6.31 a) Integrator: $\hat{H}(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$

$$\omega > 0 \quad \begin{cases} 20\log|\hat{H}| = -20\log\omega \\ \angle \hat{H}(j\omega) = -\frac{\pi}{2} \end{cases}$$

$$20\log_{10}\left|\frac{1}{j\omega}\right| = 20\log_{10}\left(\left|\frac{1}{\omega}\right|\underbrace{\left|\frac{1}{j}\right|}_{1}\right) = 20\log_{10}\frac{1}{\omega} = -20\log_{10}\omega$$

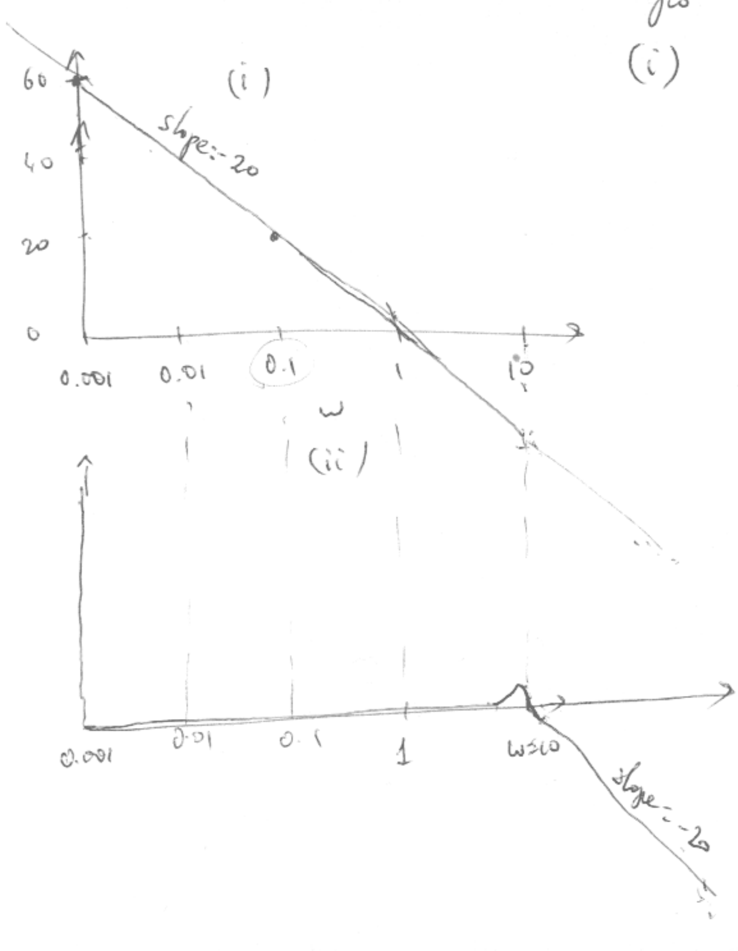
$$j = e^{j\frac{\pi}{2}}$$

$$\hat{H}(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{10}\right)} + \pi\delta(\omega)$$

Bode Plot for $\omega > 0.001$

$$\rightarrow \quad 20\log_{10}\left|\frac{1}{j\omega\left(1+j\frac{\omega}{10}\right)}\right| = \underbrace{-20\log_{10}|j\omega|}_{\substack{-20\log_{10}\omega \\ (i)}} \underbrace{-20\log_{10}\left|1+j\frac{\omega}{10}\right|}_{(ii)}$$

Magnitude



$$|1+j| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\log\frac{A}{B} = \log A - \log B$$

small $\omega \rightarrow \frac{\omega}{10} \ll 1$

$\rightarrow -20\log_{10} 1 = 0$

large $\omega \rightarrow \frac{\omega}{10} \gg 1$

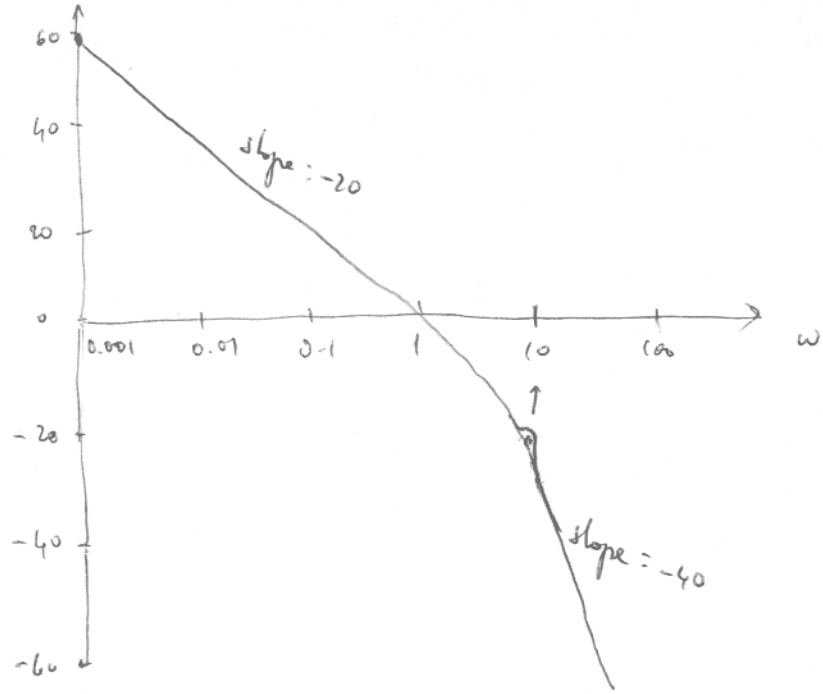
$-20\log_{10}\frac{\omega}{10} =$

$-20\log_{10}\omega + 20$

intermediate ω: $\frac{\omega}{10} \simeq 1$

$-20\log_{10}\sqrt{2} = 3\ dB$

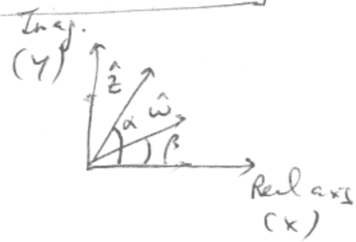
Magnitude Bode plot



$$\hat{H}(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{\omega}\right)} \qquad \omega > 0.001$$

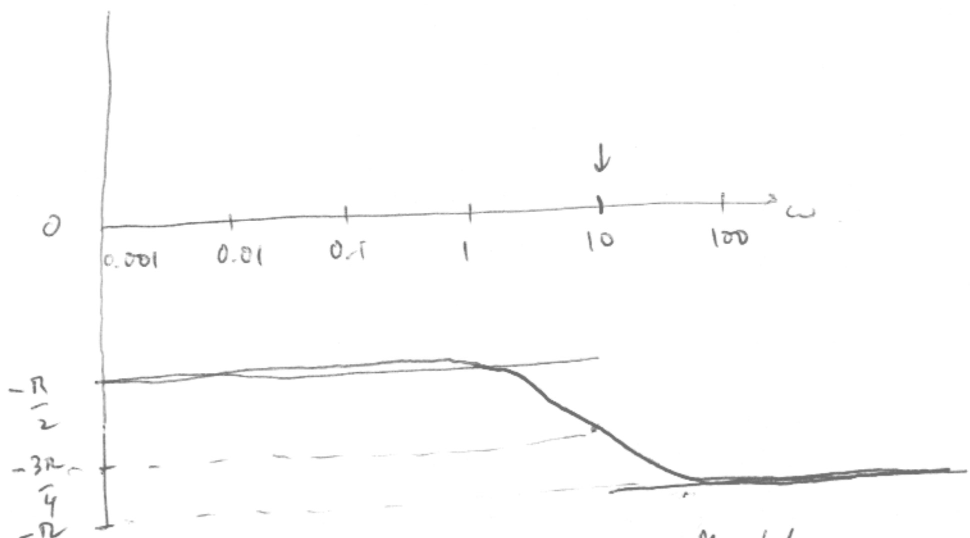
$$\angle \hat{H}(j\omega) = \left(+\angle \frac{1}{j\omega}\right) + \left(-\angle 1+\frac{j\omega}{\omega}\right) = -\frac{\pi}{2} + \boxed{\begin{cases} \text{small } \omega: & 0 \\ \text{intermediate } \omega: & -\frac{\pi}{4} \\ \text{large } \omega: & -\frac{\pi}{2} \end{cases}}$$

$$\left.\begin{aligned} &\left(\hat{z} = e^{j\alpha} \rightarrow \angle \hat{z} = \alpha \;;\; \angle \frac{1}{\hat{z}} = -\alpha\right) \\ &\hat{w} = e^{j\beta} \rightarrow \angle \hat{z}\hat{w} = \alpha+\beta \end{aligned}\right\}$$

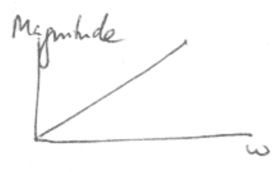


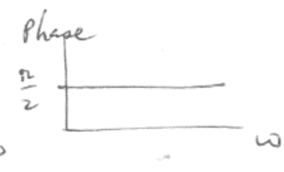
w = real # → angle is 0 ⟶ Real axis

Phase Bode plot



Part b) : Bode plot for a differentiator is



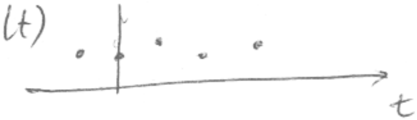


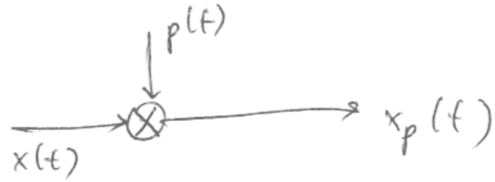
Ch. 7 Sampling

HW 7: 7.7 ✓; 7.9 ✓; 7.15; 7.24; 7.30; 7.31; 7.40; 7.45; 7.49; 7.50

Continuous-time signal $x(t)$ 

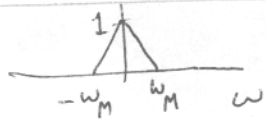
"Chopper" $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ with period T 

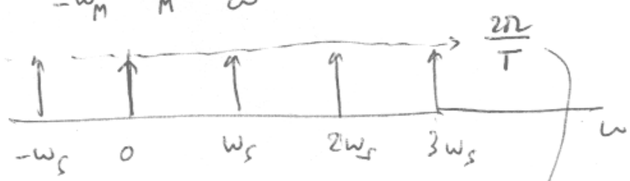
Sampled signal: $x_p(t) = x(t)\,p(t)$ 



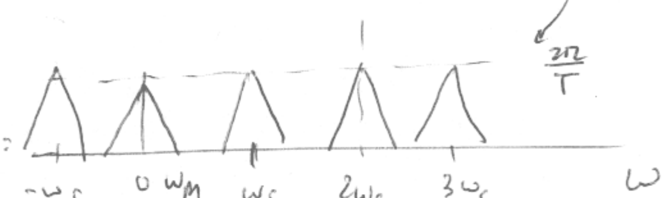
$T \rightarrow$ sampling period.

If $x(t)$ is periodic with period T_x

If $\hat{X}(j\omega)$ is 

Since $\hat{P}(j\omega)$ is 

$\left(\omega_s = \frac{2\pi}{T}\right)$ sampling frequency.

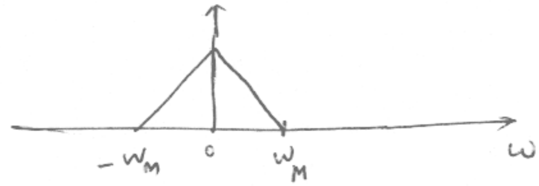
Then: $\hat{X}_p(j\omega) =$ 

if $\boxed{\omega_s \geq 2\omega_M}$ $\rightarrow$ no overlap of spectra (no aliasing).

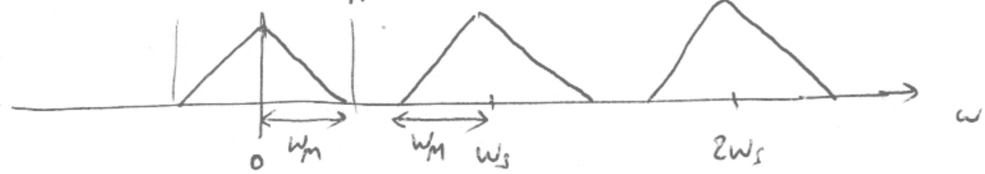
$\rightarrow$ perfect reconstruction.

Sampling Theorem or Nyquist Theorem.

Spectrum of original signal $\hat{X}(j\omega)$:

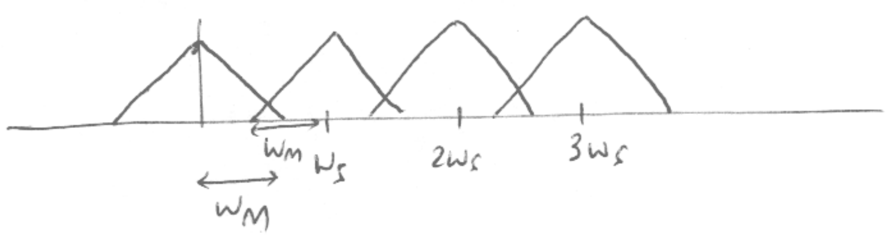


Spectrum of sampled signal $\hat{X}_p(j\omega)$:



In this case ($\omega_s \geq 2\omega_M$) can apply a low-pass filter to get back original spectrum → perfect reconstruction.

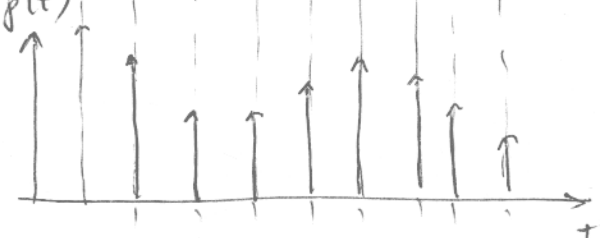
Spectrum of sample signal $\hat{X}_p(j\omega)$



In this case ($\omega_s < 2\omega_M$) cannot recover original spectrum → aliasing effect. (Fig. 7.16 c) & d))

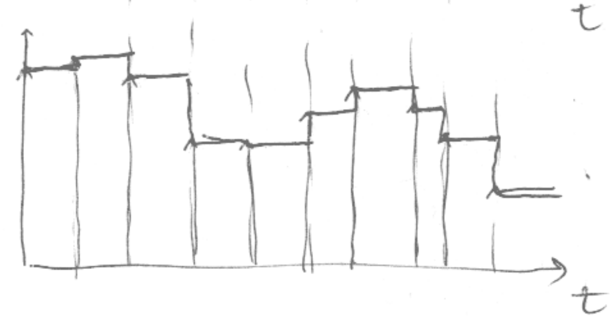
Signal: $x(t)$

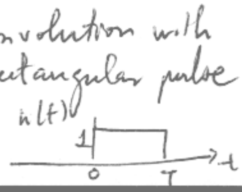
Sampled signal: $x_p(t) = x(t) \cdot p(t)$



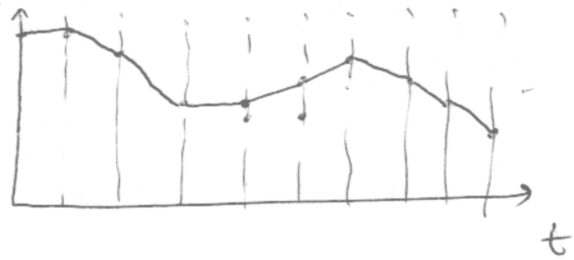
zero-order hold $x_0(t)$ (hold value until next sample)

= convolution with a rectangular pulse

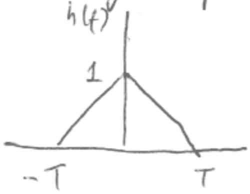




first-order hold
(linear interpolation)



or convolution with a triangular pulse



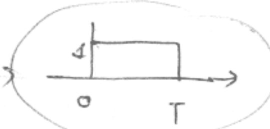
7.7

$x_0(t) \rightarrow \boxed{\hat{H}\,?} \rightarrow x_1(t)$

zero-order hold

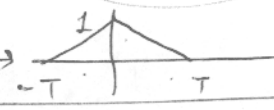
first-order hold

$x_0(t) = x_p(t) * h_0(t)$ ⟶



$\Rightarrow \hat{X}_0 = \hat{X}_p \cdot \hat{H}_0$

$x_1(t) = x_p(t) * h_1(t)$ ⟶



$\Rightarrow \hat{X}_1 = \hat{X}_p \cdot \hat{H}_1$

$$\hat{X}_1 = \underbrace{\frac{\hat{H}_1}{\hat{H}_0}}_{\hat{H}}\hat{X}_0$$

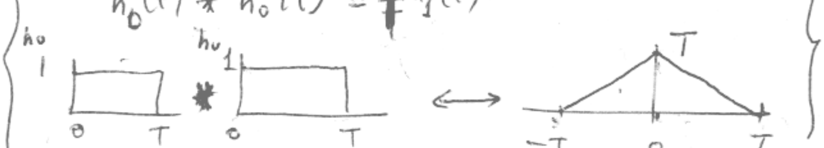
So $\hat{H} = \dfrac{\hat{H}_1}{\hat{H}_0} =$ Now relate $\hat{H}_1$ & $\hat{H}_0$

$\hat{H}_1$: a) Apply F-T def. and do integral

or b) Use some connection with $\hat{H}_0$

$\hat{H}_0$: From table

$$h_0(t) * h_0(t) = T h_1(t)$$



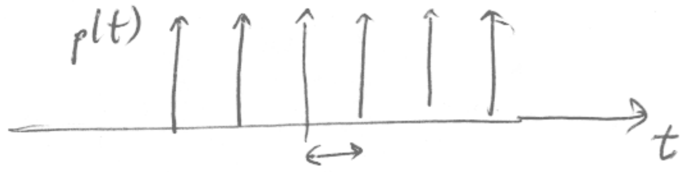
$\rightarrow \hat{H}_0 \hat{H}_0 = T \hat{H}_1$

$$\hat{H} = \frac{\hat{H}_0 \hat{H}_0}{T\,\hat{H}_0} = \frac{\hat{H}_0}{T} = \boxed{\frac{2\sin w\frac{T}{2}}{wT}\,e^{-jw\frac{T}{2}}}$$

b/c our interval is T instead of 2T as in Table 4.2

b/c our pulse is centered at $\frac{T}{2}$ instead of 0

7-9) $x(t) = \left(\frac{\sin 50\pi t}{\pi t}\right)^2$, sampled with $w_s = 150\pi$.
sampling frequency



$g(t) = x(t)\,p(t)$

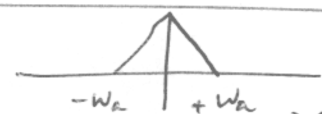
$\overset{?}{T} = \frac{2\pi}{w_s} = \frac{1}{75}\,s$
sampling period

Max value of w_0 such that

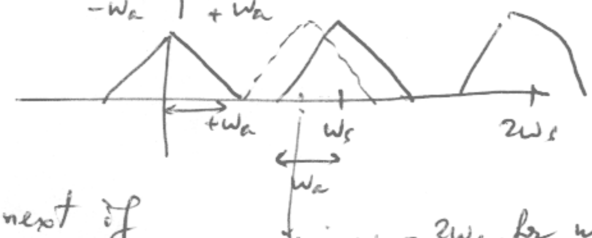
$\hat{G}(jw) = 75\,\hat{X}(jw) \quad |w| \le w_0$ } → left edge of second spectrum

In general

Sampling Theory: $x(t) \longrightarrow \hat{X}(jw) =$



$\longrightarrow \hat{X}_p(jw) =$

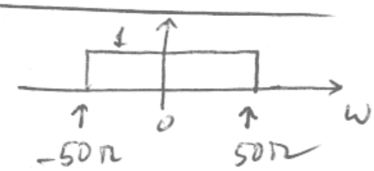


No overlap of one spectrum and the next if

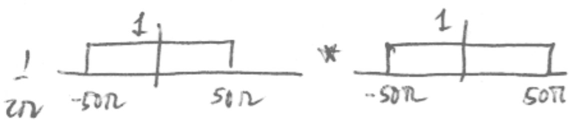
min $W_s = 2W_a$ for no aliasing

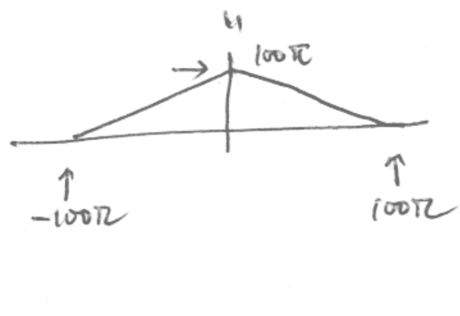
In another words: the left edge of the second spectrum is the max value for W_a for no aliasing.

→ $\hat{X}(jw) \overset{?}{=}$ Table 4.2: $\frac{\sin \overset{W}{50\pi} t}{\pi t} \longrightarrow$



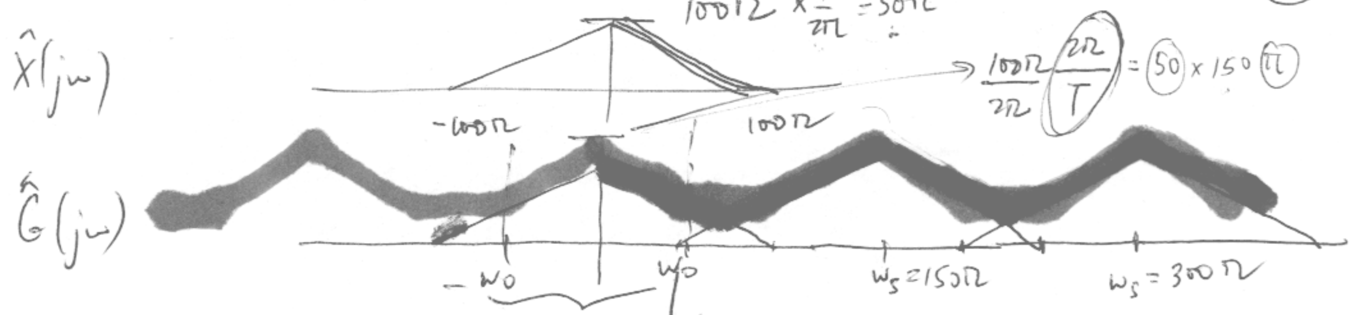
$\left(\frac{\sin 50\pi t}{\pi t}\right)^2 \longrightarrow \frac{1}{2\pi}$





Max w_0: left edge of 2nd spectrum. ?

→ Max w_0 is $150\pi - 100\pi = 50\pi$



max ω_0
or left edge
of 2nd spectrum.

error

$$\hat{G}(j\omega) = 75\, X(j\omega) \qquad (|\omega| \leq \omega_0)$$

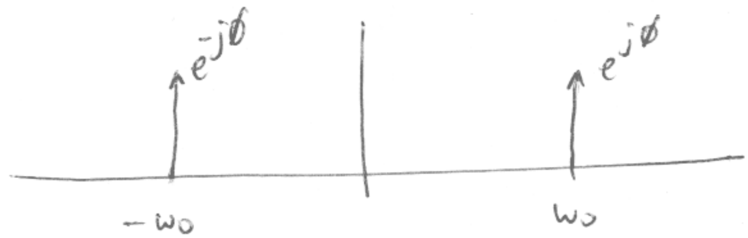
150

7.40

$v(t) = \cos(\omega_0 t + \phi)$ $\qquad i(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$

$r(t) = v(t)\, i(t)$.

a) $\hat{V}(j\omega)$



$$\cos(\omega_0 t + \phi) = \cos\left[\omega_0\left(t + \frac{\phi}{\omega_0}\right)\right]$$

t_0

Property : shift in time $\rightarrow$ $e^{j\omega_0 t_0}$ in freq.

b) $\hat{I}(j\omega)$