http://www.faculty.umb.edu/tomas_materdey/321F05/

Ch 1: Intro to Signals and Systems

HW1: 1.10; 1.11; 1.36; 1.39
1.15; 1.16; 1.27; 1.28; 1.31; 1.46

Signal: a function of time

$f(t)$: $= \sin(2\pi f t)$, $= \exp(-at)$

$f[n]$: $= \sin[2\pi f n]$, $= \exp[-an]$

→ **Properties**:

- Transformation of independent variable $t \to t' = \underset{\text{scaling}}{a}t + \underbrace{b}_{\text{shift}}$

- Periodic (sinusoids) & Non-periodic (exponentials)

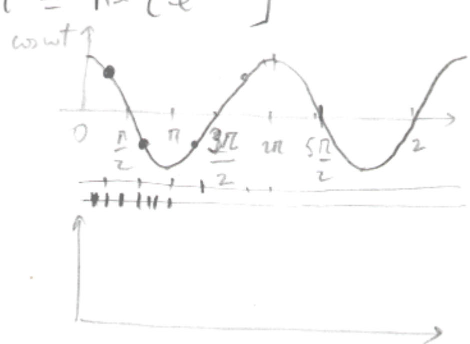
Complex representation of a periodic signal:

$\sin(\underbrace{2\pi f}_{\omega} t) = Im[\underset{\text{complex number}}{e^{j\omega t}}] = Im[\cos\omega t + j\sin\omega t]$

$\cos\omega t = Re[e^{j\omega t}]$

$e^{j\omega t}$

$e^{j\omega n}$



continuous (periodic)

discrete (periodic ?)

→ Discrete-time signal is a series of values that are picked out from the continuous signal at regular time interval (discretization of a continuous-time signal)

↳ if $\Delta t = \frac{\pi}{3} \to$ yes)

if $\Delta t = 0.5 \to$ no)

$e^{j3\pi t}$

$\omega = 3\pi \Rightarrow 2\pi\frac{1}{T} \to T = \frac{3}{2} = 1.5$

parentheses. ↓ square brackets ↓

Periodic signal: $x(t+T) = x(t)$ ‖ $x[n+N] = x[n]$

→ What is the period for $x(t) = e^{j\omega t}$?

$$e^{j\omega(t+T)} = \cancel{e^{j\omega t}}$$

$$\cancel{e^{j\omega t}}\, \underbrace{e^{j\omega T}}_{1} \rightarrow \underline{\omega T = 2\pi} \text{ or } n2\pi$$

$$T = \frac{2\pi}{\omega}$$

yes! b/c $\omega = 2\pi f$.

→ What is the period for $e^{j3\pi t}$? → $\boxed{T = \frac{2\cancel{\pi}}{3\cancel{\pi}} = \frac{2}{3}}$

→ What is the period for $e^{j3\pi n}$?

$$e^{j3\pi[n+N]} = e^{j3\pi n}$$

$$\cancel{e^{j3\pi n}} \cdot e^{j3\pi N} = \cancel{e^{j3\pi n}}$$

$$e^{j3\pi N} = 1$$

$$3\pi N = m2\pi$$

$$N = m\frac{2\cancel{\pi}}{3\cancel{\pi}} = m\frac{2}{3}$$

For N to be an integer, $m = 3$

$\boxed{N = 2}$

→ Period for e^{j3t} ? → $T = \frac{2\pi}{3}$

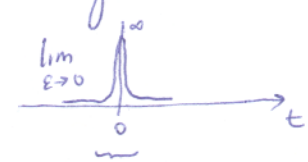
$3T = 2\pi$

→ Period for e^{j3n} ? → $N =$ none, e^{j3n} is NOT periodic!

$3N = m\,2\pi \rightarrow N = m\,\frac{2\pi}{3}$

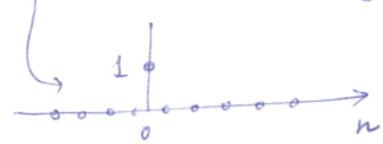
→ Conclusion: $e^{j\omega t}$ is always periodic but $e^{j\omega n}$ is not periodic if ω doesn't contain π (a difference b/w continuous time & discrete-time signal.

Unit impulse: δ (delta) $\begin{cases} \delta(t) \\ \delta[n] \end{cases}$



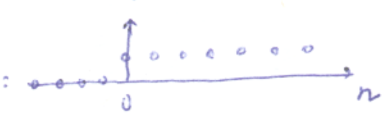
"zero-width" impulse centered at $t=0$

(huge bump at $t=0$, and zero otherwise)



$\delta(t) = \frac{d\,u(t)}{dt}$

Unit step function: u $\begin{cases} u(t) \\ u[n] \end{cases}$



Integrals involving $\delta(t)$:

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

center of impulse function

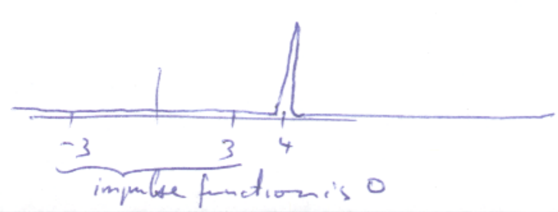
$$\int_{-\infty}^{\infty} f(t)\,\underbrace{\delta(t-a)}\,dt = f(+a)$$

center of impulse function

$\delta(t)$ shifted by a

Question: $\int_{-3}^{3} f(t)\,\delta(t)\,dt = f(0)$

$\int_{-3}^{3} f(t)\,\delta(t-4)\,dt = 0$



Systems

$x(t) \xrightarrow{\text{input}} \boxed{\text{System}} \xrightarrow{\text{output}} y(t)$

can be modelled with an equation relating input to output.

$$y(t) = 2x(t) - x^2(t+1)$$

Properties:
↓
qualitative
(HW for quantitative aspects)

1) Memory (if ~~future~~ current outputs depend on past inputs)

2) Invertibility:

$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow \boxed{\text{System}^{-1}} \longrightarrow x(t)$

system — Inverse system

3) Causality (if output just depends on current and past inputs)

Causality

4) Stability: small inputs would not produce ~~oo~~ xtremely large outputs.

5) Time invariance: same input gets same output regardless of time

6) Linearity:

$ax \longrightarrow \boxed{\text{linear System}} \longrightarrow ay$

$x_1 + x_2 \longrightarrow \boxed{\text{linear system}} \longrightarrow y_1 + y_2$

$y(t) = x^2(t)$ is a system represented by this equation linear?

$ax \longrightarrow \boxed{y = x^2} \longrightarrow a^2x^2 = a^2y \rightarrow$ Nonlinear System

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$x \longrightarrow \boxed{\sin(x)} \longrightarrow y$

$ax \longrightarrow \boxed{\sin(x)} \longrightarrow \sin(ax) \neq a\sin x \rightarrow$ Nonlinear