$$\underline{\sum_{n=-\infty}^{\infty} e^{-j\omega n T} = \int}$$

$$S = 1 + a + a^2 + \cdots a^{\infty} = \sum_{n=0}^{\infty} a^n \quad \left(a = e^{-j\omega T}\right)$$

$$S-1 = a + a^2 + \cdots a^{\infty}$$

$$= a\underbrace{\left(1 + a + \cdots \underbrace{a^{\infty-1}}_{a^{\infty}}\right)}_{S}$$

$$S-1 = aS \rightarrow S(1-a) = 1 \rightarrow S = \frac{1}{1-a}$$

$$S' = \sum_{n=-\infty}^{\infty} a^n = \underbrace{\sum_{n=-\infty}^{-1} a^n}_{\frac{1}{1-\frac{1}{a}} - 1} + \underbrace{1 + \underbrace{\sum_{n=1}^{\infty}}_{\frac{1}{1-a} - 1}}$$

$$= \frac{1}{1-\frac{1}{a}} - 1 + \frac{1}{1-a}$$

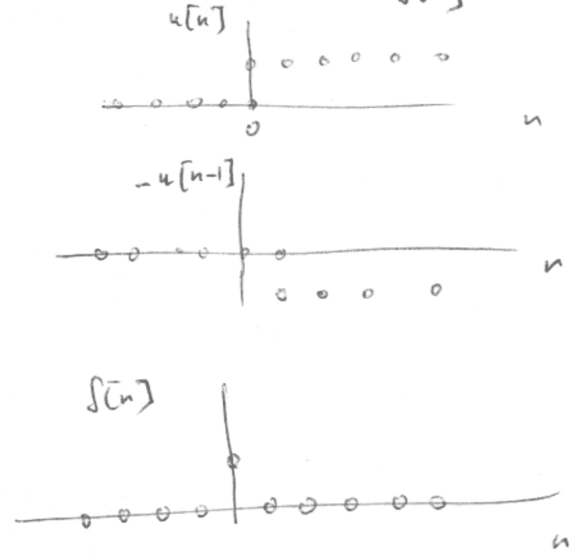
$$= \frac{a}{a-1} - \frac{1}{a-1} - 1$$

4/ $x[n] \equiv e^{an} u[n]$ (1)

Pls show: $x[n] - e^{a} x[n-1] = e^{an} \delta[n]$ ✓

(1) $\Rightarrow$ $e^{an} u[n] - \underbrace{e^{a} e^{a(n-1)}}_{e^{an}} u[n-1] = e^{an} \big(\underbrace{u[n] - u[n-1]}_{\delta[n]} \big)$

$e^{A} e^{B} = e^{A+B}$



1.2b/ Continuous-time: $e^{j\omega t}$

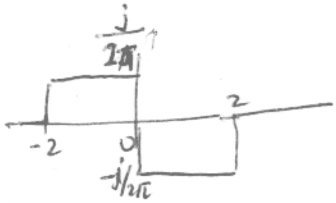
$e^{j(\omega+2\pi)t} = \underbrace{e^{j2\pi t}}_{\neq 1} e^{j\omega t}$

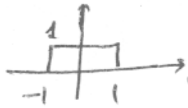
Discrete-time signal: $e^{j\omega n}$

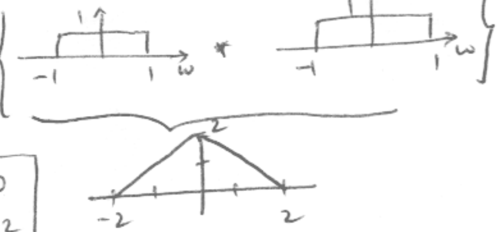
$e^{j(\omega+2\pi)n} = \underbrace{e^{j2\pi n}}_{1 \text{ always.}} e^{j\omega n}$

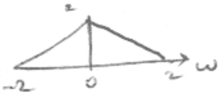
(for n: integers)

4.10 Use tables 4.1 & 4.2 :

a) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2 \longrightarrow \hat{X}(j\omega) =$ 

$\frac{\sin t}{\pi t} \xrightarrow[T4.2]{}$ 

; $\left(\frac{\sin t}{\pi t}\right)^2 \xrightarrow[T4.1]{} \frac{1}{2\pi}\Big\{$  $\Big\}$

$t\left(\frac{\sin t}{\pi t}\right)^2 \xrightarrow[T.4.1]{} j\frac{d}{d\omega}\Big(\frac{1}{2\pi}\cdot$  $\Big) = \boxed{\frac{j}{2\pi}\begin{cases} 1 & -2<\omega\le 0 \\ -1 & 0<\omega\le 2\end{cases}}$

b) Use Parseval's relation and a) to find A:

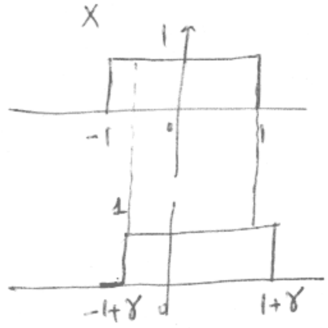
$$A = \int_{-\infty}^{\infty} t^2\left(\frac{\sin t}{\pi t}\right)^4 dt = \frac{1}{2\pi}\int_{-2}^{2}\frac{1}{4\pi^2}\,d\omega = \frac{1}{8\pi^3}\left[\omega\right]_{-2}^{2} = \frac{1}{2\pi^3}$$

Parseval's relation:

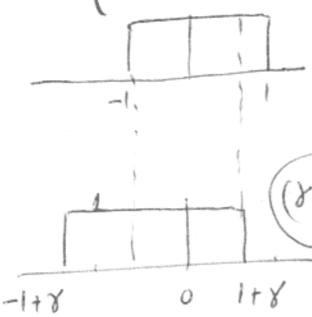
$$\int_{-\infty}^{\infty}|x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\hat{X}(j\omega)|^2 d\omega$$

Convolution of 2 rectangular pulses

In general: $\hat{X}(j\omega) * \hat{Y}(j\omega) = \int_{-\infty}^{\infty} d\gamma\, \hat{X}(j\omega)\,\hat{Y}(\gamma - j\omega)$
(reversed and shifted by γ)



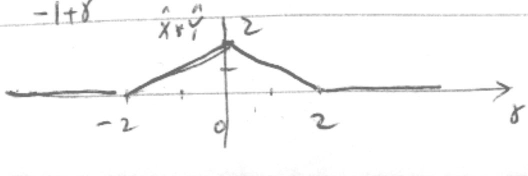
$(\gamma>0)$



$(\gamma<0)$

$$\hat{X}*\hat{Y} = \int_{-1}^{1+\gamma} d\gamma\, 1.1 = 1+\gamma-(-1) = 2+\gamma \quad (\gamma<0)$$

$$\hat{X}*\hat{Y} = \int_{-1+\gamma}^{1} d\gamma\, 1.1 = \gamma\Big]_{-1+\gamma}^{1} = 1-(-1+\gamma) = 2-\gamma \quad (\gamma>0)$$



This the convolution of with itself.

4.14

1) $x(t)$ real and nonnegative

2) $\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$

3) $\int_{-\infty}^{\infty} |\hat{X}(j\omega)|^2 d\omega = 2\pi$

What is $x(t)$?

$\hookrightarrow \int_{-\infty}^{\infty} |x(t)|^2 dt = 1$ (Find A)

2) $\Rightarrow$ $(1+j\omega)\hat{X}(j\omega) = F\{Ae^{-2t}u(t)\} = A\frac{1}{2+j\omega}$

$$\hat{X}(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)} = A\left\{\frac{1}{j\omega+1} - \frac{1}{j\omega+2}\right\}$$

Partial Fraction Expansion.

Then $x(t) = A\{e^{-t} - e^{-2t}\}u(t)$

$$\int_0^{\infty} \underbrace{(A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t})}_{x^2(t)} dt = A^2\frac{\left[e^{-2t}\right]_0^{\infty}}{-2} - 2A^2\frac{\left[e^{-3t}\right]_0^{\infty}}{-3} + A^2\frac{\left[e^{-4t}\right]_0^{\infty}}{-4}$$

$$= \frac{A^2}{2} - \frac{2A^2}{3} + \frac{A^2}{4} = \frac{6-8+3}{12}A^2$$

$$= \frac{A^2}{12} = 1 \Rightarrow A = \sqrt{12}$$

$$\Rightarrow \boxed{x(t) = \sqrt{12}\left(e^{-t} - e^{-2t}\right)u(t)}$$

4.19 LTI & causal $\hat{H}(j\omega) = \dfrac{1}{3+j\omega}$ $\left(h(t) = e^{-3t}u(t) \right)$
impulse response.

T.D $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = e^{-3t}u(t) - e^{-4t}u(t)$

What is $x(t)$?

F.D $\hat{X}(j\omega) \rightarrow \boxed{\hat{H}(j\omega)} \longrightarrow \hat{Y}(j\omega) = \dfrac{1}{3+j\omega} - \dfrac{1}{4+j\omega}$

$y(t) = x(t) * h(t) \longrightarrow \hat{Y} = \hat{H}\hat{X}$

$\rightarrow \hat{X} = \dfrac{\hat{Y}}{\hat{H}} = 1 - \dfrac{3+j\omega}{4+j\omega}$

$$\boxed{\hat{X} = \frac{1}{4+j\omega}}$$

$$\boxed{x(t) = e^{-4t}u(t)}$$