Ch 1: Intro to Signals and Systems

Signal: a function of time \( f(t) \)
- \( f(t) = \sin(\omega t + \phi) \)
- \( f(t) = \exp(-at) \)
- \( f[n] = \sin(2\pi fn) \)
- \( f[n] = \exp(-an) \)

Properties:
- Transformation of independent variable: \( t \rightarrow t' = at + b \)
- Shift
- Periodic (Sinusoids) & Non-periodic (Exponentials)

Complex representation of a periodic signal:

\[
\sin(\omega t + \phi) = \Im[e^{j\omega t}] = \Im[e^{j\omega t} + j\sin\omega t]
\]

\[
c_{\omega t} = R_e[e^{j\omega t}]
\]

\[
e^{j\omega t} = \begin{cases} e^{j\omega t} & \text{continuous (periodic)} \\ e^{j\omega nT} & \text{discrete (periodic?)} \end{cases}
\]

- Discrete-time signal is a series of values that are picked out from the continuous signal at regular time intervals (discretization of a continuous-time signal)

\[
\text{if } \Delta t = \frac{\pi}{3} \rightarrow \text{yes} \quad \text{if } \Delta t = 0.5 \rightarrow \text{no}
\]

\[
\omega = 3\pi \rightarrow 2\pi \frac{n}{T} \rightarrow T = \frac{3}{2} = 1.5
\]
Periodic signal: \( x(t+T) = x(t) \) \[
\sum_{n=N}^{N+M} x[n] = x[n]
\]

\[ e^{j\omega t} = e^{j\omega (t+T)} \]
\[ e^{j\omega t} = e^{j\omega T} \]
\[ e^{j\omega t} e^{j\omega T} \]
\[ e^{j\omega t} \]
\[ e^{j\omega (t+T)} \]
\[ e^{j\omega T} \]
\[ e^{j\omega T} \]
\[ e^{j\omega T} \]
\[ e^{j\omega T} = 1 \]

\[ 3\pi N = m\pi \]
\[ N = m \frac{\pi}{3\pi} = m \frac{2}{3} \]

For \( N \) to be an integer, \( m = 3 \)

\[ N = 2 \]
\[ T = \frac{2\pi}{3} \]

- Period for \( e^{j3t} \):
  \[ 3T = 2\pi \]

- Period for \( e^{j3n} \):
  \[ 3N = m2\pi \]
  \[ N = m\frac{2\pi}{3} \]

- Conclusion: \( e^{j3t} \) is always periodic but \( e^{j3n} \) is not periodic because it does not contain \( T \) (a difference \( \delta \) in continuous time & discrete-time signal).

- Unit impulse:
  \[ \delta(t) \]
  \[ \delta(t) = \frac{d\delta(t)}{dt} \]

- Unit step function:
  \[ u(t) \]
  \[ u(n) \]

Integrals involving \( \delta(t) \):
\[ \int_{-\infty}^{\infty} f(t) \delta(t) \, dt = f(0) \]
\[ \int_{-\infty}^{\infty} f(t-a) \delta(t-a) \, dt = f(a) \]

Question:
\[ \int_{-3}^{3} f(t) \delta(t) \, dt = f(0) \]
\[ \int_{-3}^{3} f(t) \delta(t-4) \, dt = 0 \]
Systems

\[ x(t) \xrightarrow{\text{system}} y(t) \]

can be modelled with an equation relating input to output:

\[ y(t) = 2x(t) - x^2(t+1) \]

Properties:

1. Memory (if outputs depend on past inputs)
2. Invertibility
   \[ \xrightarrow{\text{system}} \xrightarrow{\text{inverse system}} x(t) \]
3. Causality (if output just depends on current and past inputs)
4. Stability: small inputs would not produce extremely large outputs.
5. Time invariance: same input gets same output regardless of time
6. Linearity: \[ ax \xrightarrow{\text{linear system}} ay \]
   \[ x_1 + x_2 \xrightarrow{\text{linear system}} f_1 + f_2 \]

\[ y(t) = x(t)^2 \]

Is a system represented by this equation linear?

...nested diagrams...

\[ x \xrightarrow{\text{Sin}(x)} y \]

\[ ax \xrightarrow{\text{Sin}(x)} \text{Sin}(ax) \neq a\text{Sin}x \rightarrow \text{Nonlinear System} \]