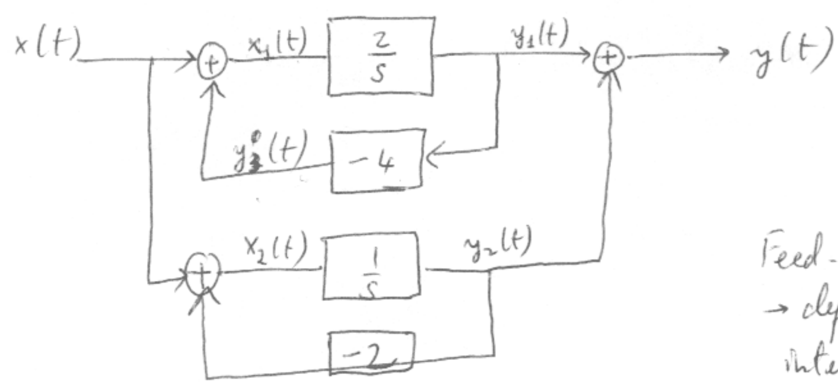
HW9: 9-17, 9-24; 9-31; 9-34; 9-38; 9-42; 9-51

9.17



Feed-back =
→ define clearly
intermediate variables.

D.E. relating x(t) to y(t) ?

1) Write equations in s :

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$L\{ \; y(t) = x(t) * h(t) \; \} \longrightarrow \hat{Y}(s) = \hat{X}(s) \cdot \hat{H}(s)$$

$$\hat{X}_1(s) = \hat{X} + \hat{Y}_3 = \hat{X} - 4\hat{Y}_1$$

$$\left(\hat{Y}_1(s) = \hat{X}_1 \cdot \frac{2}{s} \right)$$

$$\hat{X}_2(s) = \hat{X} - 2\hat{Y}_2$$

$$\left(\hat{Y}_2(s) = \frac{\hat{X}_2}{s} \right)$$

$$\hat{Y} = \hat{Y}_1 + \hat{Y}_2 = \hat{Y}_1 + \frac{\hat{X}_2}{s} = \frac{2\hat{X}_1}{s} + \frac{\hat{X}_2}{s} = \frac{2\hat{X}}{s+8} + \frac{\hat{X}}{s+2} \quad (*)$$

2) Now write $\hat{X}_1$ & $\hat{X}_2$ in term of $\hat{X}$

$$\hat{X}_1 = \frac{1}{1+\frac{8}{s}} \hat{X} \checkmark \qquad \left\{ \hat{X}_1 = \hat{X} - 4\hat{X}_1 \frac{2}{s} \rightarrow \hat{X}_1 \left(1 + \frac{8}{s}\right) = \hat{X} \right.$$

$$\hat{X}_2 = \frac{\hat{X}}{1+\frac{2}{s}} \checkmark \qquad \left\{ \hat{X}_2 = \hat{X} - \frac{2}{s}\hat{X}_2 \right.$$

$$(*) \rightarrow \quad \hat{Y} = \hat{X}\left(\frac{2}{s+8} + \frac{1}{s+2}\right) = \hat{X}\left(\frac{3s+12}{s^2+10s+16}\right)$$

$$s^2\hat{Y} + 10s\hat{Y} + 16\hat{Y} = 3s\hat{X} + 12\hat{X}$$

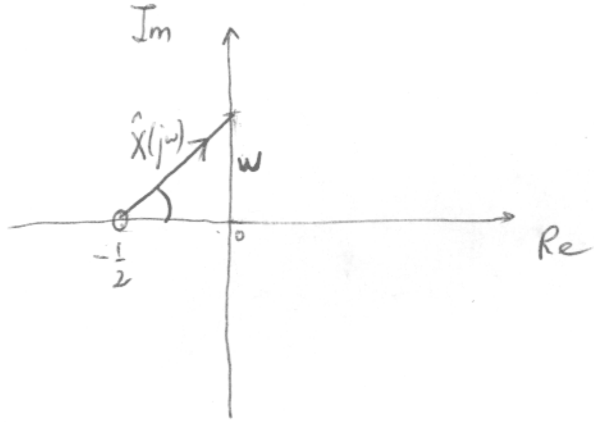
$$\downarrow$$

$$\boxed{\frac{d^2}{dt^2}y + 10\frac{dy}{dt} + 16y = 3\frac{dx}{dt} + 12x}$$

9.24 "pole-zero plot"

a) $x(t) \rightarrow \hat{X} = s + \frac{1}{2}$: Draw pole-zero plot for $\hat{X}(s)$

pole $\downarrow$ X, zero $\downarrow$ O



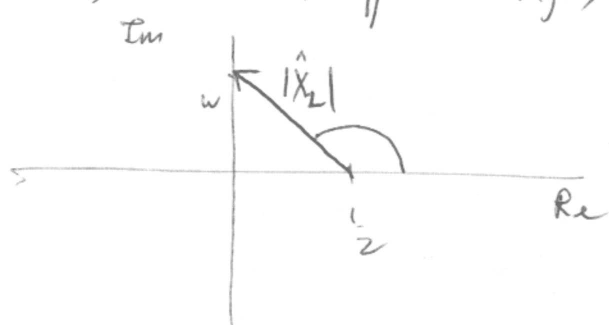
* s can have Re & Im parts. $(s=\sigma+j\omega)$
* pole is a zero of denominators
zero is a zero of numerator.

- Draw a vector representing $\hat{X}(j\omega)$ (for a given ω) $\begin{cases} |\hat{X}(j\omega)| \text{ by length of vector} \\ \angle\hat{X}(j\omega) \text{ by angle of vector w.r.t. Re axis} \end{cases}$

$$|\hat{X}(j\omega)| = \left| j\omega + \frac{1}{2} \right|$$

$$\angle\hat{X}(j\omega) = \tan^{-1}\left(\frac{\omega}{\frac{1}{2}}\right)$$

b) Determine a different $\hat{X}_1(j\omega)$ $\begin{cases} |\hat{X}_1| = |\hat{X}| \\ x_1(t) \neq x(t). \end{cases}$

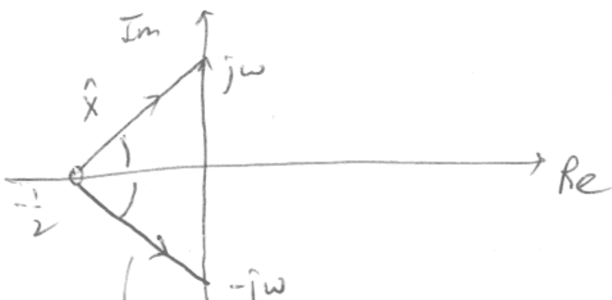


$\rightarrow \hat{X}_2(j\omega) = j\omega - \frac{1}{2}$ (second quadrant ✓)

c) Phase relationship $\angle\hat{X}_1(j\omega)$ and $\angle\hat{X}(j\omega)$

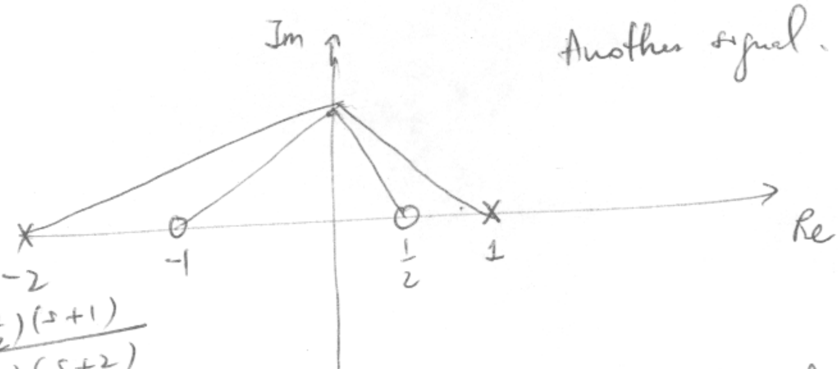
$$\angle\hat{X}_1(j\omega) = \pi - \angle\hat{X}(j\omega)$$

d) Determine $\hat{X}_2(s)$ such that $\angle\hat{X}_2(j\omega) = \angle\hat{X}(j\omega)$ but $x_2(t)$ is not proportional to $x(t)$ - Show pole-zero plot for $\hat{X}_2(s)$



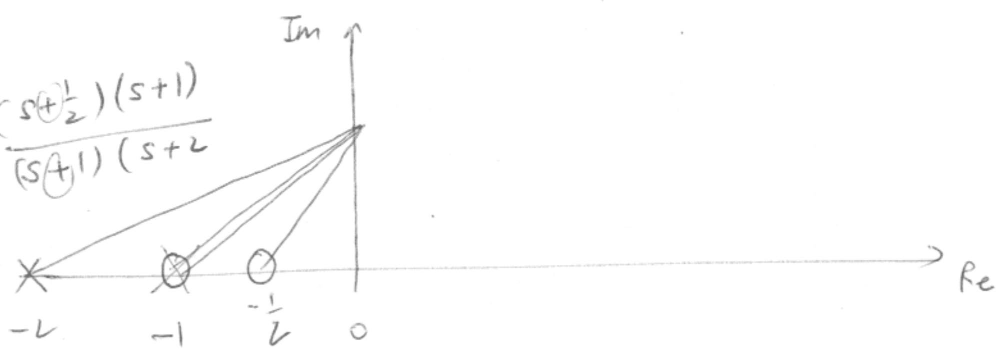
$$\hat{X}_2(j\omega) = -j\omega + \frac{1}{2}$$

f) Another signal. $x(t) \rightarrow \hat{X}(s)$



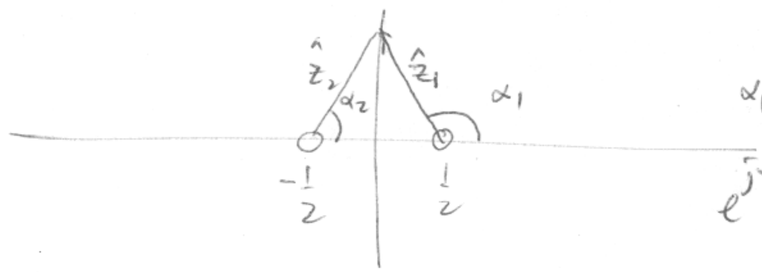
$$\hat{X} = \frac{(s-\frac{1}{2})(s+1)}{(s-1)(s+2)}$$

Determine another $\hat{X}_1(s)$ but all poles & zeros are in the LHP ($Re(s) < 0$) $\rightarrow |\hat{X}_1| = |\hat{X}|$



$$\hat{X}_1 = \frac{(s+\frac{1}{2})(s+1)}{(s+1)(s+2)}$$

Determine another $\hat{X}_2(s)$ $\left(\angle \hat{X}_2 = \angle \hat{X} \right)$ but all poles and zeros are in LHP $\left(Re(s) < 0 \right)$

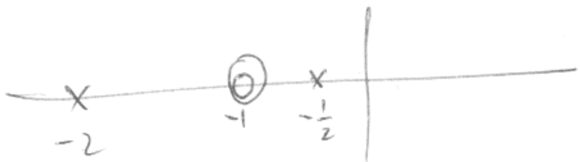


$\alpha_1 = \pi - \alpha_2$

$e^{j\alpha_1} = e^{j\pi} e^{-j\alpha_2}$

switch of sign.

cancel by switch pole $\leftrightarrow$ zero



$$\hat{X}_2 = \frac{(s+1)^2}{(s+2)(s+\frac{1}{2})}$$

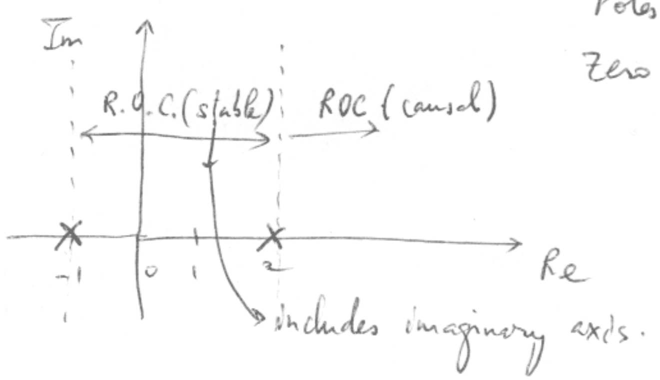
4.31

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = x(t) \rightarrow s^2\hat{Y} - s\hat{Y} - 2\hat{Y} = \hat{X}$$

a) $$\hat{H}(s) = \frac{\hat{Y}}{\hat{X}} = \frac{1}{s^2 - s - 2} = \frac{1}{(s+1)(s-2)}$$

Poles: X

Zero: O



b) $h(t)$ $\begin{cases} \text{stable} \\ \text{causal} \\ \text{neither stable nor causal} \end{cases}$

Inverse L.T.

Example 9.1: $x(t) = e^{-at}u(t) \rightarrow \hat{X}(s) = \frac{1}{s+a}$; $\boxed{Re(s) > -a}$

Why? $\hat{X}(s) = \int_0^{\infty} dt \; e^{-at} u(t) \, e^{-st}$

$$= \int_0^{\infty} dt \; e^{-(s+a)t} = \int_0^{\infty} dt \; \underbrace{e^{-(Re[s]+a)t}}_{} e^{-j\omega t}$$

$s = \sigma + j\omega$ $\begin{cases} Re(s) = \sigma \\ Im(s) = \omega \end{cases}$

needs to converge at $t \rightarrow \infty$

↓

neg. exponent

↓

$Re(s) + a > 0$

or $\boxed{Re(s) > -a}$

Example 9.2: $x(t) = -e^{-at}u(-t)$

$$\hat{X}(s) = -\int_{-\infty}^{\infty} dt \; e^{-at} u(-t) \, e^{-st} = -\int_{-\infty}^{0} dt \; e^{-(a+s)t}$$

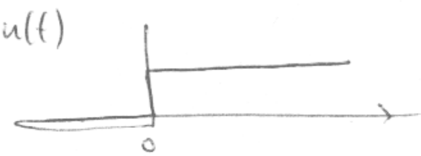
R.O.C.: $Re(s) + a < 0$

or $\boxed{Re(s) < -a}$

$$= \int_0^{-\infty} dt \; e^{-(a+s)t}$$

$$= \left. \frac{e^{-(a+s)t}}{-(a+s)} \right]_0^{-\infty} = \frac{1}{s+a}$$

u(t)



Summary:

$$\frac{1}{s+a} \longrightarrow \begin{cases} e^{-at}u(t) & Re(s) > -a \\ -e^{-at}u(-t) & Re(s) < -a \end{cases}$$

9.31 (cont.)

$$\frac{1}{(s+1)(s-2)} \rightarrow \begin{cases} \text{Causal: } Re(s) > 2 \rightarrow h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t) \\ \text{stable: } -1 < Re(s) < 2 \rightarrow h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t) \\ \text{neither: } Re(s) < -1 \rightarrow h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t) \end{cases}$$

$$\downarrow$$

$$\frac{\overset{B}{\frac{1}{3}}}{s-2} \quad \frac{-\overset{A}{1/3}}{s+1}$$

$$\hookrightarrow \frac{1}{(s+1)(s-2)} = \underbrace{\frac{A}{s+1} + \frac{B}{s-2}}_{\frac{A(s-2)+B(s+1)}{(s+1)(s-2)}}$$

Partial Fraction Expansion.

$$\begin{cases} (A+B)s = 0 \Rightarrow A = -B \\ -2A + B = 1 \Rightarrow 3B = 1 \rightarrow B = \frac{1}{3} \end{cases}$$

$$A = -\frac{1}{3}$$