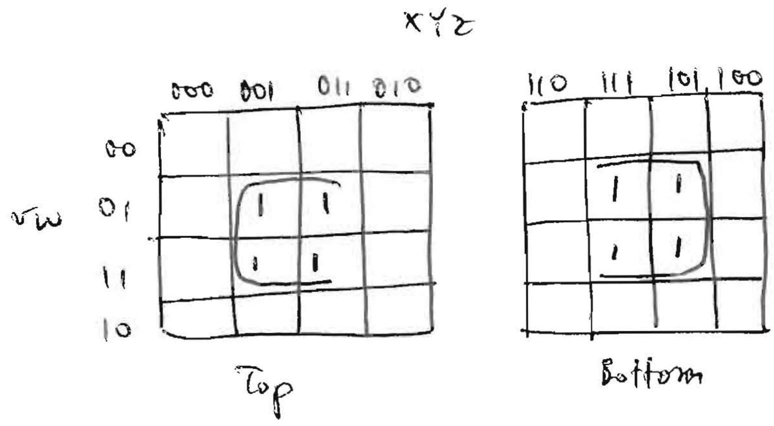
2D K-map for a 5 variable Boolean function:



Note: these eight 1's are consecutive!
(Flip the bottom map & put it below the top)

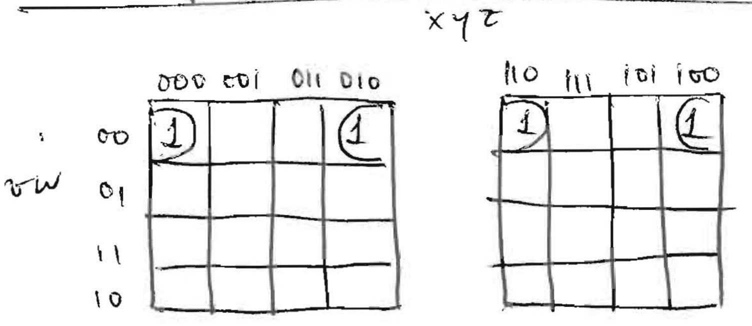
Let's eliminate those variables that change value in this group:

$v, y, x \quad \rightarrow \quad f(v,w,x,y,z) = wz$

Eight consecutive 1's $\longrightarrow$	Product terms: Simplify: eliminate 3 variables

We've seen examples of
- 4 variables → 1 variable
- 5 variables → 2 variables

2D Kmap for a 5 variable Boolean function:



Note: these 1's are part of a group of four!

This is an effective 3 variable function (eliminated: x, y)

$f(v,w,x,y,z) = \overline{v}\,\overline{w}\,\overline{z}$

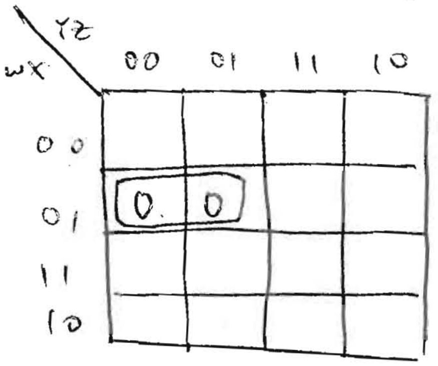
$f(w,x,y,z) = x + y$ $\qquad$ $f(w,x,y,z) = y$

K-Map Rules for Sum Terms or Product of Sums (POS)

K-Map for four-variable Boolean functions with

2 , 4 , 8 zeroes

$$\begin{array}{ccc} 2 & 4 & 8 \\ \downarrow & \downarrow & \downarrow \\ \text{Eliminate} & \text{Eliminate} & \text{Eliminate} \\ ①\text{ variable} & ②\text{ variable} & ③\text{ variable} \\ 2 = 2^{①} & 4 = 2^{②} & 8 = 2^{③} \end{array}$$



Max Term Canonical or POS:

$$f(w,x,y,z) = (\underbrace{w+\bar{x}+y}_{A}+\underbrace{z}_{B})(\underbrace{w+\bar{x}+y}_{A}+\underbrace{\bar{z}}_{C})$$

$$= (A+B)\cdot(A+C)$$

Distributive property $= A + B\cdot C$

$$= w+\bar{x}+y+\underbrace{z\cdot\bar{z}}_{0}$$

$$= w+\bar{x}+y$$

f is an effective 3-variable Boolean function, one variable, z (changes value in the group) got eliminated because of the group of 2 consecutive 0's

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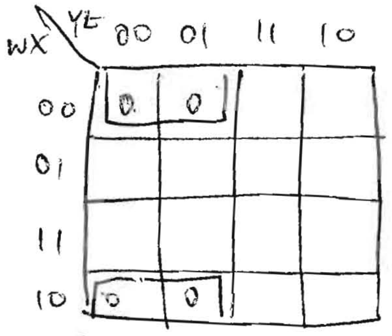
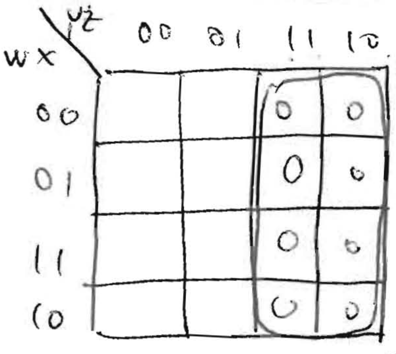
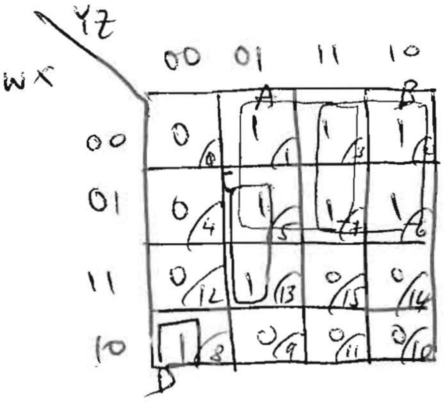


$f(w,x,y,z) = x + y$



$f(w,x,y,z) = \bar{y}$

Simplifying a Boolean Function using Visual Method of K-Map:



wx \ yz	00	01	11	10
00	0 (0)	1 (1)	1 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	1 (13)	0 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	0 (10)

This 4-variable Boolean function:

MinTC or SOP : 8 terms

$\sum m(1, 2, 3, 5, 6, 7, 8, 13)$

MaxTC or POS : 8 terms

$\prod M(0, 4, 9, 10, 11, 12, 14, 15)$

Simplified version of $f(w, x, y, z)$:

1) Biggest group of ~~Co~~nsecutive 1's : 1 2 ④ 8

$$\underset{\bar{w}z \qquad \bar{w}y}{\overset{A \,\&\, B}{\swarrow \quad \searrow}}$$

2) Look for next smaller groups of consecutive 1's

1 ② 4 8

that are not entirely included the the previous bigger groups

$$\begin{array}{c} C \\ \downarrow \\ x\bar{y}z \end{array}$$

3) Look for next smaller groups of consecutive 1's not entirely included in the previous bigger groups.

① 2 4 8

$$\begin{array}{c} D \\ \downarrow \\ w\bar{x}\bar{y}\bar{z} \end{array}$$

$$f(w, x, y, z) = \underset{A}{\bar{w}z} + \underset{B}{\bar{w}y} + \underset{C}{x\bar{y}z} + \overset{D}{w\bar{x}\bar{y}\bar{z}}$$

Can implement this reduced version into a logic circuit!

Six-variable K-Map: $f(u,v,w,x,y,z) \rightarrow 2^6 = 64$ combinations.

2D:

xyz

uvw	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	41	43	42	46	47	45	44
100	32	33	35	34	38	39	37	36

Cell number are shown for decimal notations.

→ Groups on consecutive ones or zeroes: 2, 4, 8, 16, 32.

- 2 ↓ 1 less variable
- 4 ↓ 2 less variables
- 8 ↓ 3 less variables
- 16 ↓ 4 less variables

K Map on 7-segment functions:

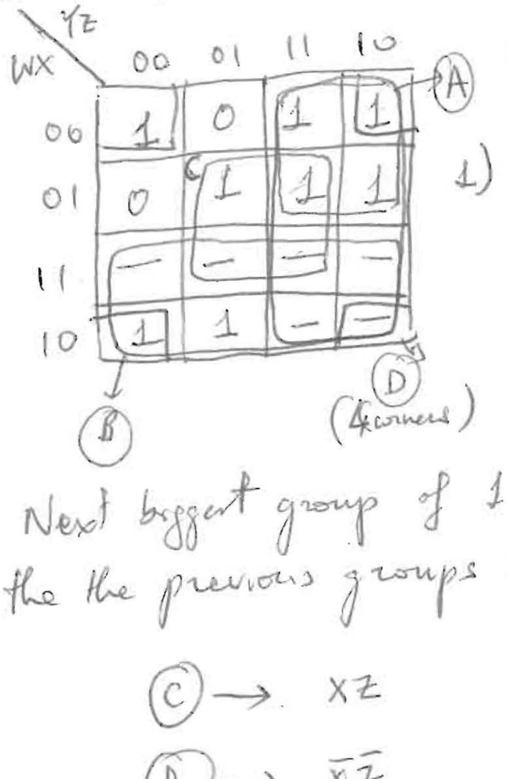
↓ Used to display alphanumeric characters. Each segment can be implemented using a logic combinational circuit/network.

Truth Table

WXYZ	Decimal	Segment a	Segment b	Segment c
0000	0	1	1	1
0001	1	0	1	1
0010	2	1	1	0
0011	3	1	1	1
0100	4	0	1	1
0101	5	1	0	1
0110	6	1	0	1
0111	7	1	1	1
1000	8	1	1	1
1001	9	1	1	1

The other 6 combinations → "don't care" or dc

K-map for segment a :



WX \ YZ	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	—	—	—	—
10	1	1	—	—

Write simplest SOP for a(w, x, y, z)

1) Biggest groups consecutive 1's & dc's : 2, ④, ⑧, ↑

Ⓐ → y ; Ⓑ → w

2) Next biggest group of 1's & dc's not entirely included the the previous groups (A & B) 2, ④, 8 ↑

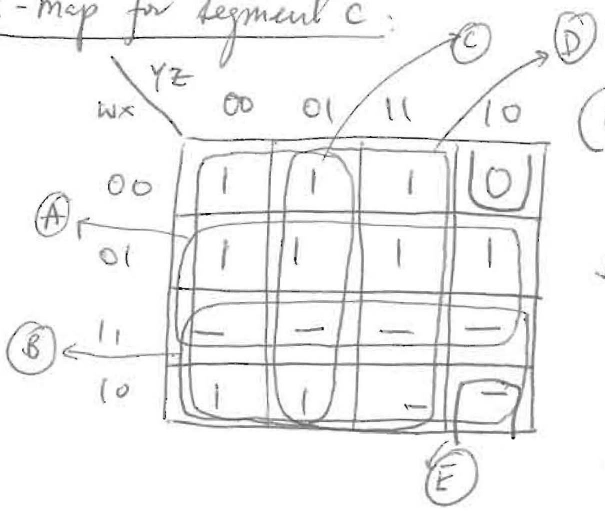
Ⓒ → xz

Ⓓ → $\bar{x}\bar{z}$

3) Next: ②, 4, 8 : all groups of two consecutive 1's & dc are included in previous A, B, C, D.

$a(w,x,y,z) = y + w + xz + \bar{x}\bar{z}$

K-map for segment c:



wx \ yz	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	–	–	–	–
10	1	1	–	–

(i) Write simplest SOP for $c(w,x,y,z)$

1) Biggest group of consecutive 1's & dc's:
2, 4, ⑧

Ⓐ → x Ⓑ → w Ⓒ → $\bar{y}$ ⓓ → z

2) Next biggest group of 4 : none (none that is not entirely included in A, B, C, D)
2, ④, 8

$\rightarrow c(w,x,y,z) = w + x + \bar{y} + z$ (Simplest SOP)

(ii) Write simplest POS : 1) Biggest group of consecutive 0's & dc's:
②, 4, 8
Ⓔ → eliminate $w \Rightarrow (x+\bar{y}+z)$

2) None (nothing less than 2 !)

$c(w,x,y,z) = x + \bar{y} + z$ (simplest POS)

Note: comparing with previous expression (SOP):
1) $w=0$ → same anyway
2) $w=1$ → dc's.
} Same! as they agree

Alternative to K-Map: Quine - McCluskey Method or QM

Example #1 $f(w,x,y,z) = \sum m(1,3,4,5,7,8,15)$

$$= \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + wxyz$$

m	w x y z	f
0	0 0 0 0	0
1	0 0 0 1	1
2	0 0 1 0	0
3	0 0 1 1	1
4	0 1 0 0	1
5	0 1 0 1	1
6	0 1 1 0	0
7	0 1 1 1	1
8	1 0 0 0	1
9	1 0 0 1	0
10	1 0 1 0	0
11	1 0 1 1	0
12	1 1 0 0	0
13	1 1 0 1	0
14	1 1 1 0	0
15	1 1 1 1	1

QM =

1) Use $AB + \bar{A}B = B$ to simplify a variable!
2) In consequence: look at entries ~~that~~ where one variable changes its value. This variable will disappear!
3) In consequence: look at entries that differ in number of 1's by one only:

m	Column#1		# of ones (index)
1	$\bar{w}\bar{x}\bar{y}z$	0001	1
3	$\bar{w}\bar{x}yz$	0011	2
4	$\bar{w}x\bar{y}\bar{z}$	0100	1
5	$\bar{w}x\bar{y}z$	0101	2
7	$\bar{w}xyz$	0111	3
8	$w\bar{x}\bar{y}\bar{z}$	1000	1
15	$wxyz$	1111	4

Reorder by on index

m	Column#1		index
1.	$\bar{w}\bar{x}\bar{y}z$	0001	1
4	$\bar{w}x\bar{y}\bar{z}$	0100	1
8	$w\bar{x}\bar{y}\bar{z}$	1000	1
3.	$\bar{w}\bar{x}yz$	0011	2
5.	$\bar{w}x\bar{y}z$	0101	2.
7	$\bar{w}xyz$	0111	3
15	$wxyz$	1111	4

→ Explanation: $AB + \bar{A}B = B$ won't be applicable if we combine entries of index 1 and index 3 or 1 & 4 or 2 & 4

It only works if we combine entries of consecutive indices!
(index 1 & 2, index 2 & 3, index 3 & 4, etc.)

For example: $\underbrace{m=1}_{\text{index1}}$ & $\underbrace{m=3}_{\text{index2}}$: $\bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}yz = \bar{w}\bar{x}z(\underbrace{\bar{y}+y}_{1})$

$\underset{0001}{\uparrow}$ $\underset{0011}{\uparrow}$ → $\boxed{\begin{matrix}00-1 & \text{(y is gone)} \\ wxyz & \end{matrix}}$

Colu $\underbrace{m=1}_{\text{index1}}$ & $\underbrace{m=5}_{\text{index2}}$: $\bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}z = \bar{w}\bar{y}z(\underbrace{\bar{x}+x}_{1})$

$\underset{0001}{\uparrow}$ $\underset{0101}{\uparrow}$ → $\boxed{\begin{matrix}0-01 & \text{(x is gone)} \\ wxyz & \end{matrix}}$

QM (continued)

... Column #1

	m	Boolean variable entry	Binary entry	Index
	1. ✓	$\bar{w}\bar{x}\bar{y}z$	0001	1
	4 ✓	$\bar{w}x\bar{y}\bar{z}$	0100	1
PI	8	$w\bar{x}\bar{y}\bar{z}$	1000	1
	3 ✓	$\bar{w}\bar{x}yz$	0011	2
	5 ✓	$\bar{w}x\bar{y}z$	0101	2
	7 ✓	$\bar{w}xyz$	0111	3
	15 ✓	$wxyz$	1111	4

Column #2

m combo	Binary combo	Index
1,3	00−1	1
1,5	0−01	1
4,3	——	
4,5	010−	1
8,3	——	
8,5	——	
3,7	0−11	2
5,7	01−1	2
7,15	−111	3

Column #2

	m combo	binary combo	index
	1,3 ✓	00−1	1
	1,5 ✓	0−01	1
PI	4,5	010−	1
	3,7 ✓	0−11	2
	5,7 ✓	01−1	2
PI	7,15	−111	3

Column #3

m combo	binary combo	index	
1,3 & 3,7	——		
1,3 & 5,7	0−−1	1	PI
1,5 & 3,7	0−−1	1	
1,5 & 5,7	——		
4,5 & 3,7	——		
4,5 & 5,7	——		
3,7 & 7,15	——		
5,7 & 7,15	——		

Important: mark which entry has been used in a combination ($AB+\bar{A}B = B$) in a subsequent column.

QM has identified 4 prime implicants:

$w\bar{x}\bar{y}\bar{z}$; $\bar{w}x\bar{y}$; xyz ; $\bar{w}z$

010-   -111   0--1

To write $f(w,x,y,z) = \sum_m m(1,3,4,5,7,8,15)$ in lowest cost form:

find <u>essential prime implicants</u>.

<u>Prime Implicant Table</u> : a PI is essential if needed to cover a minterm

		m_1	m_3	m_4	m_5	m_7	m_8	m_{15}
m_8	$A = w\bar{x}\bar{y}\bar{z}$						X	
m_4, m_5	$B = \bar{w}x\bar{y}$			X	X			
m_7, m_{15}	$C = xyz$					X		X
m_1, m_3, m_5, m_7	$D: \bar{w}z$	X	X		X	X		

{ Minterms in columns
{ Prime Implicants in rows

<u>Essential PI's</u>

- for m_1 : D
- for m_3 : D
- for m_4 : B
- for m_5 : B or D ✓
- for m_7 : C or D ✓
- for m_8 : A ✓
- for m_{15} : C ✓

All four PI's are essential in this example.

→ $\boxed{f_{min}(w,x,y,z) = w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y} + xyz + \bar{w}z}$

between groups of consecutive indices. (QM method)

2nd Example Using QM but with don't care entries (DC).

$$f(v,w,x,y,z) = \sum m(4,5,9,11,12,14,15,27,30) + dc(1,17,25,26,31)$$

5 variables : $2^5 = 32$ entries $\begin{cases} 9 \text{ ones} \\ 18 \text{ zeros} \\ 5 \text{ dc's} \end{cases}$

KM: dc's were considered as 1 → same in QM!

→ We have here 9+5 = 14 product terms to simplify to lowest cost.

Use columns to find PI's & essential PI's

Classify entries by indices: → next page

	m	binary entry	index
ones	4	00100	1
	5	00101	2
	9	01001	2
	11	01011	3
	12	01100	2
	14	01110	3
	15	01111	4
	27	11011	4
	30	11110	4
dc's	1	00001	1
	17	10001	2
	25	11001	3
	26	11010	3
	31	11111	5

Column # 1

m	binary entry	index
4 ✓	00100	1
1 ✓	00001	1
5 ✓	00101	2
9 ✓	01001	2
12 ✓	01100	2
17 ✓	10001	2
11 ✓	01011	3
14 ✓	01110	3
25 ✓	11001	3
26 ✓	11010	3
15 ✓	01111	4
27 ✓	11011	4
30 ✓	11110	4
31 ✓	11111	5

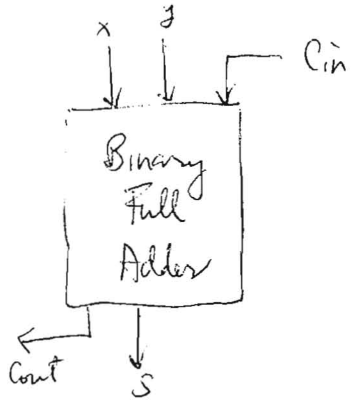
Column # 2

5

m combo	binary combo	index
4,5	0010–	1
~~4,9~~		
4,12	0–100	1
~~4,17~~		
1,5	00–01	1
1,9 ✓	0–001	1
~~4,12~~		
1,17 ✓	–0001	1
9,11 ✓	010–1	2
9,25 ✓	–1001	2
12,14	011–0	2
17,25 ✓	1–001	2
11,15 ✓	01–11	3
11,27 ✓	–1011	3
14,15 ✓	0111–	3
14,30 ✓	–1110	3
25,27 ✓	110–1	3
26,27 ✓	1101–	3
26,30 ✓	11–10	3
15,31 ✓	–1111	4
27,31 ✓	11–11	4
30,31 ✓	1111–	4

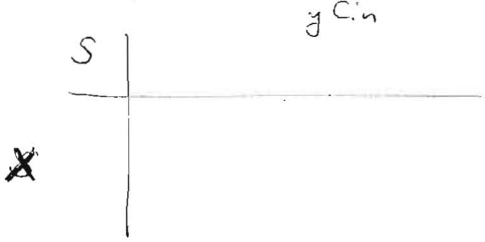
ignore bl

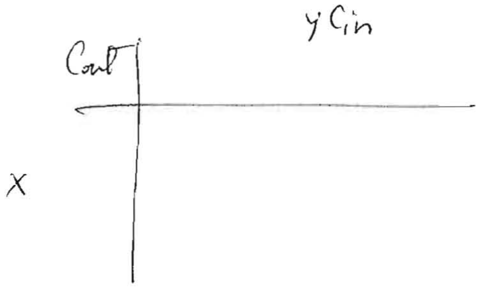
Quiz #2 3-24-11



x	y	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

1) KM for S & Cout





2) Simplify using these KM's

S =

Cout =

3) Use XOR's to simplify further :

$ab' + a'b = a \oplus b$

$a'b' + ab = \overline{a \oplus b}$

4) Draw logic diagram for S & Cout.

Looking at possible combinations in the sense of $AB' + AB = A$ between groups of consecutive indices. (QM method)

Column # 3

m combo	binary combo	index
1, 9, 17, 25	--001	1) same !
1, 17, 9, 25	--001	1
9, 11, 25, 27	-10-1	2) same
9, 25, 11, 27	-10-1	2
11, 15, 27, 31	-1-11	3)
11, 27, 15, 31	-1-11	3
14, 30, 15, 31	-111-	3)
14, 15, 30, 31	-111-	3
26, 27, 30, 31	11-1-	3)
26, 30, 27, 31	11-1-	3

Column #4

No more combo's

Important step: go back to all columns, mark which entry has been used in a combination ($AB' + AB = A$)

→ Total 9 prime implicants: (boxed)

0010- ($\bar{v}\bar{w}x\bar{y}$); 0-100 ($\bar{v}x\bar{y}\bar{z}$), 00-01 ($\bar{v}\bar{w}\bar{y}z$); 011-0 ($\bar{v}wx\bar{z}$), --001 ($\bar{x}\bar{y}z$); -10-1 ($w\bar{x}z$), -1-11 (wyz); -111- (wxy); 11-1- (vwy). Now to find essential PI's: build PI Table: m terms on columns (no dc's), PI on rows:

dc dc dc

			m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}
①, 9, ⑰, ㉕	A	$\bar{x}\bar{y}z$			X						
9, 11, ㉕, 27	B	$w\bar{x}z$			X	X				X	
11, 15, 27, ㉛	C	wyz				X			X	X	
14, 15, 30, ㉛	D	wxy						X	X		X
㉖, 27, 30, ㉛	E	vwy								X	X
①, 5	F	$\bar{v}\bar{w}\bar{y}z$		X							
4, 5	G	$\bar{v}\bar{w}x\bar{y}$	X	X							
4, 12	H	$\bar{v}x\bar{y}\bar{z}$	X				X				
12, 14	I	$\bar{v}wx\bar{z}$					X	X			