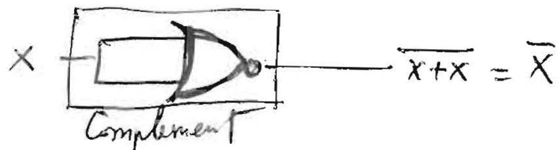
NOR as a universal gate:

a) Complement using NOR gate:

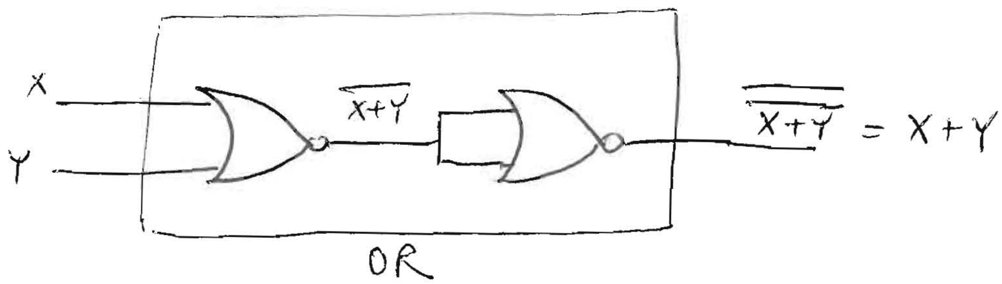
Absorption Law:

$$X + X = X$$

$$\rightarrow \overline{X + X} = \overline{X}$$



b) OR using 2 NOR's:

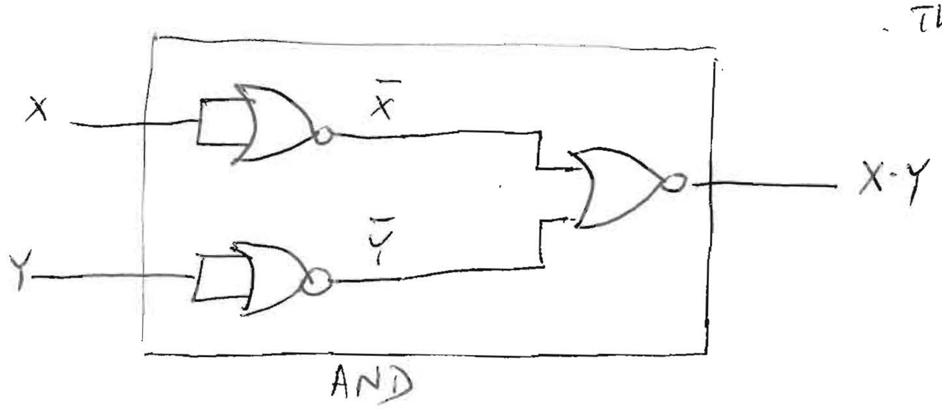


c) AND using 3 NOR's:

De Morgan's Law:

$$\overline{\overline{X} + \overline{Y}} = \overline{\overline{X}} \cdot \overline{\overline{Y}} = X \cdot Y$$

- Produce $\overline{X}$ & $\overline{Y}$ using 2 NORs
- Then a third NOR to produce $X \cdot Y$



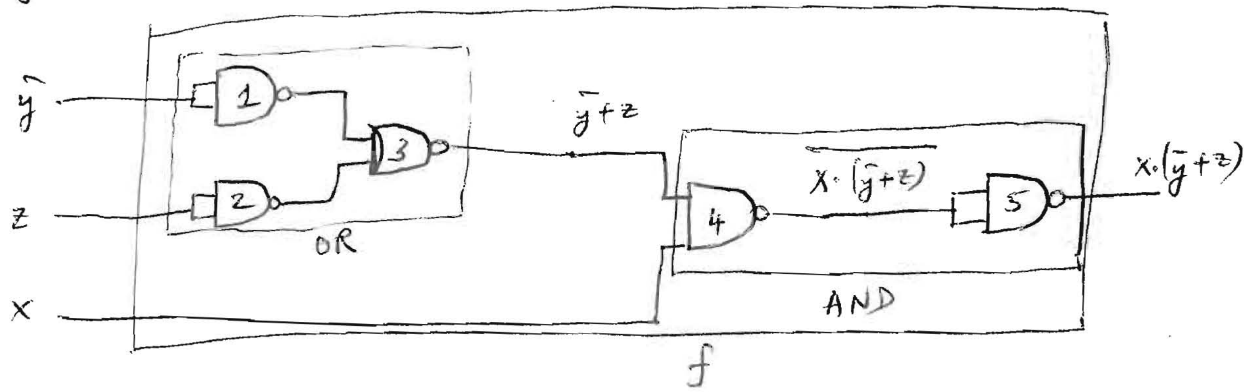
Create a Logic Circuit (or Combinational Network) using only NAND's

$$f(x,y,z) = x \cdot (\bar{y} + z)$$

AND (2 NAND's)

OR (3 NAND's)

1) Using 5 NAND's:



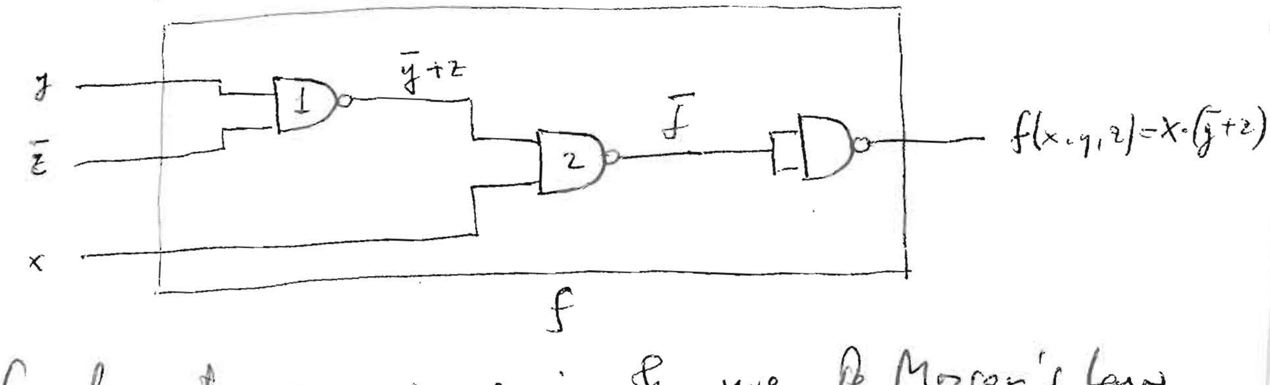
2) Using 3 NAND's:

Note: a) $\bar{f} = \overline{x\cdot(\bar{y}+z)} = \text{NAND}(x, \bar{y}+z)$

b) $\bar{y} + z = \overline{y \cdot \bar{z}}$ (de Morgan's Law)

$= \text{NAND}(y, \bar{z})$

c) Third NAND to get $f = \bar{\bar{f}}$

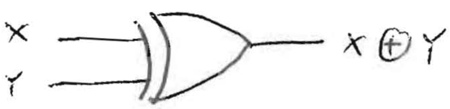


* Complement your expression & use De Morgan's Law when possible!

Exclusive-OR or XOR:

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

Logic gate symbol:



Interesting property:

X	Y	$\bar{X}Y + X\bar{Y}$
0	0	0
0	1	1
1	0	1
1	1	0

By comparing truth tables:

$$\boxed{X \oplus Y = \bar{X}Y + X\bar{Y}}$$

Properties for XOR (Table 3.16, pg. 112)

(i) a) $\boxed{X \oplus Y = \bar{X}Y + X\bar{Y} \overset{?}{=} (X+Y)\cdot(\bar{X}+\bar{Y})}$ ✓

Distributive property:

$$(X+Y)\cdot(\bar{X}+\bar{Y}) = (X+Y)\cdot\bar{X} + (X+Y)\cdot\bar{Y} = Y\cdot\bar{X} + X\cdot\bar{Y}$$ ✓

$X\cdot\bar{X} = Y\cdot\bar{Y} = 0$

Proof 2

b) $\boxed{\overline{X \oplus Y} = \bar{X}\cdot\bar{Y} + X\cdot Y = (X+\bar{Y})\cdot(\bar{X}+Y)}$

Proof 1

$$\overline{X \oplus Y} = \overline{\bar{X}\cdot Y + X\cdot\bar{Y}} = \overline{\bar{X}\cdot Y} \cdot \overline{X\bar{Y}} = (X+\bar{Y})\cdot(\bar{X}+Y)$$

De Morgan's Law: $\overline{A+B} = \bar{A}\cdot\bar{B}$

De Morgan's Law: $\overline{A\cdot B} = \bar{A}+\bar{B}$

Proof 2: $(X+\bar{Y})\cdot(\bar{X}+Y) = (X+\bar{Y})\cdot\bar{X} + (X+\bar{Y})\cdot Y$

Distributive Property

$= \bar{Y}\cdot\bar{X} + X\cdot Y$

$= \bar{X}\cdot\bar{Y} + X\cdot Y$ ✓

commutative

- XOR is commutative since AND & OR are commutative

$$X\oplus Y = \bar{X}Y + X\bar{Y} = \bar{Y}X + Y\bar{X} = Y\oplus X$$

AND & OR are commutative.

- XOR is associative (by the same reason!).

$$X\oplus(Y\oplus Z) = (X\oplus Y)\oplus Z$$

- Neutral element for XOR is 0:

$$X\oplus 0 = \bar{X}\cdot 0 + X\cdot 1 = 0 + X = X$$

- XOR is distributive (AND & OR are distributive)

(iv) a) $\boxed{\bar{X}\oplus\bar{Y} = X\oplus Y}$

Proof: $\bar{X}\oplus\bar{Y} = X\cdot\bar{Y} + \bar{X}\cdot Y = X\oplus Y$

b) $\boxed{\bar{X}\oplus Y = X\oplus\bar{Y} = \overline{X\oplus Y}}$

Proof:

$$\begin{cases} \bar{X}\oplus Y = X\cdot Y + \bar{X}\cdot\bar{Y} = \overline{X\oplus Y} \\ X\oplus\bar{Y} = \bar{X}\cdot\bar{Y} + X\cdot Y \overset{(i)b)}{=} \overline{X\oplus Y} \end{cases}$$

Odd-Parity Bit Formula : Total number of 1's should be odd (including the parity bit), for error detection.

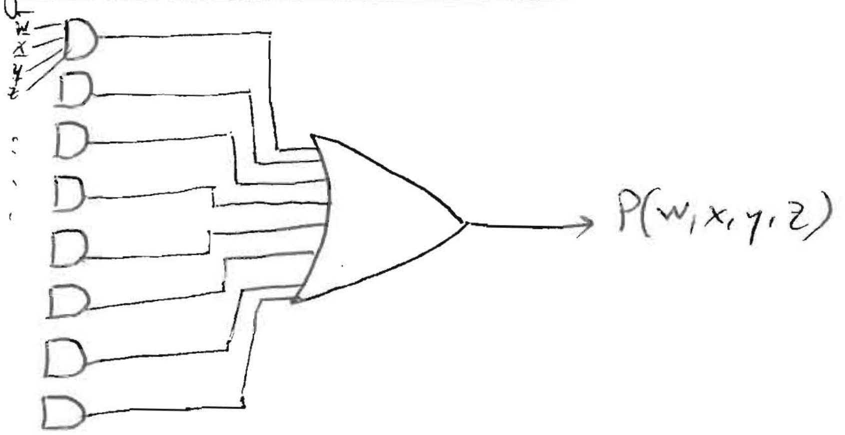
1) Truth Table : 4 variables $\longrightarrow 2^4 = 16$ entries.

8 W	4 X	2 Y	1 Z	P (odd-parity bit)	
0	0	0	0	1	$\bar{w}\bar{x}\bar{y}\bar{z}$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	$\bar{w}\bar{x}yz$
0	1	0	0	0	
0	1	0	1	1	$\bar{w}x\bar{y}z$
0	1	1	0	1	$\bar{w}xy\bar{z}$
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	$w\bar{x}\bar{y}z$
1	0	1	0	1	$w\bar{x}y\bar{z}$
1	0	1	1	0	
1	1	0	0	1	$wx\bar{y}\bar{z}$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	$wxyz$

2) Minterm Canonical: focus on the 1's $\rightarrow$ 8 terms.

$$P(w,x,y,z) = \overset{1}{\bar{w}\bar{x}\bar{y}\bar{z}} + \overset{2}{\bar{w}\bar{x}yz} + \overset{3}{\bar{w}x\bar{y}z} + \overset{4}{\bar{w}xy\bar{z}} + \overset{5}{w\bar{x}\bar{y}z} + \overset{6}{w\bar{x}y\bar{z}} + \overset{7}{wx\bar{y}\bar{z}} + \overset{8}{wxyz}$$

3) Logic gate or combinational network :



4) Reducing combinational network by using XOR to simplify the odd-parity bit formula:

$$P(w,x,y,z) = \bar{w}\cdot\bar{x}\cdot\overset{1+2}{\underbrace{(\bar{y}\bar{z} + yz)}_{\overline{y\oplus z}}} + \bar{w}x\cdot\overset{3+4}{\underbrace{(\bar{y}z + y\bar{z})}_{y\oplus z}} + w\bar{x}\overset{5+6}{\underbrace{(\bar{y}z + y\bar{z})}_{y\oplus z}} + wx\overset{7+8}{\underbrace{(\bar{y}\bar{z} + \ldots}_{\overline{y\oplus \ldots}}}$$

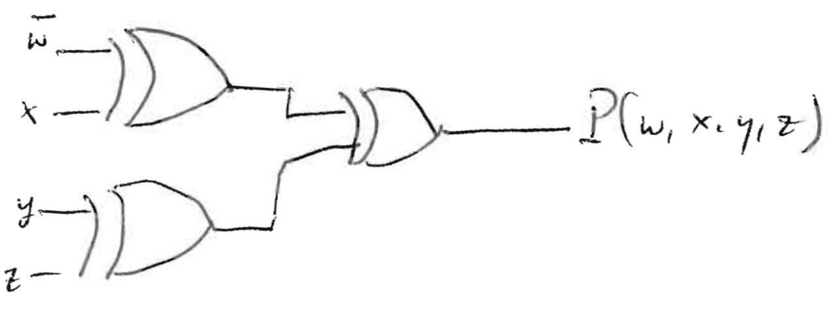
$$= \overset{1+2+7+8}{\underbrace{(\bar{w}\cdot\bar{x} + wx)}_{(\overline{w\oplus x})}\cdot(\overline{y\oplus z})} + \overset{3+4+5+6}{\underbrace{(\bar{w}x + w\bar{x})}_{(w\oplus x)}\cdot(y\oplus z)}$$

$$= (\overline{w\oplus x})\cdot(\overline{y\oplus z}) + (w\oplus x)\cdot(y\oplus z)$$

$$= \overline{(w\oplus x)\oplus(y\oplus z)}$$

$$\overset{(IV)b}{=} \overline{(w\oplus x)} \oplus (y\oplus z)$$

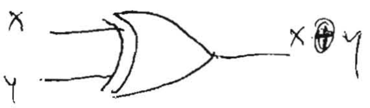
$$\overset{(IV)b.}{=} \underset{1}{(\bar{w}\oplus x)} \underset{2}{\oplus} (y \underset{3}{\oplus} z)$$



XOR's → 7486 IC's (3 gates versus original 9!)

Quiz #1 2/24/11

XOR properties:



$$x \oplus y = \bar{x} \cdot y + x \cdot \bar{y}$$

$$\overline{x \oplus y} = \bar{x} \cdot \bar{y} + x \cdot y$$

$$\bar{x} \oplus \bar{y} = x \oplus y$$

$$\bar{x} \oplus y = x \oplus \bar{y} = \overline{x \oplus y}$$

Even-parity bit generator for 8421 code: a four-variable Boolean function:

1) Truth Table 2) Min Term Canonical 3) Simplify using XOR properties 4) Sketch Logic Circuit using XOR's.

1) $2^4 = 16$

Even-parity: # of ones including parity bit is even.

w	x	y	z	P(w,x,y,z)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

2) $P(w,x,y,z) = \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}x\bar{y}\bar{z} + \ldots$

Quiz#1' (Continued)

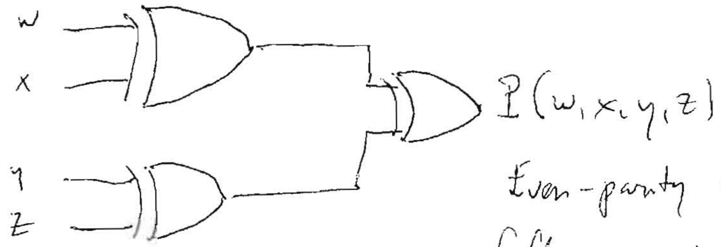
$$P(w,x,y,z) = \overset{1}{\bar{w}\bar{x}\bar{y}z} + \overset{2}{\bar{w}\bar{x}y\bar{z}} + \overset{3}{\bar{w}x\bar{y}\bar{z}} + \overset{4}{\bar{w}xyz} + \overset{5}{w\bar{x}\bar{y}\bar{z}} + \overset{6}{w\bar{x}yz} + \overset{7}{wx\bar{y}z} + \overset{8}{wxy\bar{z}}$$

$$= \overset{1\,\&\,2}{\bar{w}\bar{x}\underbrace{(\bar{y}z + y\bar{z})}_{y \oplus z}} + \overset{3\,\&\,4}{\bar{w}x\underbrace{(\bar{y}\bar{z} + yz)}_{\overline{y \oplus z}}} + \overset{5\,\&\,6}{w\bar{x}\underbrace{(\bar{y}\bar{z} + yz)}_{\overline{y \oplus z}}} + \overset{7\,\&\,8}{wx\underbrace{(\bar{y}z + y\bar{z})}_{y \oplus z}}$$

$$= (\underbrace{\bar{w}\bar{x} + wx}_{\overline{w \oplus x}})(y \oplus z) + (\underbrace{\bar{w}x + w\bar{x}}_{(w \oplus x)})(\overline{y \oplus z})$$

$$= (w \oplus x) \oplus (y \oplus z)$$

3)



Even-parity bit

(Change w to $\bar{w}$ for odd-parity bit!)

Ch4 Simplification of Boolean Expressions:

Imply: f_i implies f_j if $\left\{ \begin{array}{l} \text{There are no entries for which} \\ f_i = 1 \ \& \ f_j = 0 \end{array} \right.$

Truth Table for 2 functions f_1 & f_2:

$2^3 = 8$

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

"f_1 implies f_2"

→ "f_2 does not imply f_1"

Note: $f_1(x,y,z) = yz + xy$

$f_2(x,y,z) = \bar{x}z + xy + yz$

(f_1 is part of f_2)

Implicants (the one which implies) & **Implicates** (the one which is implied)

Product term:

x	y	z	f(x,y,z)	
0	0	0	0	
0	0	1	1	← ①
0	1	0	0	
0	1	1	1	← ②
1	0	0	1	← ③
1	0	1	0	
1	1	0	0	
1	1	1	0	

Min Term Canonical:

$f(x,y,z) = \underset{①}{\bar{x}\bar{y}z} + \underset{③}{x\bar{y}\bar{z}} + \underset{②}{\bar{x}yz}$

Product terms: ①, ②, ③

Note: if a product term is 1, f is 1!
There is no entry for which a product term is 1 and f is 0!

→ $\boxed{\text{Product term implies } f \text{ or a product term is an Implicant of the function}}$

Sum term:

Same function:

x	y	z	f(x,y,z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Max term Canonical:

$$f(x,y,z) = \underset{a}{(x+y+z)} \cdot \underset{b}{(x+\bar{y}+z)} \cdot \underset{c}{(\bar{x}+y+\bar{z})} \cdot \underset{d}{(\bar{x}+\bar{y}+z)} \cdot \underset{e}{(\bar{x}+\bar{y}+\bar{z})}$$

Sum terms: a, b, c, d, e

Note: due to the "AND" or "·" relationships b/w sum terms: when f is 1, each sum term will be 1

$\boxed{\begin{array}{l} \rightarrow \text{f implies each sum term} \\ \rightarrow \text{Each sum term is an implicate of the function} \end{array}}$

Prime implicant: is a product term that if any further literal is removed, it no longer implies the function

↓

Simplest form of a Boolean Function.

Ch 4: Simplification of Boolean Expressions (Cont.)

"Irredundant disjunctive & conjunctive normal expressions"
(IDNF)

Product term: if it's 1, the function is 1 → it's an $\boxed{\text{implicant}}$ of f

$f(x,y,z) = \underbrace{\bar{x}\bar{y}z}_{\text{product term}} + \bar{x}yz + x\bar{y}\bar{z}$

Sum term: if f is 1 → any sum term will be 1 → sum term is an $\boxed{\text{implicate}}$ of f

Same function $f(x,y,z) = (x+y+z)(x+\bar{y}+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

↳ will have 5 sum terms since there was 3 product terms

Can be obtained using Prime Implicants or Implicates:

Prime Implicant: is the simplest product term: if any further literal is removed then it no longer implies the function.

Example:

x	y	z	f
0	0	0	1 ←
0	0	1	1 ←
0	1	0	1 ←
0	1	1	1 ←
1	0	0	0
1	0	1	1 ←
1	1	0	0
1	1	1	0

$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z$

f has 5 implicants.

Prime implicants?

$x\bar{y}z$ $\begin{cases} \bar{y}z = 1 \ (YZ = 01) \\ xz \\ x\bar{y} \end{cases}$ → f is 1 in both cases.
→ $\bar{y}z$ is an implicant of f
→ $x\bar{y}z$ is not a prime implicant f

$\bar{x}yz$ $\begin{cases} yz = 1 \ (YZ=11) \text{ not an implicant} \\ \bar{x}y = 1 \ (XY=01) \text{ → an implicant} \\ \bar{x}z \end{cases}$

f.

$$\bar{x}y\bar{z}\begin{cases} y\bar{z}=1 & (YZ=10) \quad \text{not an implicant.}\\ \bar{x}y=1 & (XY=01) \quad \text{an implicant.}\\ \bar{x}z & \end{cases}$$

$$\bar{x}\bar{y}z\begin{cases} \bar{y}z=1 & \text{an implicant.}\end{cases}$$

$$\bar{x}\bar{y}\bar{z}\begin{cases} \bar{y}\bar{z}=1 & (YZ=00) \quad \text{not an implicant.}\\ \bar{x}\bar{y}=1 & (XY=00) \quad \text{implicant.}\end{cases}$$

None of the product terms in this example are prime.

$$f(x,y,z)= \bar{x}\bar{y}\underbrace{(\bar{z}+z)}_{1} + \bar{x}y\bar{z} + \underbrace{(\bar{x}y+x\bar{y})}_{(x\oplus y)}z$$

$$= \bar{x}\bar{y} + \bar{x}y\bar{z} + (x\oplus y)z$$

<u>Prime implicants:</u>

$$\bar{x}\bar{y} \to \begin{cases} \bar{x}=1 \ (X=0). & \text{implicant.}\\ \bar{y}=1 \ (Y=0) & \text{not an implicant} \to \boxed{\text{Prime}}\end{cases}$$

IDNF is achieved if it consists of product terms that are prime implicants and if any of these terms is dropped then we don't have the same function.

<u>Prime implicate</u>: a sum term that if any further literal is removed it is no longer implied (or no longer an implicate) by the function

ICNF is achieved if it consists of sum terms that are prime implicates and if any of these terms is dropped we don't have the same function.

1
1
1

Karnaugh Maps: just a different representation of the truth table which defines a Boolean function. But it will allow graphical methods to simplify the function.

2 variable Karnaugh map:

X	Y	f(x,y)
0	0	f(0,0)
0	1	f(0,1)
1	0	f(1,0)
1	1	f(1,1)

$\xrightarrow{\text{K-map}}$

X \ Y	0	1
0	f(0,0)	f(0,1)
1	f(1,0)	f(1,1)

3 variable K-map.

X	Y	Z	f(x,y)
0	0	0	f(000)
0	0	1	f(001)
0	1	0	f(010)
0	1	1	f(011)
1	0	0	f(100)
1	0	1	f(101)
1	1	0	f(110)
1	1	1	f(111)

$\xrightarrow{\text{K-map}}$

Note!

X \ YZ	00	01	11	10
0	f(000)	f(001)	f(011)	f(010)
1	f(100)	f(101)	f(111)	f(110)

Note the switch of columns 3 & 4! So there will always be only one change of value between cells

4 variable K-map:

W	X	Y	Z	f(w,x,y,z)
0	0	0	0	f(0000)
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1				
1				
1				
1				
1				
1				
1				
1				

$\xrightarrow{\text{K-map}}$

Note!

WX \ YZ	00	01	11	10
00	f(0000)	.	.	.
01	.	f(0101)	.	.
11	.	.	f(1111)	.
10	.	.	.	f(1010)

Note!

K-map & Canonical Formulas: $f(w,x,y,z)$.

m	w	x	y	z	$f(w,x,y,z)$
0	0	0	0	0	1
1		0	0	1	0
2		0	1	0	0
3		0	1	1	1
4		1	0	0	0
5		1	0	1	1
6		1	1	0	1
7		1	1	1	0
8	1	0	0	0	0
9		0	0	1	1
10		0	1	0	1
11		0	1	1	0
12		1	0	0	1
13		1	0	1	0
14		1	1	0	0
15		1	1	1	1

wx \ yz	00	01	11	10
00	1 (0)	0 (1)	1 (3)	0 (2)
01	0 (4)	1 (5)	0 (7)	1 (6)
11	1 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	0 (11)	1 (10)

Minterm canonical for f: $f(w,x,y,z) = \begin{cases} = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} \\ \quad + w\bar{x}\bar{y}z + w\bar{x}y\bar{z} + wx\bar{y}\bar{z} + wxyz \\ = \sum m(0,3,5,6,9,10,12,15) \end{cases}$

Note: both expressions can be written down from either the truth table or the K-map!

Maxterm canonical for f using K-map:

$$f(w,x,y,z) = \begin{cases} = (w+x+y+\bar{z})(w+x+\bar{y}+z)(w+\bar{x}+y+z)(w+\bar{x}+\bar{y}+\bar{z}) \\ \quad (\bar{w}+\bar{x}+y+\bar{z})(\bar{w}+\bar{x}+\bar{y}+z)(\bar{w}+x+y+z)(\bar{w}+x+\bar{y}+\bar{z}) \\ = \prod M(1,2,4,7,13,14,8,11) \end{cases}$$

Simplification: Boolean algebra properties & graphical situations on K map!

$$AB + \bar{A}B = (A+\bar{A})B = B$$

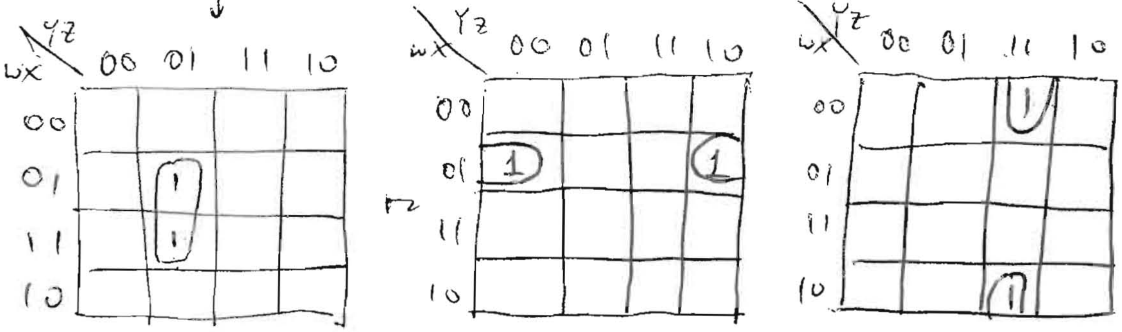
By the way we set up K-map ('only one change of value between consecutive cells') $\longrightarrow$ This property can be applied when there are two consecutive 1's!

⇒ Two consecutive 1's →

Product Terms

Simplify 2 variables combination → 1

4 variables → 3



$x\bar{y}z$ $\quad$ $\bar{w}x\bar{z}$ $\quad$ $\bar{x}yz$

The variable that changes value from one 1 to the other 1 is eliminated.

$x\bar{y}z$

(the two 1's allowed us to simplify two four literal terms into one three literal term!)

We can predict:

Four consecutive 1's

Product term:

Simplify eliminating 2 variables

(two 1's → eliminate one variable

four 1's → " two variables)

Possible scenarios with four consecutive 1's: (on four-variable Boolean functions)

wx \ yz	00	01	11	10
00				
01		1	1	
11		1	1	
10				

xz

Product terms or $\boxed{\text{Sum of products (SOP)}}$:

$$f(w,x,y,z) = \underbrace{\bar{w}x\bar{y}z + \bar{w}xyz}+\underbrace{wx\bar{y}z + wxyz}$$

$$= \bar{w}x\underbrace{(\bar{y}+y)}_{1}z + wx\underbrace{(\bar{y}+y)}_{1}z$$

$$= \underbrace{(\bar{w}+w)}_{1}xz = xz$$

Observation: 1) This group of four consecutive 1's allows elimination of w & y.

2) w & y change value ($0 \to 1$) within the group!

w changes value vertically while y changes value horizontally

wx \ yz	00	01	11	10
00	1			1
01	1			1
11				
10				

$\bar{w}\bar{z}$

wx \ yz	00	01	11	10
00	1			1
01				
11				
10	1			1

$\bar{x}\bar{z}$

wx \ yz	00	01	11	10
00		1		
01		1		
11		1		
10		1		

$\bar{y}z$

wx \ yz	00	01	11	10
00				
01				
11	1	1	1	1
10				

wx

etc...

Eight consecutive 1's:

$wx \backslash yz$	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

$\bar{z}$

$$f(w,x,y,z) = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + wx\bar{y}\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}xy\bar{z} + wxy\bar{z} + w\bar{x}y\bar{z}$$

$$= (\bar{w}\bar{x} + \bar{w}x + wx + w\bar{x})\bar{y}\bar{z} + (\bar{w}\bar{x} + \bar{w}x + wx + w\bar{x})y\bar{z}$$

$$= \Big(\bar{w}\underbrace{(\bar{x}+x)}_{1} + w\underbrace{(x+\bar{x})}_{1}\Big)\bar{y}\bar{z} + 1\ y\bar{z}$$

$$= \underbrace{(\bar{y}+y)}_{1}\bar{z} = \bar{z}$$

$wx \backslash yz$	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11				
10				

$\bar{w}$

K-Map: 5 variable Boolean function: $f(\underbrace{v,w}_{\text{Rows}}, \underbrace{x,y,z}_{\text{Columns}})$

3D

$vw \backslash xyz$	000	001	011	010
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$vw \backslash xyz$	100	101	111	110
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

180°

2D:

$vw \backslash xyz$	000	001	011	010
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

xyz	110	111	101	100	vw
	18	19	17	16	00
	22	23	21	20	01
	30	31	29	28	11
	26	27	25	24	10