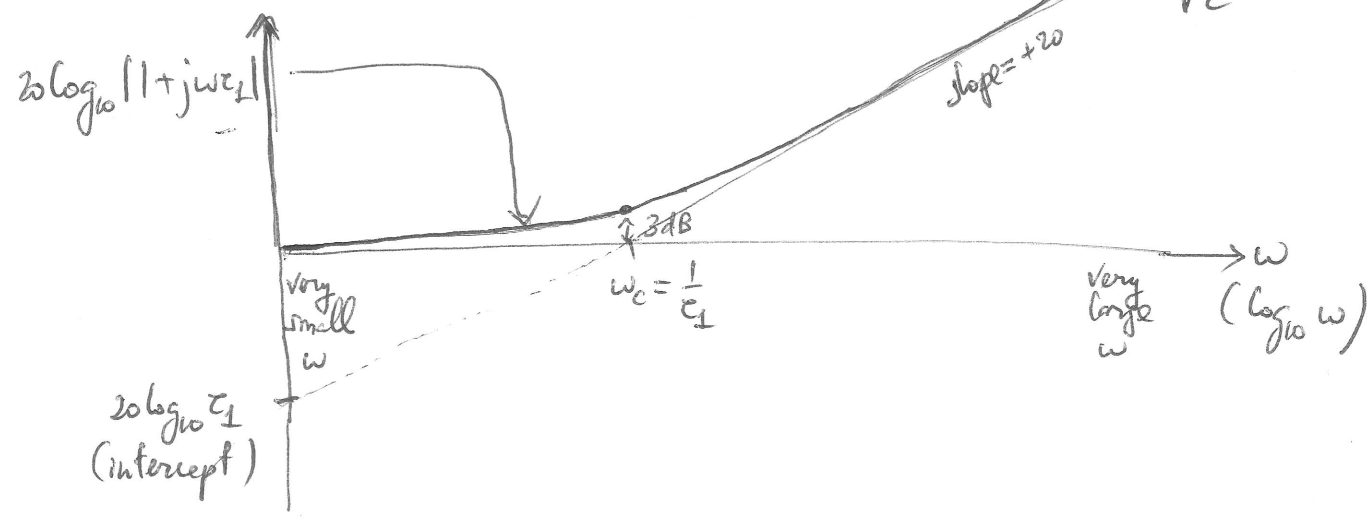
3) $\cancel{\pm}\, 20\log_{10}|1 + j\omega\tau_1|$ vs $\log_{10}\omega$ (or ω in log. scale)

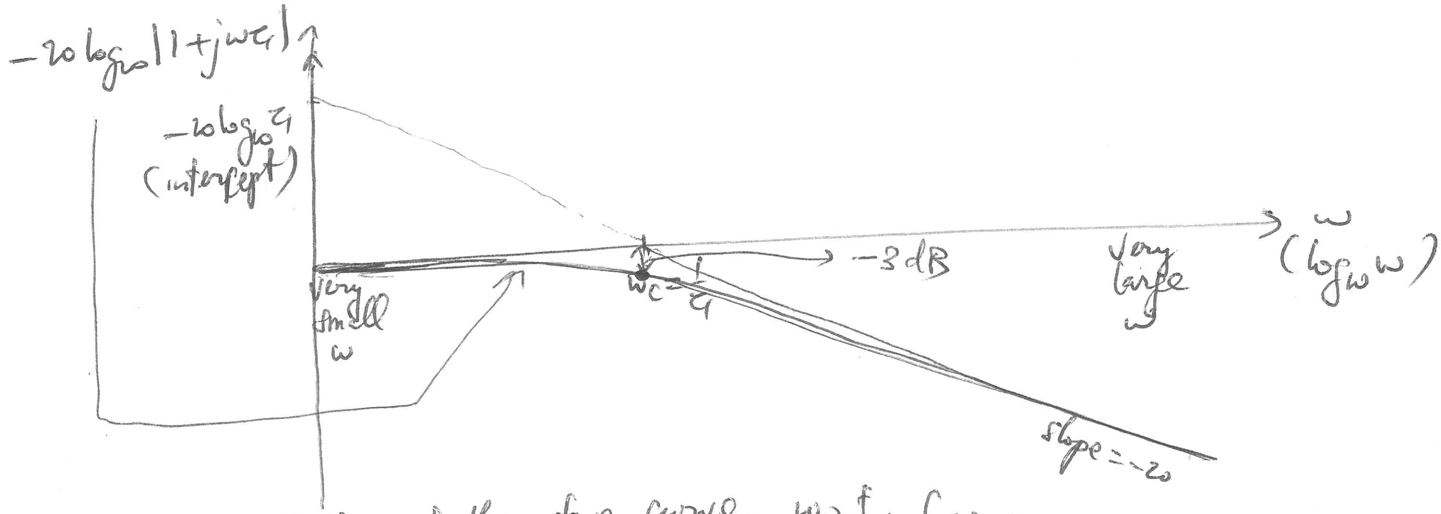
→ Asymptotic analysis or extreme analysis (very small ω, very large ω, critical ω)

- Very small frequency $\omega \to 0 \Rightarrow j\omega\tau_1 \ll 1 \to 20\log_{10}|1| = 0$
- Very high/large frequency $\omega \to \infty \Rightarrow j\omega\tau_1 \gg 1 \to 20\log_{10}|j\omega\tau_1|$

$$= \underbrace{20}_{\text{slope}}\log_{10}\omega + \underbrace{20\log_{10}\tau_1}_{\text{Intercept}}$$

- A the critical frequency $\omega = \omega_c = \frac{1}{\tau_1} \to 20\log_{10}\underbrace{|1+j|}_{\sqrt{2}} = 3\text{ dB}$





(A reflection of the top curve w.r.t. frequency axis)

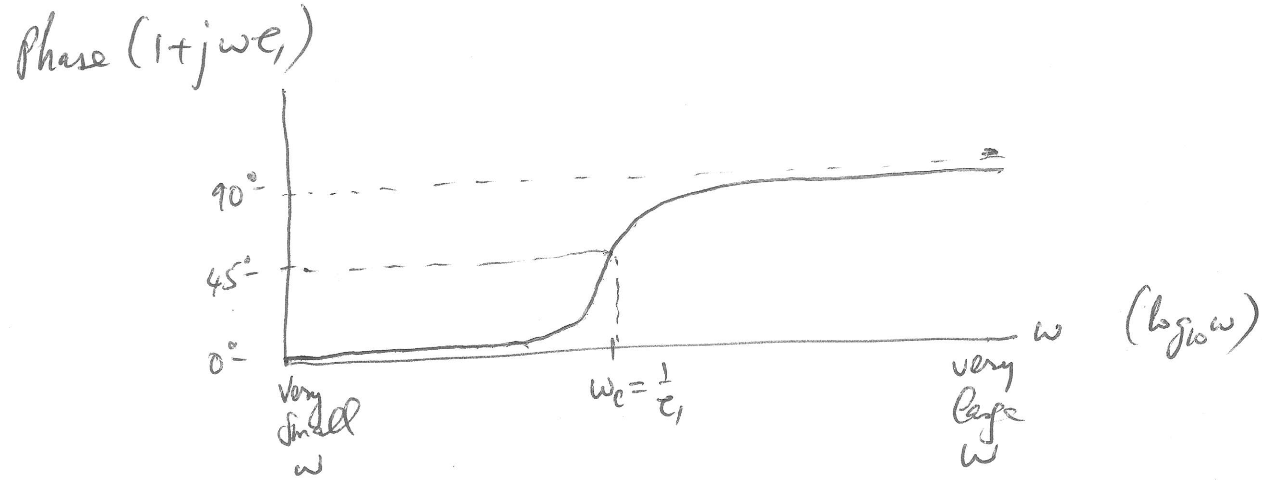
Phase Bode Plot for $(1+j\omega\tau_1)$ vs. ω ($\log_{10}\omega$)

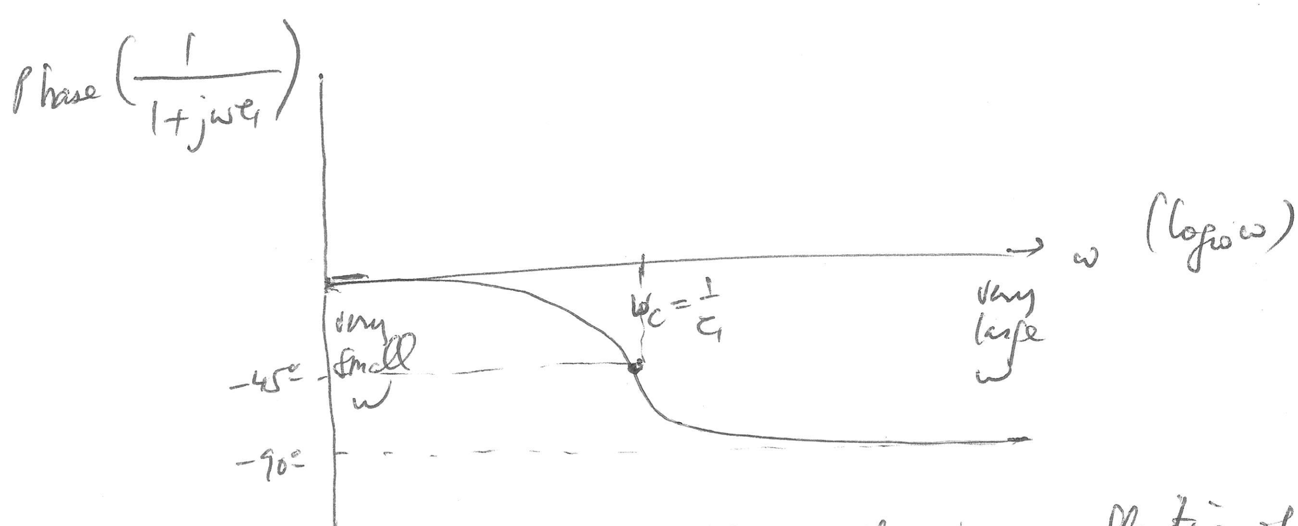
<u>Asymptotic Analysis:</u>

Very small ω : $1+j\omega\tau_1 \simeq 1 \rightarrow$ Phase$(1) = 0$

Very large ω : $1+j\omega\tau_1 \simeq j\omega\tau_1 \rightarrow$ Phase$(j\omega\tau_1) = 90°$

Critical ω $\quad \omega = \omega_c = \frac{1}{\tau_1} \rightarrow 1+j \rightarrow$ Phase$(1+j) = 45°$





(As with the magnitude Bode Plot, this is a reflection of the top curve w.r.t. frequency axis)

4) $+ 20\log_{10} \left| 1 + 2\xi_b j\omega\tau_b + (j\omega\tau_b)^2 \right|$

Asymptotic Analysis:

1) Very small ω: $(\omega\tau_b)^2 \ll \omega\tau_b \ll 1$

$$20\log_{10}\left|1 + 2\xi_b j\omega\tau_b + (j\omega\tau_b)^2\right| \simeq 20\log_{10}|1| = 0$$

2) Very large ω: $(\omega\tau_b)^2 \gg \omega\tau_b \gg 1$

$$20\log_{10}\left|1 + 2\xi_b j\omega\tau_b + (j\omega\tau_b)^2\right| \simeq 20\log_{10}(\omega\tau_b)^2$$

$$= \underbrace{40}_{\substack{\text{Line of}\\ \text{slope } 40}} \log_{10}\omega + \underbrace{40\log_{10}\tau_b}_{\text{intercept}}$$

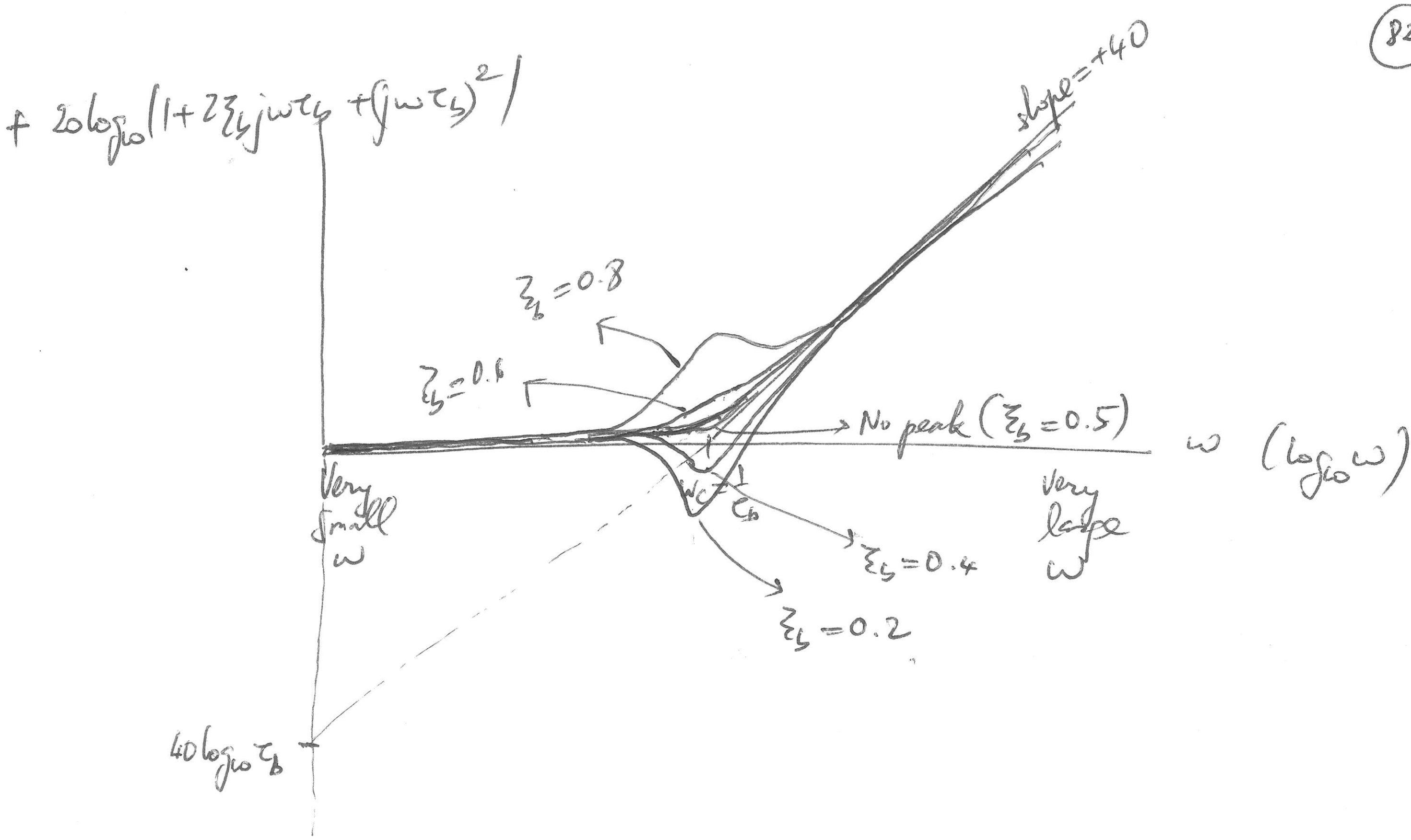
3) Critical $\omega = \omega_c = \frac{1}{\tau_b}$

$$20\log_{10}\left|1 + 2\xi_b j\omega\tau_b + (j\omega\tau_b)^2\right|$$

$$= 20\log_{10}\left|\cancel{1} + 2\xi_b j + \underbrace{\cancel{j^2}}_{\cancel{-1}}\right| = 20\log_{10}|2\xi_b j|$$

$$= 20\log_{10}(2\xi_b)$$

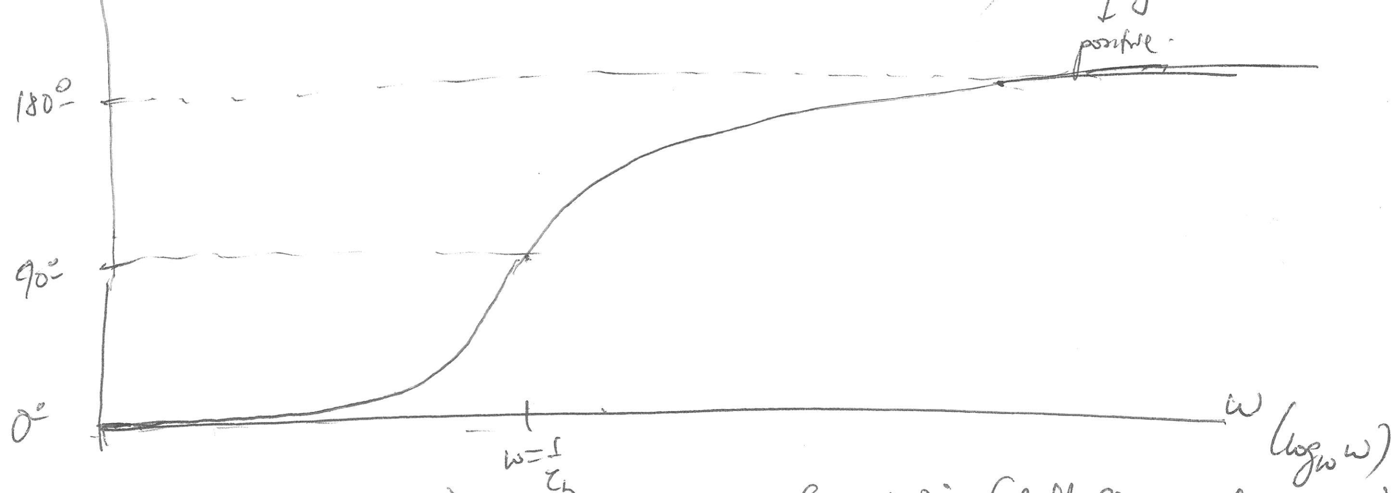
ξ_b	$-20\log_{10}(2\xi_b)$	
0.2	7.96	(larger peak)
0.4	1.94	
0.5	0	(no peak)
0.6	-1.58	
0.8	-4.1	(valley)



For $-20\log_{10}|1+2\xi_b j\omega\tau_b+(j\omega\tau_b)^2|$ → <u>reflection of this wrt. frequency axis</u>

Phase$\left[1+2\xi_b j\omega\tau_b+(j\omega\tau_b)^2\right]$

Asymptotic Analysis:
1) Very small ω: → Phase$[1]=0°$
2) Very large ω → Phase$[\underbrace{-\omega^2\tau_b^2}_{\text{positive}}]=180°$
3) At $\omega=\frac{1}{\tau_b}$ → Phase$[\cancel{1}+\underbrace{2\xi_b}_{\text{positive}}j+(\cancel{-1})]=90°$



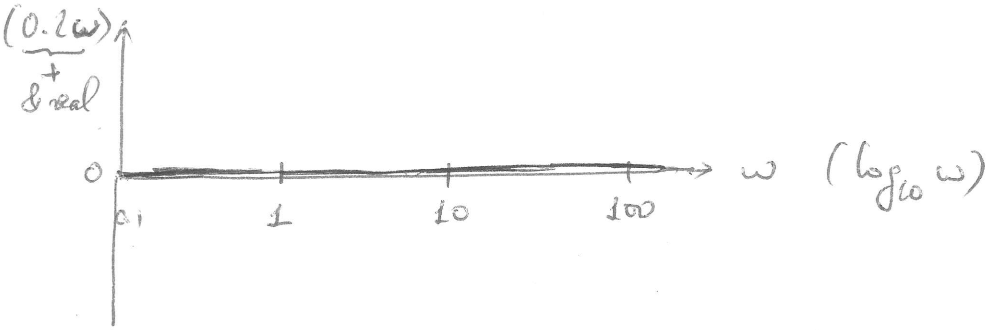
... (...) lies between 0 & −180° (Reflection wrt. ω axis)

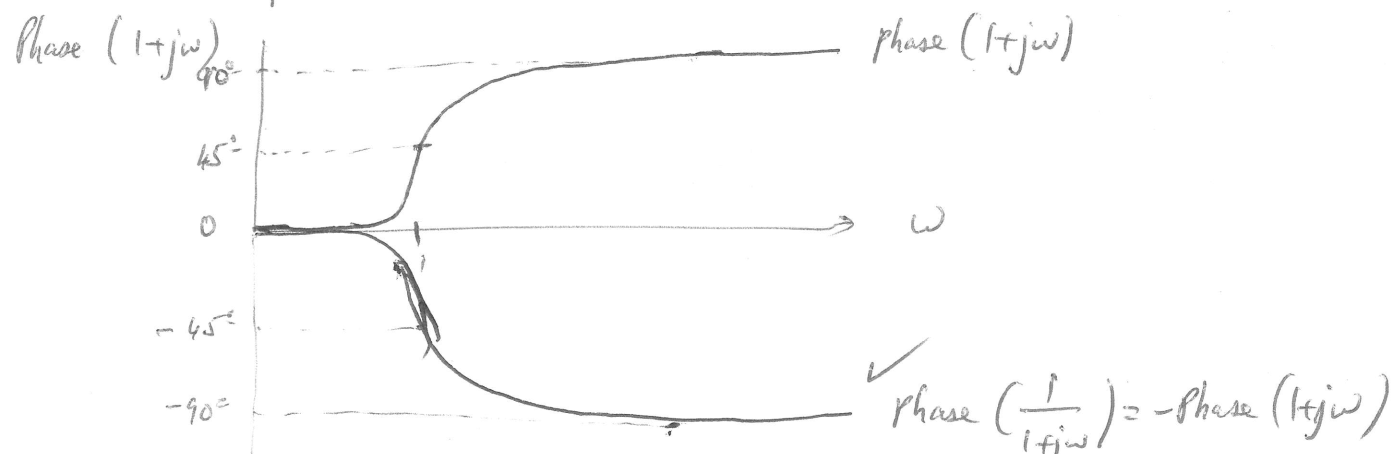
Example: Sketch the Bode Plots for: $\hat{H}(j\omega) = \dfrac{100j\omega}{(1+j\omega)(10+j\omega)(50+j\omega)}$

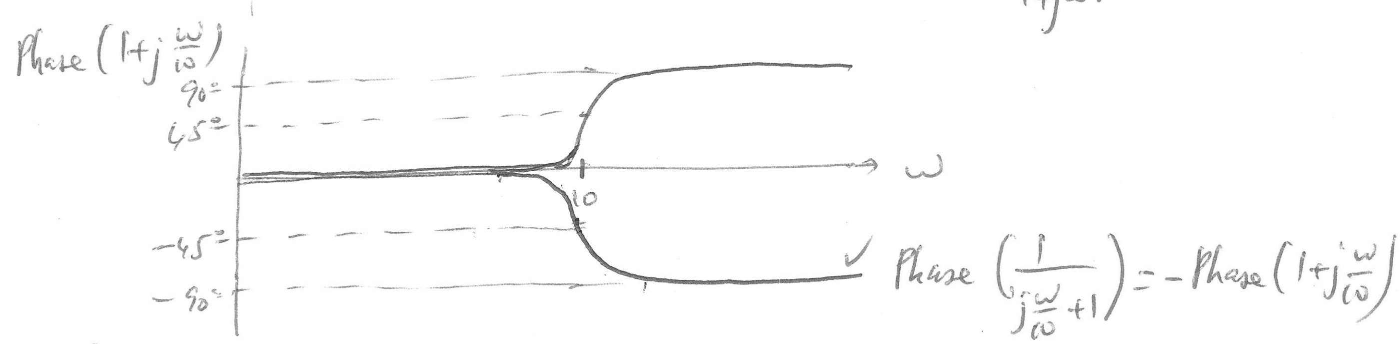
HW9 = 11.13, 11.18, 11.29, 11.50 due 4/20.

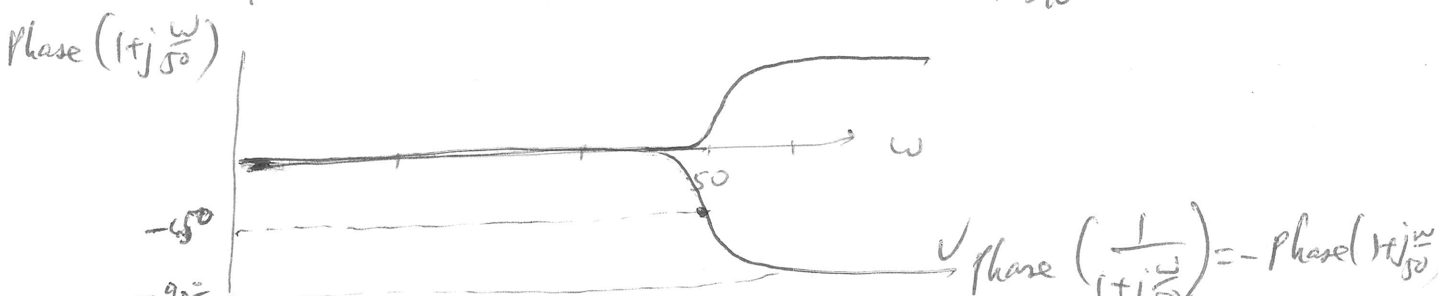
$$\hat{H}(j\omega) = \frac{0.2\omega}{(1+j\omega)(1+j\frac{\omega}{10})(1+j\frac{\omega}{50})}$$

$$\text{Phase}(\hat{H}(j\omega)) = \text{Phase}(0.2\omega) - \text{Phase}(1+j\omega) - \text{Phase}(1+j\tfrac{\omega}{10}) - \text{Phase}(1+j\tfrac{\omega}{50})$$









Example: Plot (Bode plots) this transfer function:

$$\hat{H}(j\omega) = \frac{100\, j\omega}{(1+j\omega)(10+j\omega)(50+j\omega)}$$

1) Rearrange into the general transfer function format $\Rightarrow$

$$\hat{H}(j\omega) = \frac{\frac{100}{10 \cdot 50}\; j\omega}{(1+j\omega)(1+j\frac{\omega}{10})(1+j\frac{\omega}{50})}$$

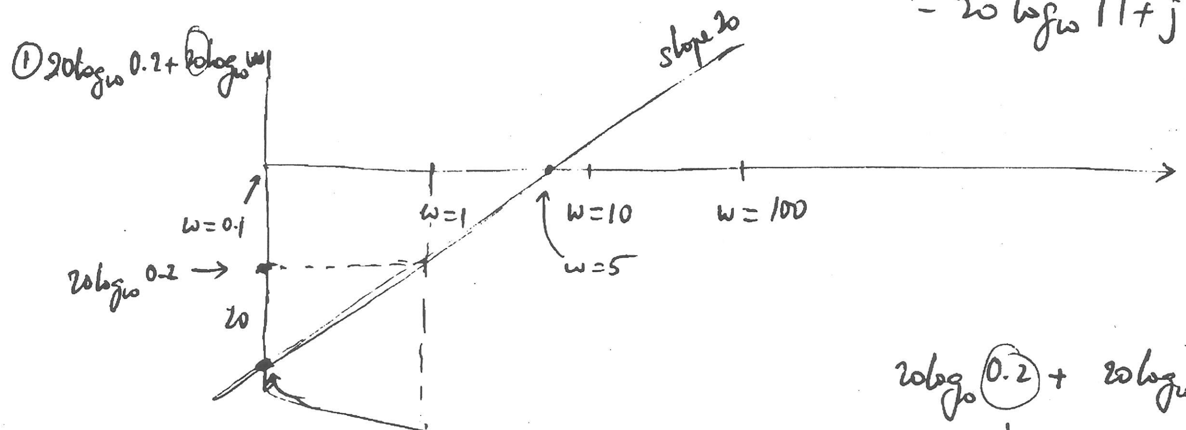
$$\rightarrow |\hat{H}(j\omega)| = \frac{0.2\,|j\omega|}{|1+j\omega|\,|1+j\frac{\omega}{10}|\,|1+j\frac{\omega}{50}|} = \frac{0.2\,\omega}{|1+j\omega|\,|1+j\frac{\omega}{10}|\,|1+j\frac{\omega}{50}|}$$

$$|j\omega| = \underbrace{|j|}_{1}|\omega| = \omega$$

2) Magnitude Bode plot:

$$20\log_{10}|\hat{H}(j\omega)| = \overbrace{20\log_{10} 0.2 + 20\log_{10}\omega}^{1} \overbrace{- 20\log_{10}|1+j\omega|}^{2} \overbrace{- 20\log_{10}|1+j\tfrac{\omega}{10}|}^{3} \overbrace{- 20\log_{10}|1+j\tfrac{\omega}{50}|}^{4}$$

1 $20\log_{10} 0.2 + 20\log_{10}\omega$

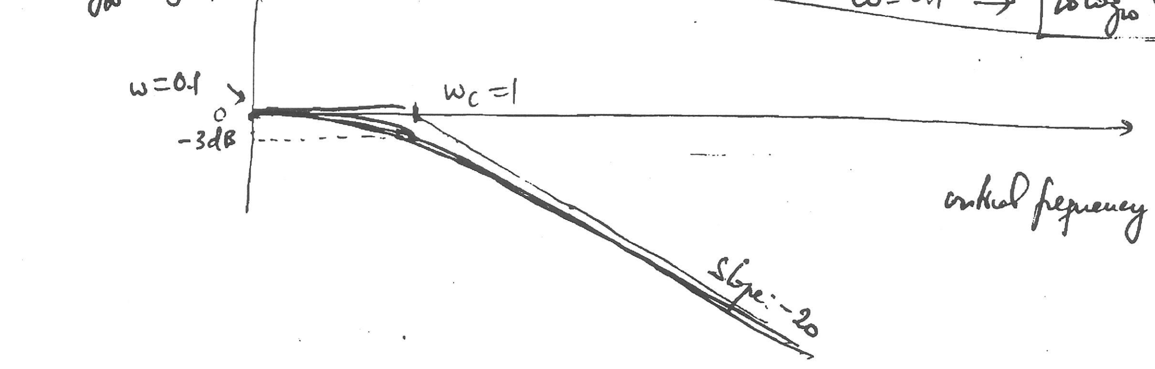


$20\log_{10} 0.2 + 20\log_{10}\omega = 0$ at $\boxed{\omega = 5}$

$0.2 = \frac{1}{5}$

$\omega = 0.1 \rightarrow \boxed{20\log_{10} 0.2 - 20}$

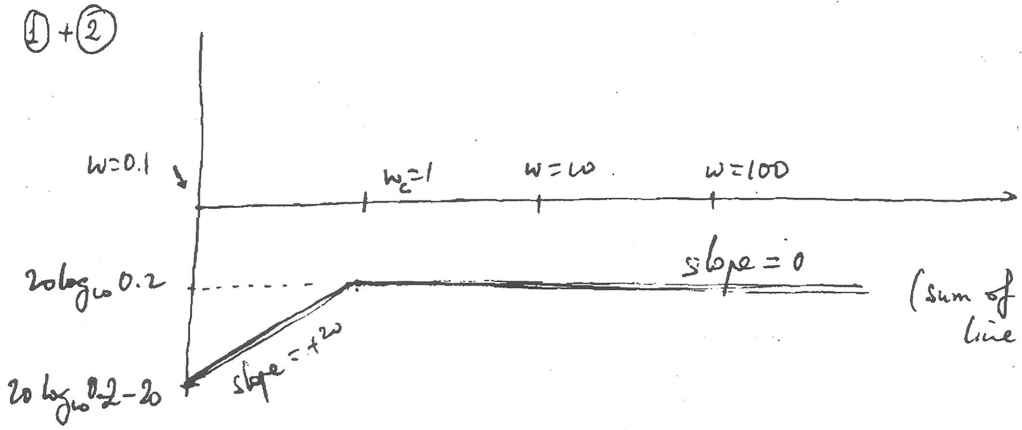
2 $-20\log_{10}|1+j\omega|$



critical frequency at $\omega_c = 1$

Let's add the plots for ① & ② : doing it in 2 pieces separated by the critical frequency $w_c = 1$

① + ②

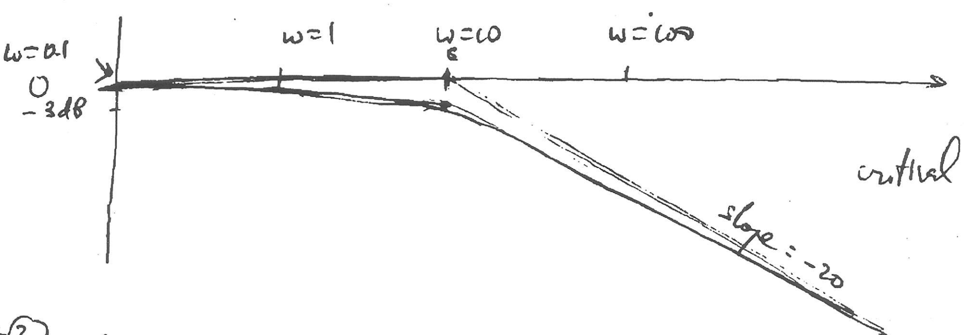


ω ($\log_{10} ω$)

(sum of line of slope +20 and line of slope − 20)

Next group:

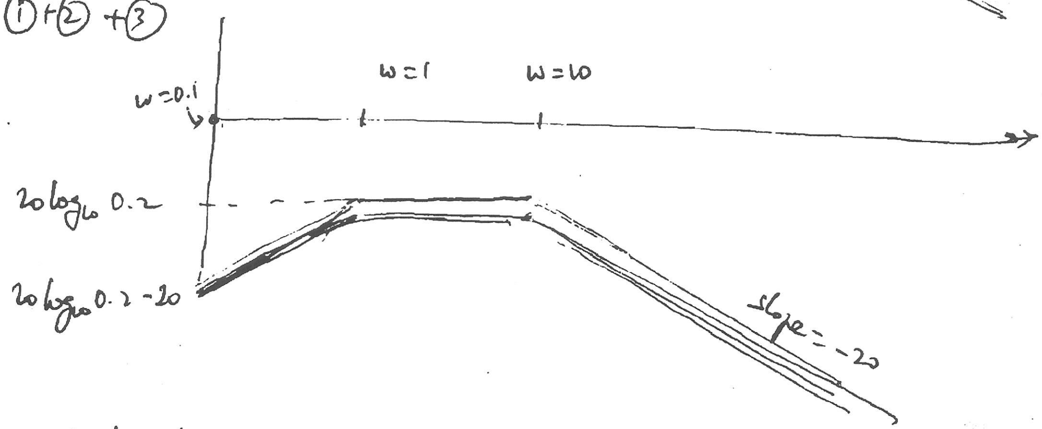
③ $-20\log_{10}\left|1+j\frac{\omega}{10}\right|$



ω ($\log_{10} ω$)

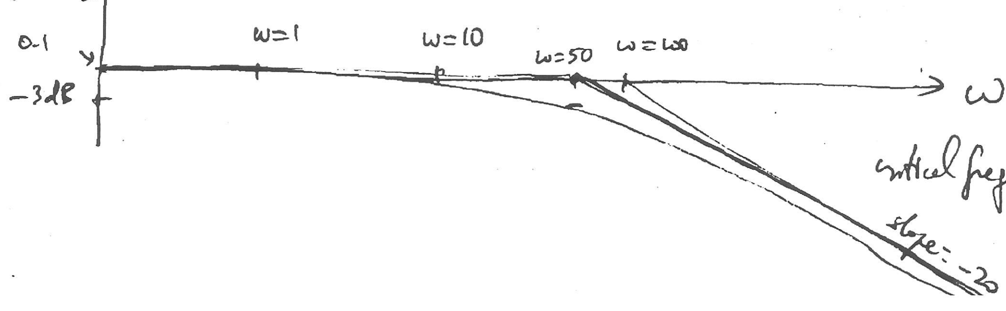
critical frequency at $w_c = 10$.

①+②+③



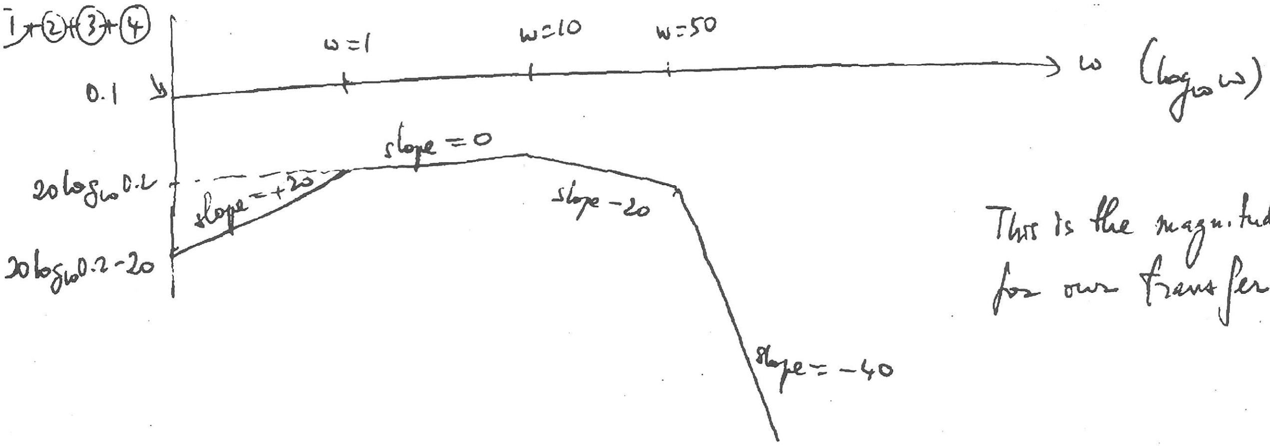
ω ($\log_{10} ω$)

④ $-20\log_{10}\left|1+j\frac{\omega}{50}\right|$



ω ($\log_{10} ω$)

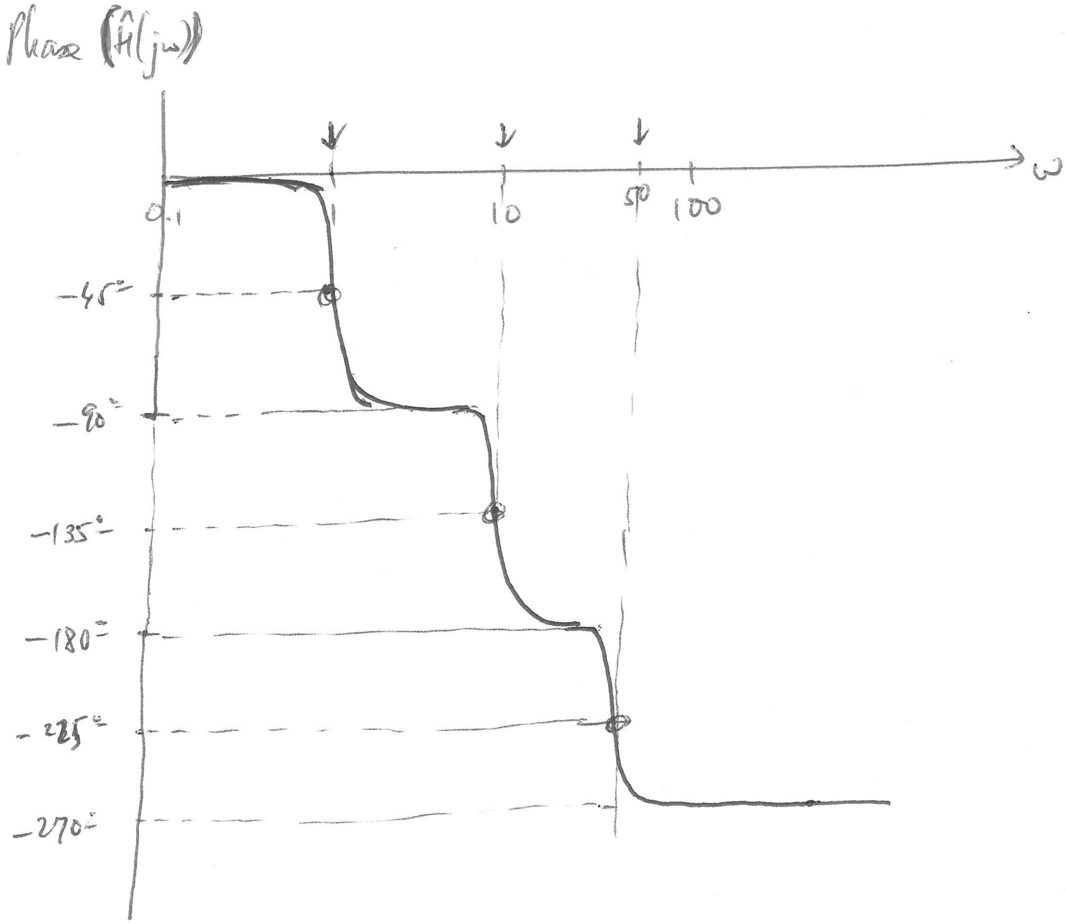
critical frequency at $w_c = 50$



This is the magnitude Bode Plot for our transfer function.

$\boxed{\text{HW9} \rightarrow \text{due } 4/20/10}$
(~~on the plane travel~~).

11.13, 11.18, 11.29, 11.50


Phase $(\hat{H}(j\omega))$
ω
0.1
1
10
50
100
$-45°$
$-90°$
$-135°$
$-180°$
$-225°$
$-270°$

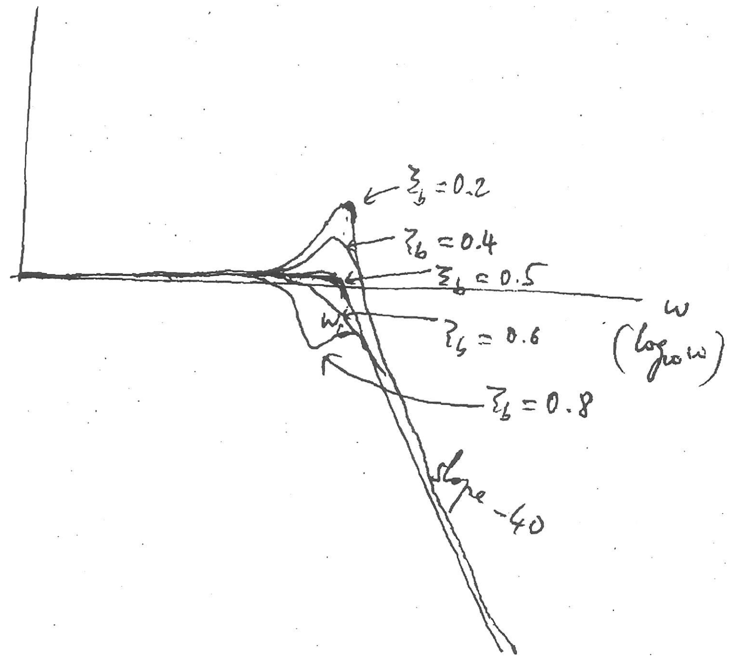
2nd Example: $\hat{H}(j\omega) = \dfrac{25j\omega}{(j\omega + 0.5)\left[(j\omega)^2 + 4j\omega + 100\right]}$

1st step = reduce to standard formats of $(1 + j\omega\tau)$, $\left[1 + 2\xi_s j\omega\tau_s + (j\omega\tau_s)^2\right]$

$\downarrow$
peak @
critical
ω

$$\hat{H}(j\omega) = \frac{0.5j\omega}{\left(1 + j\frac{\omega}{0.5}\right)\left[\left(j\frac{\omega}{10}\right)^2 + 4j\frac{\omega}{100} + 1\right]}$$

Magnitude Bode Plot
2nd order pole



Example: $\hat{H}(j\omega) = \dfrac{25j\omega}{(j\omega+0.5)\left[(j\omega)^2+4j\omega+100\right]}$

standard form $(1+\cdots)$ $\longrightarrow$ $= \dfrac{0.5j\omega}{\left(j\frac{\omega}{0.5}+1\right)\underbrace{\left[\left(j\frac{\omega}{10}\right)^2+4j\frac{\omega}{100}+1\right]}}$

$w_c = 0.5$. $\qquad w_c = 10$

$2\zeta_b = \frac{4}{10} \rightarrow \boxed{\zeta_b = 0.2}$ high peak

$\left(\begin{array}{c} 2\zeta_b j\omega\tau_b = 4j\frac{\omega}{100} \\ \tau_b = \frac{1}{w_c} = \frac{1}{10} \end{array}\right)$

2 critical frequencies $\rightarrow$ 3 intervals
a) b/w 0 & $w_c = 0.5$ $\rightarrow$
b) b/w $w_c = 0.5$ & $w_c = 10$ $\rightarrow$
c) beyond $w_c = 10$ $\rightarrow$

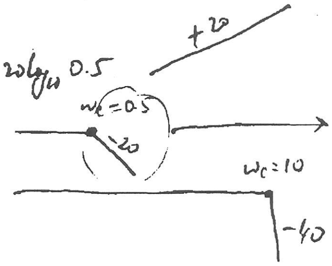
$$\boxed{20\log_{10}|\hat{H}(j\omega)| = 20\log_{10}\omega + 20\log_{10}0.5 - 20\log_{10}\left|1+j\frac{\omega}{0.5}\right| - 20\log_{10}\left|1+\frac{4}{10}j\frac{\omega}{10}+\left(j\frac{\omega}{10}\right)^2\right|}$$

a) B/w $\omega = 0$ & $\omega = 0.5$:

- Numerator : line of slope +20 and intercept w/ vertical axis is $20\log_{10} 0.05$ (negative).
- 1st order pole : } 0
- 2nd order pole : }

b) B/w $\omega = 0.5$ & $\omega = 10$

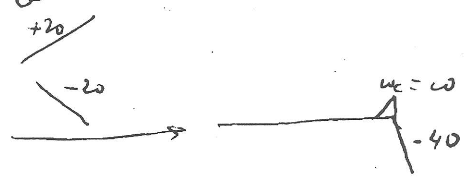
- Numerator : $20\log_{10}\omega + 20\log_{10} 0.5$
- 1^{st} order pole : $\omega_c = 0.5$
- 2^{nd} order pole : zero



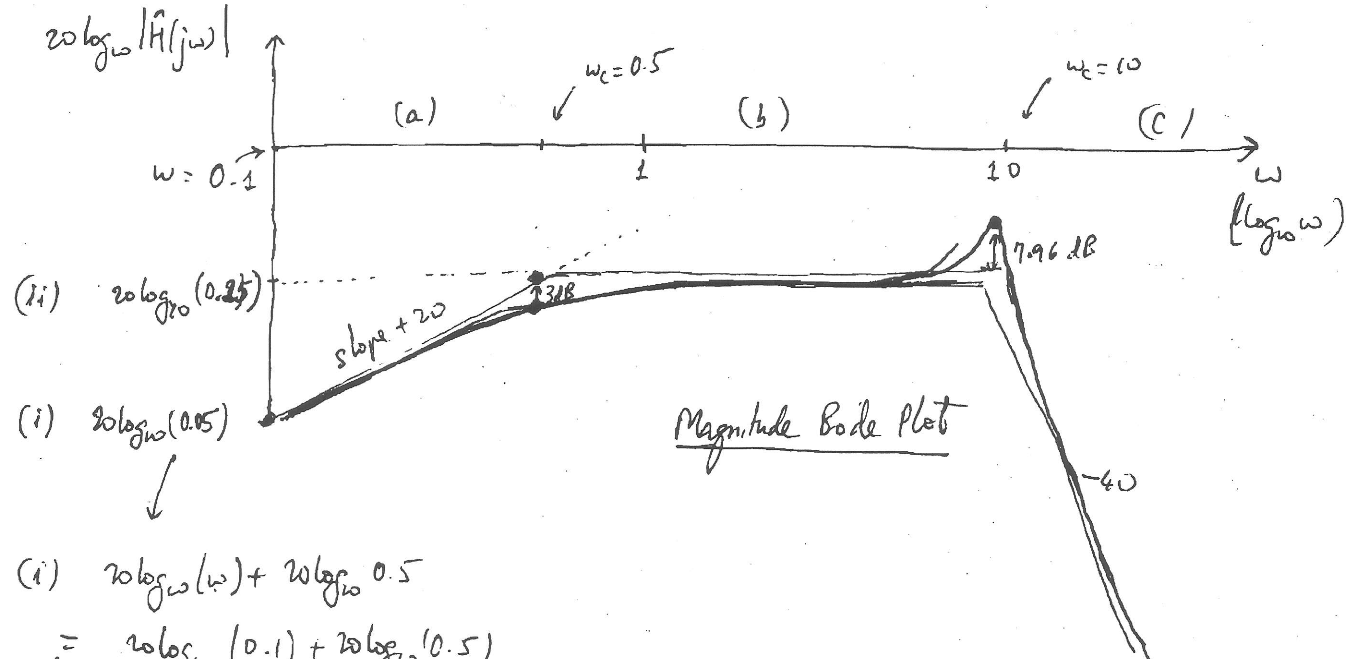
This cancels the +20 slope line from numerator. Value at ω_c gives $-20\log_{10} \sqrt{2} = -3d$

c) B/w $\omega = 10$ and ∞

- Numerator :
- 1^{st} order pole :
- 2^{nd} order pole :



$\zeta_b = 0.2$ (high peak) → 7.96



Magnitude Bode Plot

(i) $20\log_{10}(\omega) + 20\log_{10} 0.5$

$\underset{\omega = 0.1}{=} 20\log_{10}(0.1) + 20\log_{10}(0.5)$

$= 20\log_{10}(0.1 \times 0.5) = 20\log_{10}(0.05)$

(ii) $20\log_{10} 0.5 + 20\log_{10} 0.5$

$= 20\log_{10}(0.5^2) = 20\log_{10}(0.25)$

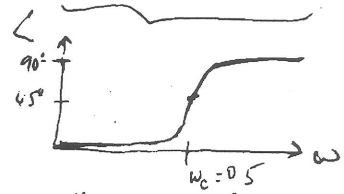
Phase Bode Plot:

$$\hat{H}(j\omega) = \frac{0.5\,j\omega}{\left(1+j\frac{\omega}{0.5}\right)\left(1+\frac{4}{10}j\frac{\omega}{10}+\left(j\frac{\omega}{10}\right)^2\right)}$$

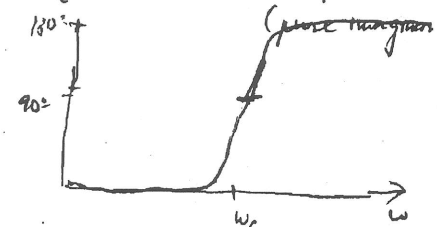
$$\angle\hat{H}(j\omega) = \angle 0.5j\omega - \angle\left(1+j\frac{\omega}{0.5}\right) - \angle\left(1+\frac{4}{10}j\frac{\omega}{10}+\left(j\frac{\omega}{10}\right)^2\right)$$

angle of the transfer function; pure imaginary; asymptotic analysis; asymptotic analysis

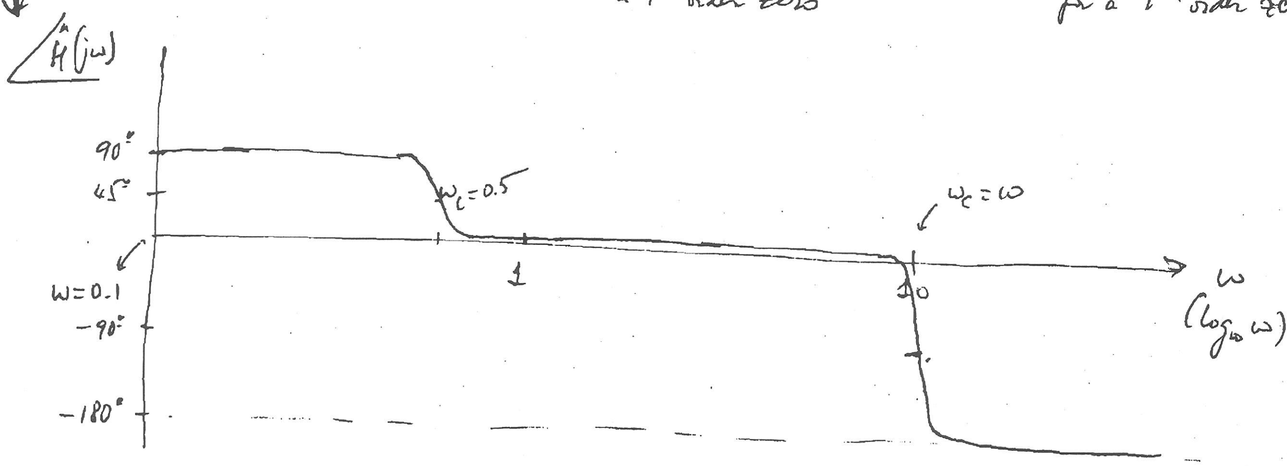
$= 90° \; - \; \begin{cases} \text{small } \omega: \angle = 0 \\ \text{large } \omega: \angle = 90° \\ \text{critical } \omega: \angle 1+j = 45° \end{cases} \; - \; \begin{cases} \text{small } \omega: \angle = 0° \\ \text{large } \omega: \angle = 180° \\ \text{critical } \omega: \angle = 90° \end{cases}$



Phase Bode plot for a 1st order zero



Phase Bode plot for a 2nd order zero



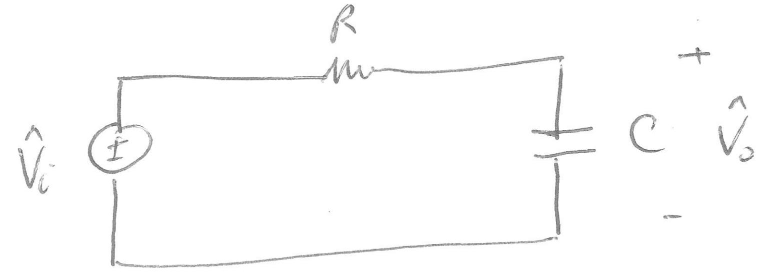
HW 8 = 10.6; 10.14; 10.25; 10.36 due 4/22

HW 9 = 11.13; 11.18; 11.29; 11.50 due 4/24.

Application of Bode Plots : Designing Filter Networks

Filters types
- **Passive** : use only R, L, C (gain is 1)
- **Active** : use op-amps (gain > 1)

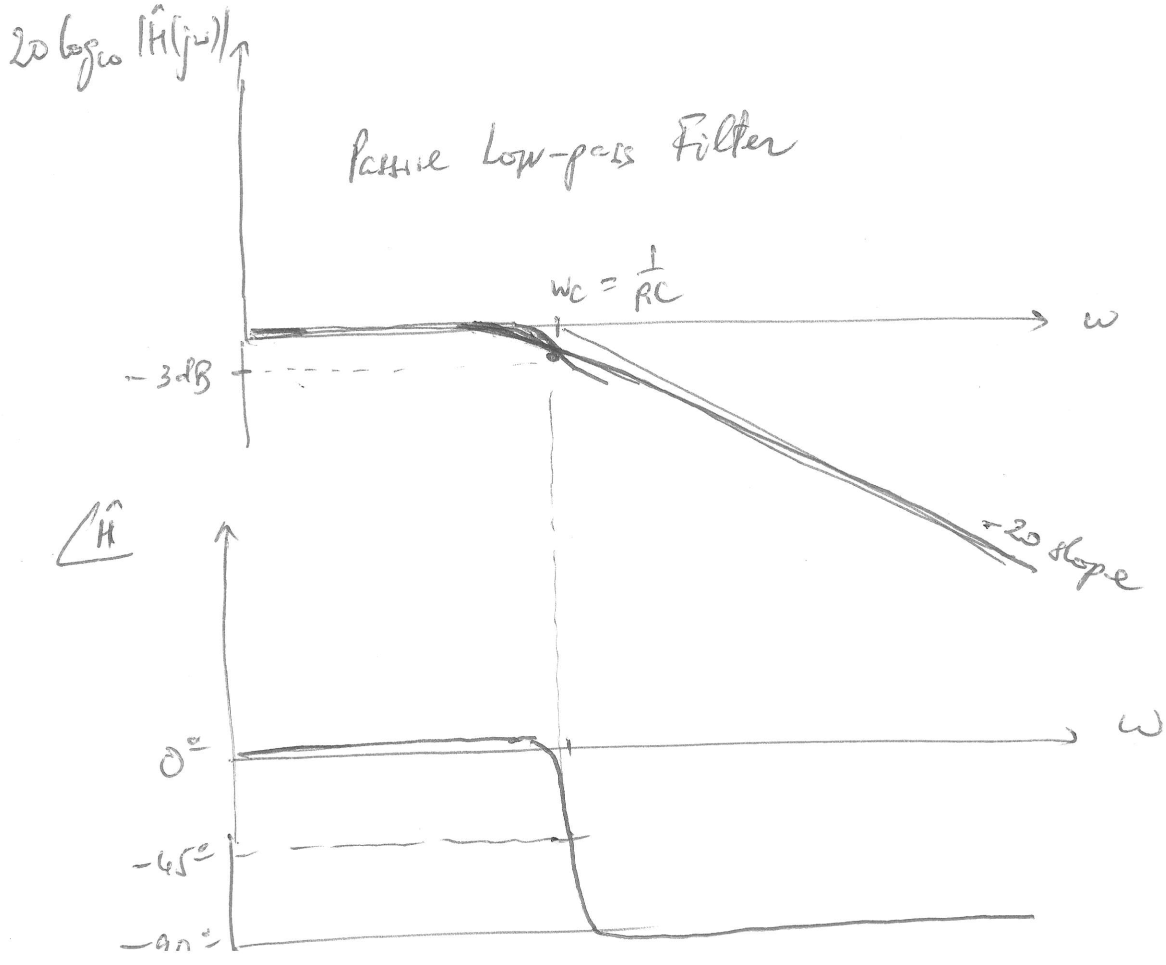
Passive low-pass filter: RC circuit:



$$\hat{H}(j\omega) = \frac{\hat{V}_o}{\hat{V}_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 + j\omega RC}$$ Pole of 1st order

$$\omega_c = \frac{1}{RC}$$



Extra credit: $\frac{1}{2}$ one HW set: due 4/27

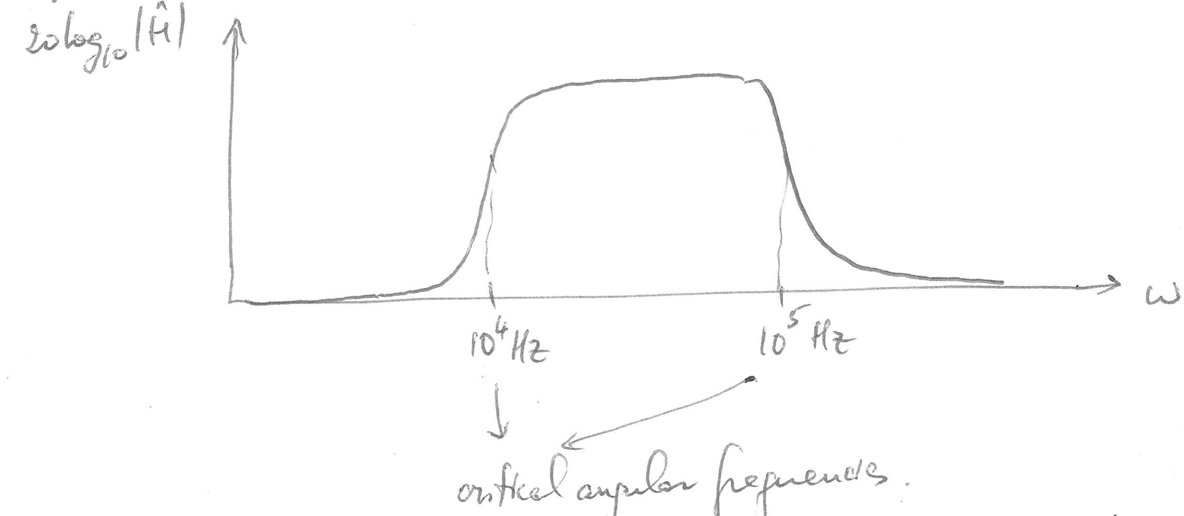
Use PSPICE to design a passive low-pass filter for $f_c = 100\text{K Hz}$ (PSPICE plots v.s. linear frequency)

$2\pi f_c = w_c = \frac{1}{RC} \rightarrow \frac{1}{RC} = 2\pi \times 10^5$. For $R = 100\Omega \rightarrow C = ?$

Provide the magnitude Bode-plot for this filter.

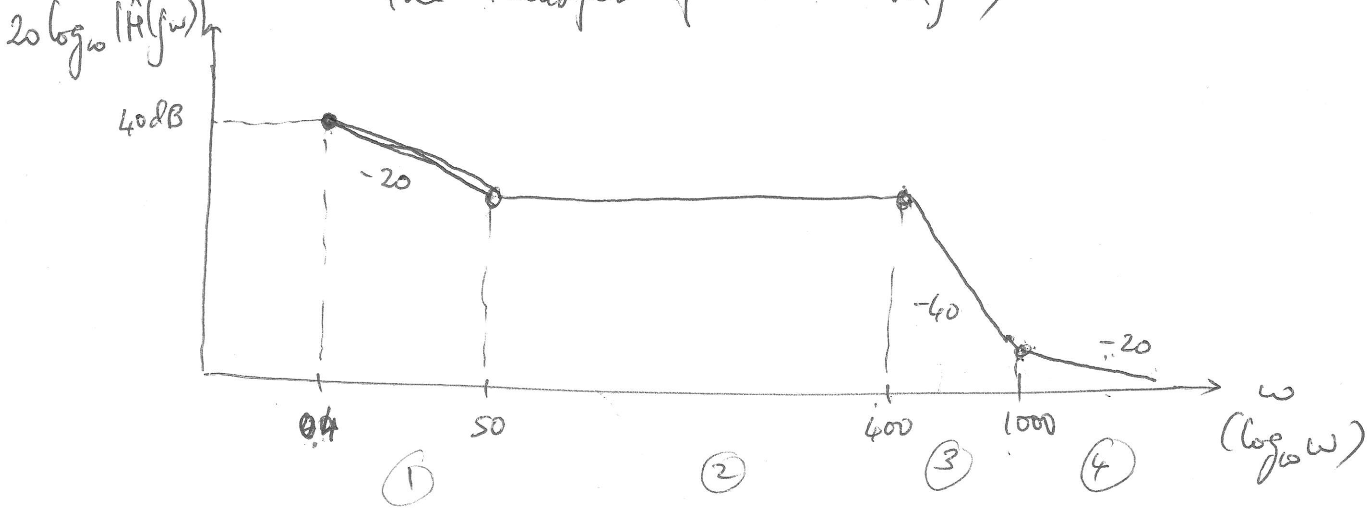
Extra credit: 1 HW set: due 4/27

Use PSPICE to design a band-pass filter that has a magnitude Bode plot of:



Turn in schematics of the filter (using R's & C's: two of each!) & Magnitude & Phase Bode Plots by PSPICE.

HW9 **11.29**. Given the magnitude Bode plot below, provide the transfer function $\hat{H}(j\omega)$



4 intervals:

1. $0.4 < \omega \le 50 \rightarrow$ line w/ slope $= -20$

$$\frac{1}{j\omega} \quad \text{or} \quad \frac{1}{1+j\omega\tau}$$

2. $50 < \omega \le 400 \rightarrow$ line w/ slope $= 0$

$\rightarrow$ some cancellation due to a zero of 1st order

$$\cancel{j\omega} \qquad 1 + j\omega\tau = 1 + j\frac{\omega}{50}$$

✓

3. $400 < \omega \le 1000 \rightarrow$ line w/ slope $= -40$

$\rightarrow$ there is a pole of order 2 @ $\omega_c = 400$

$$\frac{1}{(1 + 2\xi j\omega\tau + (j\omega\tau)^2)} \quad \text{or} \quad \frac{1}{(1+j\omega\tau)^2} = \frac{1}{\left(1+j\frac{\omega}{400}\right)^2}$$

peak depending on ξ ✓

There is no peak

$$\hat{H}(j\omega) = \frac{A\left(1+j\frac{\omega}{50}\right)\cdot\left(1+j\frac{\omega}{1000}\right)}{j\omega\left(1+j\frac{\omega}{400}\right)^2}$$

$20\log_{10}|\hat{H}(\omega=0.4)| = 40$ → allow an additional parameter A to satisfy this requirement!

To find A:

$$40 = 20\log_{10}|A| + \cancel{20\log_{10}\left|1+j\frac{\omega}{50}\right|_{\omega=0.4}}^{\approx 0} + \cancel{20\log_{10}\left|1+j\frac{\omega}{1000}\right|_{\omega=0.4}}^{\approx 0}$$

$$- 20\log_{10}|\omega|_{\omega=0.4} - \cancel{40\log_{10}\left|1+j\frac{\omega}{400}\right|_{\omega=0.4}}^{\approx 0}$$

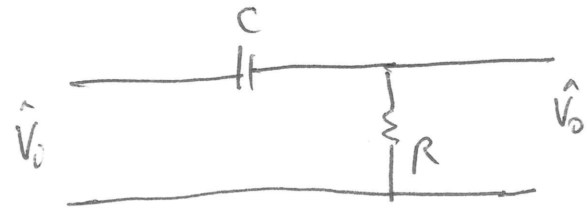
$$40 = 20\log_{10}|A| - \underbrace{20\log_{10} 0.4}_{-7.96} \rightarrow 40-7.96 = 20\log_{10}(A)$$

$$\rightarrow A = 10^{\frac{32.04}{20}}$$

$$= 39.99 \approx 40$$

$$\rightarrow \boxed{\hat{H}(j\omega) = \frac{40\left(1+j\frac{\omega}{50}\right)\left(1+j\frac{\omega}{1000}\right)}{j\omega\left(1+j\frac{\omega}{400}\right)^2}}$$

Passive High Pass Filter :



$$\hat{H}(j\omega) = \frac{\hat{V}_o}{\hat{V}_i} = \frac{R}{R + \frac{1}{j\omega C}}$$

line

$$= \frac{j\omega RC}{1 + j\omega RC}$$

Pole of 1st order.

$$\omega_c = \frac{1}{RC}$$

a number   +20

$$20\log_{10}|\hat{H}(j\omega)| = 20\log_{10} RC + 20\log_{10}\omega - 20\log_{10}|1 + j\omega RC|$$