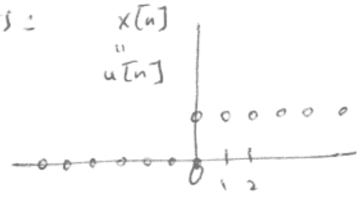
Product: $\delta[n]\,\delta[n-2]$



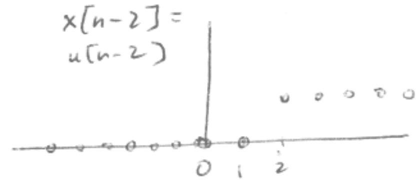
c) What is the inverse system S^{-1}: input/output equation?

No.

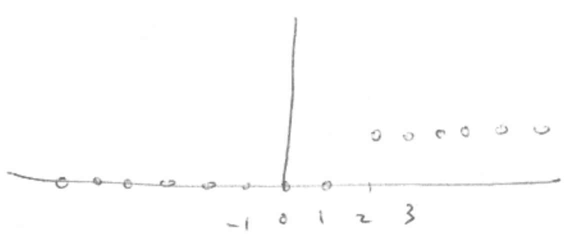
For other inputs: $x[n] = u[n]$



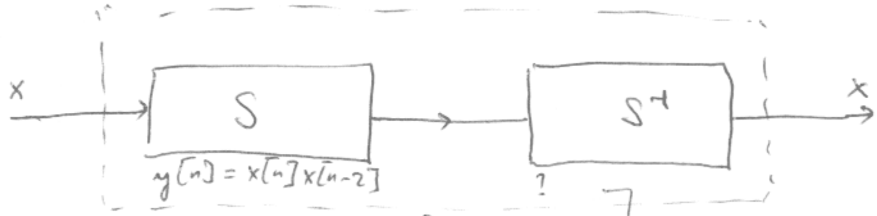
$x[n-2] = u[n-2]$



Product: $u[n] \cdot u[n-2]$



inverse system S^{-1}:



neutral system

an equation that gives $x[n]$ from $\underbrace{x[n]x[n-2]}$?

Not possible in general.

→ $\boxed{\text{No } S^{-1}}$

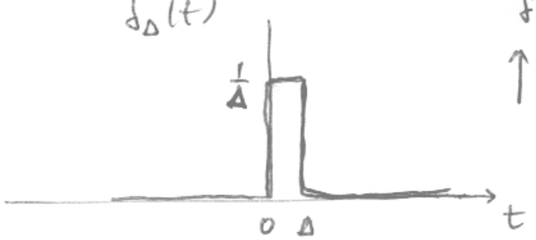
$\boxed{1.39}$ step function $u(t)$, impulse function $\delta(t)$

1.38 ↳ $u_\Delta(t) = \int_{-\infty}^{t} \delta_\Delta(\tau)\,d\tau$; $u(t) = \lim_{\Delta \to 0} u_\Delta(t)$

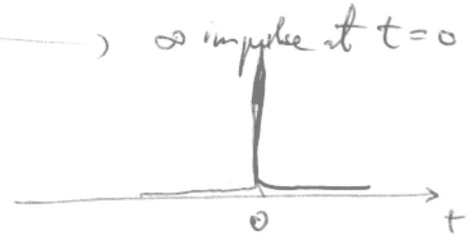
Recall: $\frac{du}{dt} = \delta(t)$ → $u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

$\delta(t) = \lim_{\Delta \to 0} \boxed{\delta_\Delta(t)}$ $\left\{\begin{array}{l} \text{. infinitesimal width} \\ \text{. infinite impulse} \end{array}\right.$

$\delta_\Delta(t)$

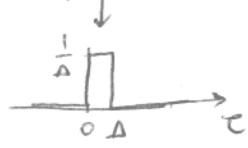


∞ impulse at $t=0$



Prove: $\lim_{\Delta \to 0} [u_\Delta(t)\,\delta(t)] = 0$ ✓ ‖ $\lim_{\Delta \to 0} [u_\Delta(t)\,\delta_\Delta(t)] = \frac{1}{2}\delta(t)$.

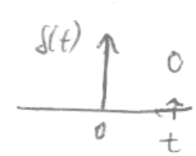
$\lim_{\Delta \to 0} \left[\delta(t) \underbrace{\int_{-\infty}^{t} \delta_\Delta(\tau)\,d\tau}\right]$



$t = 0 \rightarrow \delta(t) \times 0 \rightarrow 0$

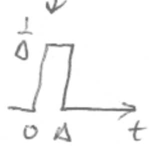
$t < 0 \rightarrow 0 \times 0 \rightarrow 0$

$t > 0$

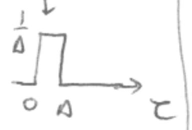


$0 \times 1 \rightarrow 0$

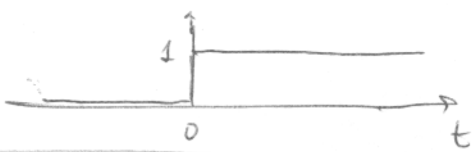
$$\lim_{\Delta\to 0}\left[u_\Delta(t)\,\delta_\Delta(t)\right] = \lim_{\Delta\to 0}\left[\right.$$



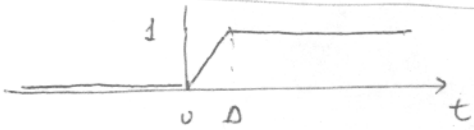
$$\left.\times \int_{-\infty}^{t} \delta_\Delta(\tau)\,d\tau\right]$$



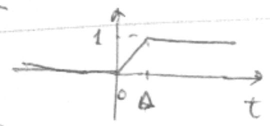
$u(t)$



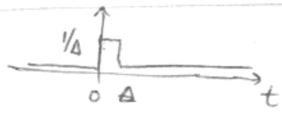
$u_\Delta(t)$



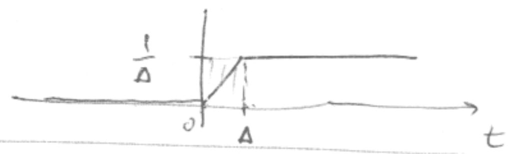
$$\lim_{\Delta\to 0}\left[u_\Delta(t)\,\delta_\Delta(t)\right] = \lim_{\Delta\to 0}\Big($$



$\times$



$\Big)$



→ Recall:

$\lim_{\Delta\to 0}$

$= \delta(t)$

So → $\frac{1}{2}\delta(t)$.

→ half area.

1.27 a) $y(t) = x(t-2) + x(2-t)$

1) Memoryless : Not. b/c current output depends on previous inputs.
2) Time-inv. : Yes : if time is shifted → system still works.
3) Linear : Yes : $ax \rightarrow \boxed{S} \rightarrow ay$
4) Causal : No : current output depends on future inputs.
5) Stable : Yes.

$t \rightarrow t + a$ $\begin{cases} \text{inputs: } x(t) \rightarrow x(t+a) \\ x(t+a-2) + x(2-a-t) = y(t+a) \checkmark \\ \text{output: } y(t+a) \end{cases}$

f) $y(t) = x\left(\frac{t}{3}\right)$

1) Memoryless ? Not ($y(t) = 3x(t)$ = memoryless)
2) Time inv. ? No : $t \rightarrow t+a$ $\begin{cases} \text{input: } t \rightarrow \frac{t+a}{3} \\ \text{output: } t \rightarrow t+a \end{cases}$

$t = 30s$: output at 30s is input at 10s

+60s ↕ 20s not 60s

$t = 30 + 60s$: output at 90s is input at 30s.

Shift of 60s for outputs corresponds to a shift of only 20s for input. → No time-inv.

3) Linear , Yes.
4) Causal Yes
5) Stable Yes.