$$\hat{H}_2 = \frac{-72 + 48 e^{-j\omega} - 8 e^{-2j\omega}}{36 - 33 e^{-j\omega} + 10 e^{-j2\omega} - e^{-j3\omega}}$$

4.53 d)

$$\hat{X}(j\omega_1, j\omega_2) = \frac{2\pi}{4 + j\omega_1} \delta(\omega_2 - 2\omega_1)$$

$$x(t_1, t_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 \, d\omega_2 \underbrace{\frac{2\pi}{4 + j\omega_1} \delta(\omega_2 - 2\omega_1)}_{\hat{X}(j\omega_1, j\omega_2)} e^{j\omega_1 t_1} e^{j\omega_2 t_2}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega_1 \frac{e^{j\omega_1 t_1}}{4 + j\omega_1} \underbrace{\int_{-\infty}^{\infty} d\omega_2 \, \delta(\omega_2 - 2\omega_1) e^{j\omega_2 t_2}}_{e^{j 2\omega_1 t_2}}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega_1 \frac{e^{j\omega_1 (t_1 + 2t_2)}}{4 + j\omega_1} = e^{-4(t_1 + 2t_2)} u(t_1 + 2t_2)$$

Table 4.2:

$$e^{-at} u(t) \longleftrightarrow \frac{1}{a + j\omega}$$

$$\begin{cases} \hat{X}(j\omega) = \dfrac{1}{a + j\omega} = \displaystyle\int_{-\infty}^{\infty} dt \; e^{-at} u(t) \, e^{-j\omega t} \\ \text{and} \\ x(t) = e^{-at} u(t) = \dfrac{1}{2\pi} \displaystyle\int_{-\infty}^{\infty} d\omega \frac{1}{a + j\omega} e^{j\omega t} \end{cases}$$

Ch 4: $\begin{cases} x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, e^{j\omega t}\hat{X}(j\omega) & (4.8) \\ \hat{X}(j\omega) = \int_{-\infty}^{\infty} dt \, e^{-j\omega t} x(t) & (4.9) \end{cases}$

Ch 5: $\begin{cases} x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega & (5.8) \\ \hat{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} & (5.9) \end{cases}$

Discrete-time Fourier Transform $\hat{X}(e^{j\omega})$

<u>Difference</u>: $\hat{X}(e^{j(\omega+2\pi)}) = \hat{X}(e^{j\omega})$ DTFT is always periodic in ω, period 2π.

Ch. 6: Time & Frequency Characterization of Signals and Systems

In connection with pg 327,

HW 6: 6.5; 6.18; 6.19; 6.23; 6.27; 6.30; 6.31; 6.33; 6.40; 6.51

6.5 : $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$h(t) = \frac{\sin \omega_c t}{\pi t}\, g(t)$?

$\hat{H}(j\omega) = \begin{cases} 1 & \omega_c \le |\omega| \le 3\omega_c \\ 0 & \text{elsewhere} \end{cases}$ →

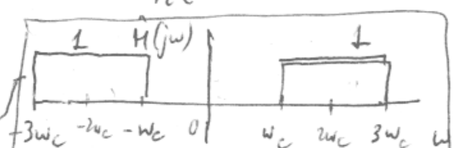
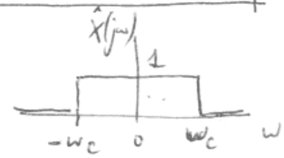


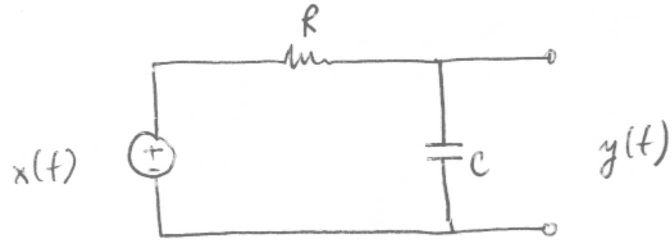
Table 4.2 : $\frac{\sin \omega_c t}{\pi t} \longleftrightarrow \hat{X}(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$



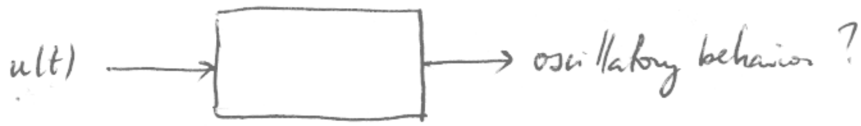
$\hat{H}(j\omega) = \hat{X}(j(\omega-2\omega_c)) + \hat{X}(j(\omega+2\omega_c))$

$(e^{j2\omega_c t} + e^{-j2\omega_c t})\frac{\sin \omega_c t}{\pi t} = \underbrace{(2\cos 2\omega_c t)}_{g(t)} \cdot \frac{\sin \omega_c t}{\pi t} \longleftarrow = e^{j2\omega_c t} \times \frac{\sin \omega_c t}{\pi t} + e^{-j2\omega_c t}\frac{\sin \omega_c t}{\pi t}$

6.18



Can step response show oscillatory behavior?



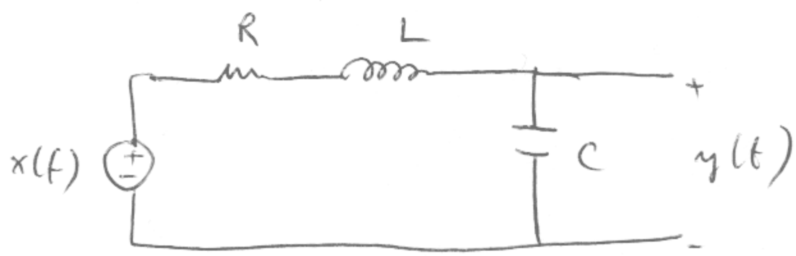
1) Frequency domain: $\hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC} = \hat{H}(j\omega)$

If $\hat{X}(j\omega) = \ ;\ \hat{Y}(j\omega) = \hat{X}\hat{H} = \frac{\frac{1}{j\omega} + \pi\delta(\omega)}{1 + j\omega RC}$ → oscillation in time?

Table 4.2: $\frac{1}{j\omega} + \pi\delta(\omega)$

NO. (denominator is of first order → in time domain it is an exponential)

6.19



Can step response have osc., how to eliminate oscillation?

$\hat{H} = \frac{\hat{Y}}{\hat{X}} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + (j\omega)^2 LC + 1}$

second order denominator → ripple in time.

How to eliminate ripple?

Quadratic pole: "Standard way to write"

$$\frac{1}{1 + 2(j\omega\tau\zeta) + (j\omega\tau)^2}$$

To eliminate osc. (ripple) $\to \zeta \geq 1$

$$1 + j\omega RC + (j\omega\sqrt{LC})^2 = 1 + 2j\omega\tau\zeta + (j\omega\tau)^2$$

$$\tau = \sqrt{LC}$$

$$RC = 2\sqrt{LC}\,\zeta$$

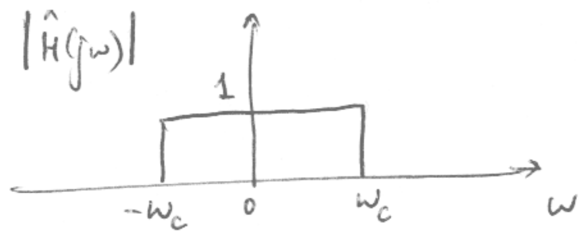
$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$\to$ no ripple: $\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$

or $\boxed{R \geq 2\sqrt{\frac{L}{C}}}$

6-23

Low-pass filter:



Sketch impulse response $h(t)$ for
a) Phase $(\hat{H}(j\omega)) = 0$
b) Phase $(\hat{H}(j\omega)) = \omega T$
c) Phase $(\hat{H}(j\omega)) = \begin{cases} \frac{\pi}{2} & \omega > 0 \\ -\frac{\pi}{2} & \omega < 0 \end{cases}$

$h(t) = F^{-1}[\hat{H}(j\omega)]$

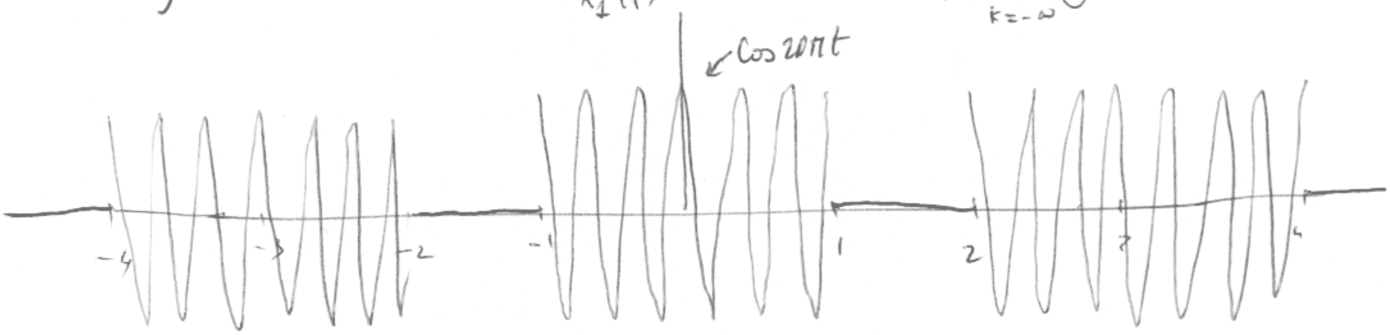
a) $\hat{H}(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases} \to h(t) = \frac{\sin \omega_c t}{\pi t}$

b) $\hat{H}(j\omega) = |\hat{H}(j\omega)|\, e^{j\omega T} \to h(t) = \frac{\sin \omega_c(t+T)}{\pi(t+T)}$ ✓

$\hat{z} = |\hat{z}|\, e^{j\,\text{phase}(\hat{z})} = (|\hat{z}| \cos[\text{phase}(\hat{z})],\ \hat{z} \sin[\text{phase}(\hat{z})])$

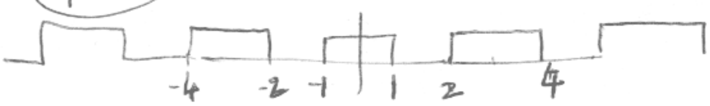
3.46 c) i)

$$x_1 = \sum_{k=-\infty}^{\infty} \overset{?}{c_k}\, e^{-jk\omega_0 t}$$



$$x_1(t) = x(t)\, y(t) \qquad y(t) \rightarrow \cos 20\pi t$$

$T = 3$



$$\left\{ \begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t} \\ y(t) &= \sum_{k=-\infty}^{\infty} b_k e^{-jk\omega_0 t} \end{aligned} \right\} \qquad \omega_0 \overset{?}{=} \frac{2\pi}{T} = \frac{2\pi}{3}$$

$$c_k = a_k * b_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}$$

---

b_k:

$$x(t) = \cos 20\pi t = \frac{e^{j20\pi t} + e^{-j20\pi t}}{2} = \frac{1}{2} e^{j30\omega_0 t} + \frac{1}{2} e^{-j30\omega_0 t}$$

$$\Rightarrow b_k = \frac{1}{2}\delta(k-30) + \frac{1}{2}\delta(k+30)$$

---

a_k:
pg26

$$\left\{ \begin{aligned} T &= 3 \\ T_1 &= 1 \end{aligned} \right. \gg a_k = \frac{\sin k\omega_0}{\pi k} = \frac{\sin \frac{2\pi k}{3}}{\pi k}$$

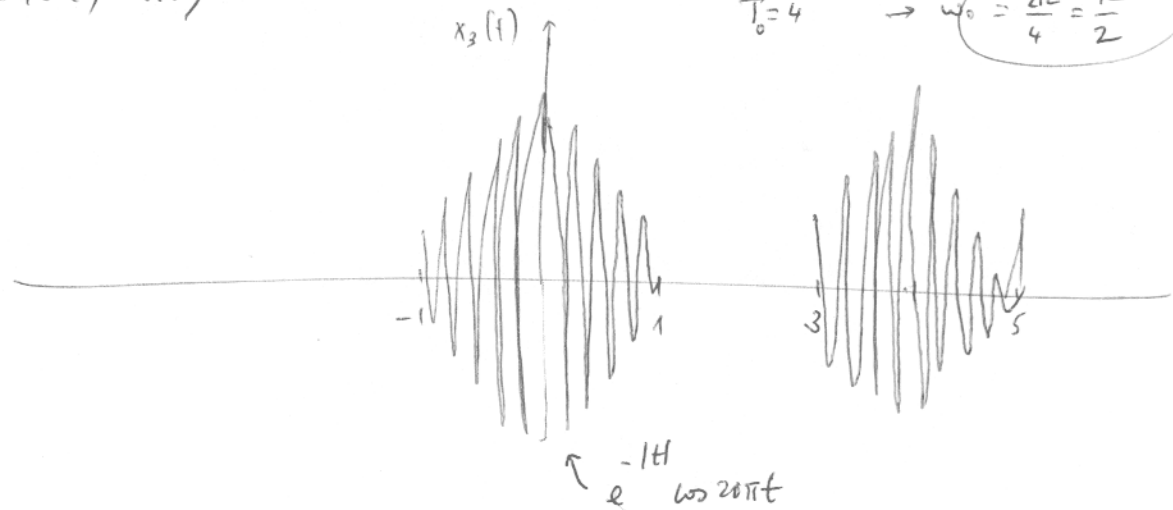
$$c_k = a_k * b_k = \left[\frac{1}{2}\delta(k-30) + \frac{1}{2}\delta(k+30)\right] * \frac{\sin\frac{2\pi k}{3}}{\pi k}$$

$$= \frac{1}{2}\,\frac{\sin\frac{2\pi}{3}(k-30)}{\pi(k-30)} + \frac{1}{2}\,\frac{\sin\frac{2\pi}{3}(k+30)}{\pi(k+30)}$$

property of convolution

3.46 c) iii)

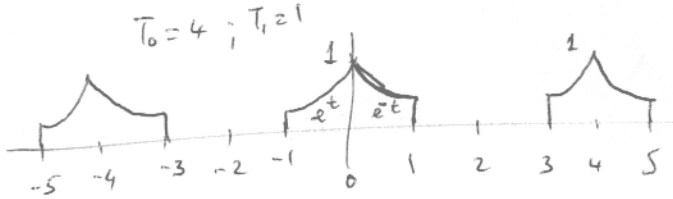
$T_0 = 4 \quad \to \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$



Can't convolute c_k in part i) with d_k in $e^{-|t|} = \sum_{k=-\infty}^{\infty} d_k e^{-jk\omega_0 t}$ b/c $e^{-|t|}$ only applies within each rectangular pulse.

$x_3(t) = x(t)\, y(t)$, $y(t) \to \cos 20\pi t \quad \longrightarrow \quad b_k = \frac{1}{2}[\delta(k-40) + \delta(k+40)]$

$T_0 = 4 \;;\; T_1 = 1$



$\to a_k = ?$

Similar to page 26: $a_k = \frac{1}{T_0}\int_{-1}^{1} dt\, e^{-jk\omega_0 t}\, e^{-|t|} = \frac{1}{4}\int_{-1}^{0} dt\, e^{-jk\frac{\pi}{2}t}\, e^{t}$

$$+ \frac{1}{4}\int_{0}^{1} dt\, e^{-jk\frac{\pi}{2}t}\, e^{-t} = \frac{1}{4}\int_{-1}^{0} dt\, e^{(1-jk\frac{\pi}{2})t} + \frac{1}{4}\int_{0}^{1} dt\, e^{-(1+jk\frac{\pi}{2})t}$$

$$= \frac{1}{4}\left[\frac{e^{(1-jk\frac{\pi}{2})t}}{1-jk\frac{\pi}{2}}\right]_{-1}^{0} + \frac{1}{4}\left[\frac{e^{-(1+jk\frac{\pi}{2})t}}{-(1+jk\frac{\pi}{2})}\right]_{0}^{1}$$

$$= \frac{1 - e^{-(1-jk\frac{\pi}{2})}}{4(1-jk\frac{\pi}{2})} + \frac{e^{-(1+jk\frac{\pi}{2})} - 1}{-4(1+jk\frac{\pi}{2})}$$

$$= \frac{1}{4}\,\frac{\left(1 - e^{-(1-jk\frac{\pi}{2})}\right)\left(1+jk\frac{\pi}{2}\right) - \left(1-jk\frac{\pi}{2}\right)\left(e^{-(1+jk\frac{\pi}{2})} - 1\right)}{\left(1 + \frac{k^2\pi^2}{4}\right)}$$

$$= \frac{1}{4}\,\frac{1}{1+\frac{k^2\Omega^2}{4}}\left[2 - \left(1+j\frac{k\Omega}{2}\right)e^{-\left(1-j\frac{k\Omega}{2}\right)} - \left(1-j\frac{k\Omega}{2}\right)e^{-\left(1+j\frac{k\Omega}{2}\right)}\right]$$

$$= \frac{1}{4}\,\frac{1}{1+\frac{k^2\Omega^2}{4}}\left[2 - \underbrace{\left(e^{j\frac{k\Omega}{2}} + e^{-j\frac{k\Omega}{2}}\right)}_{2\cos\frac{k\Omega}{2}} e^{-1} + \underbrace{j\frac{k\Omega}{2}\left(-e^{-j\frac{k\Omega}{2}} + e^{j\frac{k\Omega}{2}}\right)e^{-1}}_{-\,k\Omega e^{-1}\sin\frac{k\Omega}{2}}\right]$$

$$a_k = \frac{1}{4}\,\frac{1}{1+\frac{k^2\Omega^2}{4}}\left[2 - e^{-1}\left\{2\cos\frac{k\Omega}{2} + k\Omega\sin\frac{k\Omega}{2}\right\}\right]$$