Circuit Analysis II

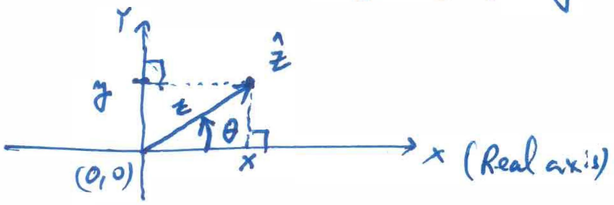
Sp '10

"Imaginary" Complex Numbers : $\hat{z}$

$j = \sqrt{-1}$ "imaginary factor"

$\hat{z} =$

- Rectangular coordinates or Cartesian coords. : $\hat{z} = x + jy$ $\begin{cases} Re[\hat{z}] = x \\ Im[\hat{z}] = y \end{cases}$



- Polar coordinates : $\hat{z} = z e^{j\theta}$ $\begin{cases} z: \text{magnitude of } \hat{z} \\ \theta: \text{phase of } \hat{z} \text{ or angle w.r.t. Real axis.} \end{cases}$

Having identified rectangular & polar coordinates on a same graph of $\hat{z}$, use trigonometry & geometry to establish connections between these two representations of a complex number $\hat{z}$.

Rect. $\xrightarrow{!}$ Polar : $\begin{cases} z = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$ (Pythagoras Theorem)

Polar $\rightarrow$ Rect. : $\begin{cases} x = z\cos\theta \\ y = z\sin\theta \end{cases}$

$\hat{z}_1 = \underbrace{-3 + j4}_{\text{in 2nd quad}} \rightarrow$ in polar form $\begin{cases} z_1 = \sqrt{9+16} = 5 \\ \theta_1 = \tan^{-1}\left(\frac{4}{-3}\right) = -53.13° \end{cases}$

$\rightarrow \theta_1 = -53.13° + 180° = 126.87°$

$\hat{z}_2 = 3 - j4$

↳ in 4^{th} quad $\longrightarrow$ $\theta_1 = -53.13°$ ✓

(No need to fix!)

<u>De Moivre formula:</u> $\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$

↳ $e^{j\frac{\pi}{2}} = \underbrace{\cos\frac{\pi}{2}}_{0} + j\underbrace{\sin\frac{\pi}{2}}_{1} = j$

so the imaginary factor j has a phase of $\frac{\pi}{2}$ or $90°$ → This is consistent with the rectangular representation of a complex number where the imaginary part is along the Y-axis

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What is the use of imaginary complex numbers in AC circuit analysis?

- AC circuits : work with sinusoids signals:

$$x(t) = \underset{\substack{\downarrow \\ \text{Amplitude} \\ \text{of sinusoid}}}{A} \cos(\underset{\substack{\downarrow \\ \text{angular} \\ \text{frequency}}}{\omega} t + \underset{\substack{\searrow \\ \text{phase}}}{\theta})$$

- AC <u>linear</u> circuits : frequency is unchanged throughout circuit → a sinusoid in linear AC circuit can be identified with only two out of three quantities : A, θ

Since imaginary complex numbers involve 2 quantities → a perfect match for a description of sinusoids in linear AC circuits.

"Phasors" = vectors with an amplitude and a phase.

→ Our imaginary complex numbers will be ~~ca~~ called "phasors"

$$x(t) = A \cdot \cos(\omega t + \theta)$$

$$= \mathrm{Re}\left[A \cdot e^{j(\omega t + \theta)} \right] = A \cdot \underbrace{\mathrm{Re}\left[e^{j(\omega t+\theta)} \right]}_{\cos(\omega t + \theta)}$$

A ↓ no imaginary factor, it's a real number

(De Moivre formula)

↓

Properties of exponentials: $e^{j \cdot (\omega t + \theta)} = e^{j \cdot \omega t} \cdot e^{j \cdot \theta}$

↓

$$= \mathrm{Re}\left[\underbrace{\boxed{A \cdot e^{j\theta}}}_{\text{phasor}} \cdot \underbrace{e^{j\omega t}}_{\text{same everywhere in a linear AC circuit.}} \right]$$

A sinusoid is associated with a phasor (or its real part)

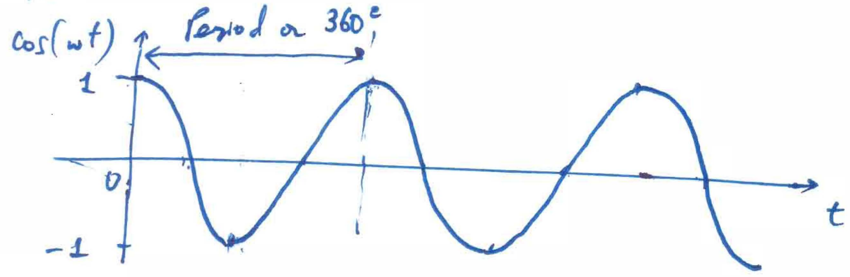
Comparing sinusoids using phasors:

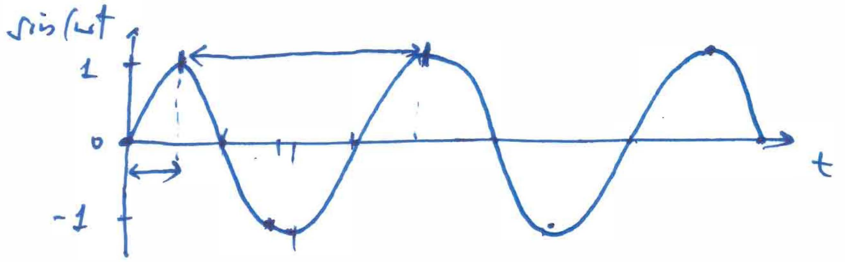
- Phasors are vectors in the complex plane with a magnitude & a phase:

$\underbrace{Ae^{j\theta}}$ is a phasor with amplitude A & phase θ

polar form of a complex number.

- From trigonometry: cos is a sin with a phase shift of $90°$:





If we shift $\sin(wt)$ a quarter of a period to the left that is $\cos(wt)$. A quarter of a period is $\frac{360°}{4} = 90°$:

$$\cos(wt) = \sin(wt + 90°)$$

Let's compare two sinusoids: $\begin{cases} v_1(t) = 12\sin(100t + 60°) \\ v_2(t) = -6\cos(100t + 30°) \end{cases}$

$= 12 \sin(100t - 30° + 90°)$

$$\rightarrow \begin{cases} v_1(t) = 12 \cos(100t - 30°) \\ v_2(t) = 6 \cos(100t + 30° - 180°) = 6 \cos(100t - 150°) \end{cases}$$

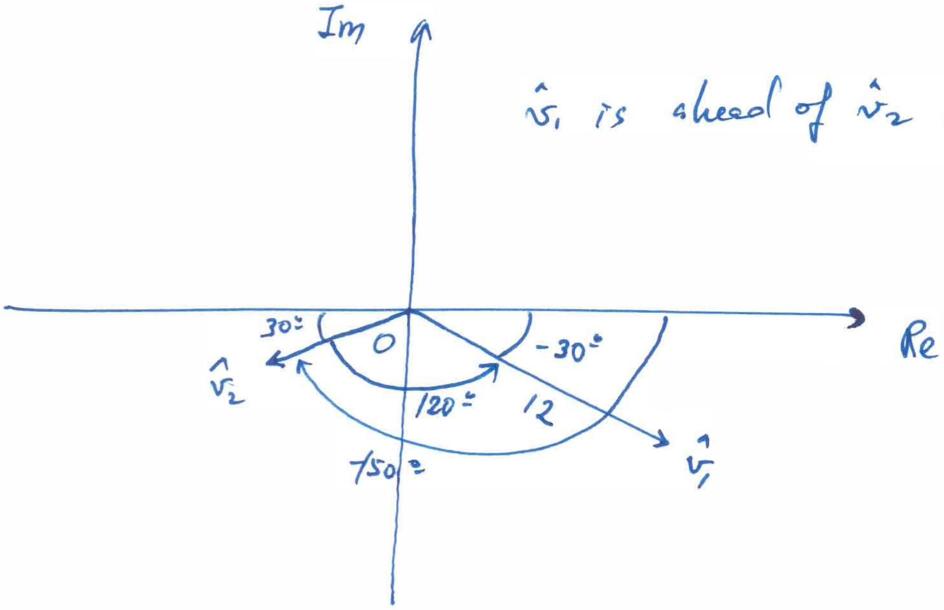
→ ~~Clock~~ Counterclockwise (standard convention):

v_1 is ahead of v_2 by 120°

<u>Using phasors</u>: sinusoids are represented by vectors in the complex plane:

$$v_1(t) \longrightarrow 12\, e^{j(-30°)} = \hat{v}_1$$

$$v_2(t) \longrightarrow 6\, e^{j(-150°)} = \hat{v}_2$$



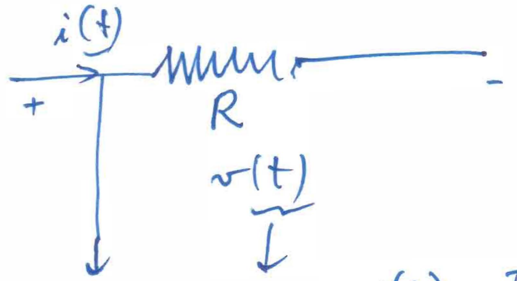
Impedances are phasors in linear AC circuits:

Impedance : extension of the resistences

$$\hat{Z} \equiv \frac{\hat{V}}{\hat{I}} \qquad\qquad R = \frac{v}{i}$$

(ratios of complex values or phasors) (ratios of real values)

What is the connection b/w $\hat{Z}$ & R ?



By Ohm's Law:

$$R = \frac{v(t)}{i(t)}$$

Linear AC circuits : $i(t) = I\cos(\omega t + \theta_I) = Re[Ie^{j\theta_I} \cdot e^{j\omega t}]$

$v(t) = V\cos(\omega t + \theta_v) = Re[Ve^{j\theta_v} \cdot e^{j\omega t}]$

$$Re[Ie^{j\theta_I} \cdot e^{j\omega t}] = Re[Ie^{j(\omega t + \theta_I)}] = I\,Re[e^{j(\omega t+\theta_I)}]$$

$$= I\,Re[\underbrace{\cos(\omega t + \theta_I)} + j\sin(\omega t + \theta_I)]$$

$$= I\cos(\omega t + \theta_I)$$

$$R = \frac{v(t)}{i(t)} = \frac{Re[Ve^{j\theta_v} \cdot \cancel{e^{j\omega t}}]}{Re[Ie^{j\theta_I} \cdot \cancel{e^{j\omega t}}]} = Re\left[\frac{Ve^{j\theta_v}}{Ie^{j\theta_I}}\right]$$

$\hat{V}$ voltage phasor

$\hat{I}$ current phasor

$$Re\left[\frac{Ve^{j\theta_v} \cdot \cancel{e^{j\omega t}}}{Ie^{j\theta_I} \cdot \cancel{e^{j\omega t}}}\right]$$

$$= Re[\hat{Z}]$$

Discussion: $R = Re[\hat{Z}]$

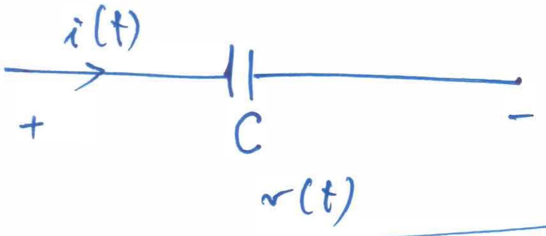
$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{V e^{j\theta_v}}{I e^{j\theta_I}} = \frac{V}{I} e^{j(\theta_v - \theta_I)}$$

θ_v: phase of voltage
θ_I: phase of current

At a resistor: voltage & current are in phase $\theta_v - \theta_I = 0$

$$\boxed{\hat{Z}_R = \frac{V}{I} = R}$$ Impedance @ a resistor is R

Impedance @ a capacitor:



$$\boxed{i(t) = C\frac{dv}{dt}}$$

Linear AC circuits → $i(t)$ & $v(t)$ are sinusoids
↓
R, L, C

$\rightarrow i(t) = I\cos(\omega t + \theta_I) = Re[I e^{j\theta_I} \cdot e^{j\omega t}]$

...

$$Re[\underbrace{I e^{j\theta_I}}_{\substack{\hat{I} \\ \text{current phasor}}} \cdot e^{j\omega t}] = C\frac{d}{dt} Re[V e^{j\theta_v} \cdot e^{j\omega t}]$$

$$= C\, Re[\underbrace{V e^{j\theta_v}}_{\substack{\hat{V} \\ \text{(voltage phasor)}}} \underbrace{\frac{d}{dt} e^{j\omega t}}_{j\omega e^{j\omega t}}]$$

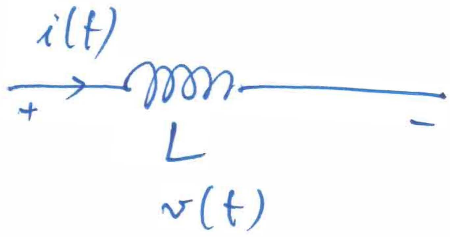
$$Re[\hat{I} \cdot e^{j\omega t}] = C\, Re[j\omega \hat{V} \cdot e^{j\omega t}] = Re[j\omega C \hat{V} e^{j\omega t}]$$

$$\hat{I}\cdot\cancel{e^{j\omega t}} = j\omega C\cdot\hat{V}\cdot\cancel{e^{j\omega t}}$$

$$\frac{\hat{V}}{\hat{I}} = \frac{1}{j\omega C} \quad \text{or} \quad \boxed{\hat{Z}_c = \frac{1}{j\omega C} = -\frac{j}{\omega C}}$$

$$j\cdot j = \sqrt{-1}\cdot\sqrt{-1} = \left[(-1)^{1/2}\right]^2 = -1$$

<u>Impedance @ an inductor</u>:



$$\boxed{v(t) = L\frac{di}{dt}}$$

Linear AC circuits:

$$Re\left[\underbrace{Ve^{j\theta_v}}_{\hat{V}}\cdot e^{j\omega t}\right] = L\frac{d}{dt}Re\left[Ie^{j\theta_I}e^{j\omega t}\right]$$

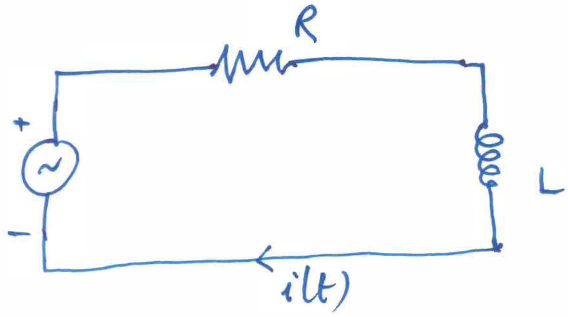
$$= Re\left[j\omega L\underbrace{Ie^{j\theta_I}}_{\hat{I}}e^{j\omega t}\right]$$

$$\hat{V}\cancel{e^{j\omega t}} = j\omega L\hat{I}\cancel{e^{j\omega t}}$$

$$\frac{\hat{V}}{\hat{I}} = j\omega L \quad \text{or} \quad \boxed{\hat{Z}_L = j\omega L}$$

Advantage in Using Phasors for Linear AC Analysis:

$v(t) = V\cos\omega t$



Solve for $i(t)$

Method #1: using time-domain information

Loop Analysis:

$$\underbrace{V\cos\omega t}_{\substack{\text{potential}\\\text{gain}\\\text{across}\\\text{AC source}}} = \underbrace{R\cdot i(t)}_{\substack{\text{potential}\\\text{drop}\\\text{across } R}} + \underbrace{L\cdot\frac{di}{dt}}_{\substack{\text{potential}\\\text{drop}\\\text{across } L}}$$

Solve for $i(t)$:

a) linear AC circuit → sinusoid w/ same frequency ω & different phase

$$i(t) = I\cos(\omega t + \theta)$$

↑magnitude, ↑phase
2 unknowns!

b) Observation (trig.)

$$i(t) = I\left[\cos\omega t\cos\theta - \sin\omega t\sin\theta\right]$$

$$= \underbrace{I\cos\theta}_{\equiv I_1}\cos\omega t + \underbrace{I(-\sin\theta)}_{\equiv I_2}\sin\omega t$$

$$= I_1\cos\omega t + I_2\sin\omega t$$

(2 unknowns: I_1 & I_2)

c) Plug $i(t)$ into loop Analysis equation:

$$\begin{cases} V\cos\omega t = R\left[I_1\cos\omega t + I_2\sin\omega t\right] + L\omega\left[-I_1\sin\omega t + I_2\cos\omega t\right] \\ V\cos\omega t = (RI_1 + \omega L I_2)\cdot\cos\omega t + (RI_2 - \omega L I_1)\cdot\sin\omega t \end{cases}$$

- Note: $\cos\omega t$ & $\sin\omega t$ are out of phase by $90°$ or perpendicular in a (phasors) complex plane.

$$\rightarrow \left.\begin{cases} RI_2 - \omega L I_1 = 0 \\ RI_1 + \omega L I_2 = V \end{cases}\right\}$$ 2 equations for 2 unknowns I_1 & I_2

$$\hookrightarrow I_2 = \frac{\omega L}{R} I_1 \rightarrow R I_1 + \frac{(\omega L)^2}{R} I_1 = V$$

$$\boxed{\begin{aligned} I_1 &= \frac{R}{R^2 + (\omega L)^2} V \\ I_2 &= \frac{\omega L}{R^2 + (\omega L)^2} V \end{aligned}}$$

- Write $i(t) = I_1\cos\omega t + I_2\sin\omega t = I\cos(\omega t + \theta)$

or write I & θ in terms of I_1 & I_2

$$\left.\begin{cases} I = \sqrt{I_1^2 + I_2^2} \checkmark \\ \theta = -\tan^{-1}\dfrac{I_2}{I_1} \checkmark \end{cases}\right\}$$ b/c $I_1 \equiv I\cos\theta$
$I_2 \equiv -I\sin\theta$

$$\frac{I_2}{I_1} = -\tan\theta \rightarrow \theta = -\tan^{-1}\frac{I_2}{I_1}$$

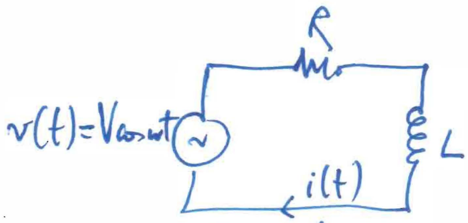
$$I = \sqrt{\frac{R^2}{[R^2+(\omega L)^2]^2}V^2 + \frac{(\omega L)^2}{[R^2+(\omega L)^2]^2}V^2} = \frac{V}{[R^2+(\omega L)^2]}[R^2+(\omega L)^2]^{1/2}$$

$$\boxed{I = \frac{V}{\sqrt{R^2+(\omega L)^2}} \qquad \theta = -\tan^{-1}\frac{I_2}{I_1} = -\tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$\boxed{i(t) = \frac{V}{\sqrt{R^2+(\omega L)^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)}$$

Current in an AC RL circuit using Loop Analysis

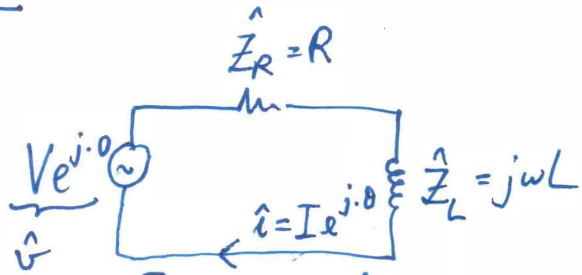
<u>Method #2</u>: using <u>Phasors</u>



Time-domain

$i(t) = \underline{I}\cos(\omega t + \underline{\theta})$

$\longrightarrow$



Frequency-domain or Phasor version of same circuit.

Note: Ohm's Law is valid in both time & frequency domain! Find current phasor, then convert back into time-domain.

$$\hat{i} = \frac{\hat{v}}{\hat{Z}_R + \hat{Z}_L} = \frac{Ve^{j\cdot 0}}{R + j\omega L} = \frac{V}{R + j\omega L} =$$

$\hat{i} = \dfrac{V}{R+j\omega L}$. To get or write $i(t) = \underline{\underline{I}} \cos(\omega t + \underline{\underline{\theta}})$

→ just need the magnitude (I) & phase (θ) of $\dfrac{V}{R+j\omega L}$:

$$I = \frac{|V|}{|R+j\omega L|} = \frac{V}{\sqrt{R^2+(\omega L)^2}}$$

$$\theta = \text{Angle}(V) - \text{Angle}(R+j\omega L)$$

$$= 0 - \tan^{-1}\frac{\omega L}{R}$$

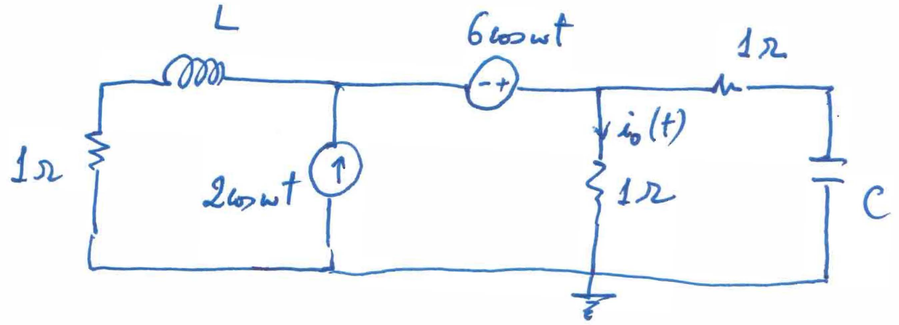
$$\boxed{i(t) = \frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)}$$

(1 page vs 2 pages)

$$\boxed{\begin{aligned} &\hat{z} = \frac{\hat{a}}{\hat{b}} \quad \text{in polar coords.} \quad z e^{j\theta_z} = \frac{a e^{j\theta_a}}{b e^{j\theta_b}} \\ &\qquad\qquad\qquad\qquad\qquad\qquad = \frac{a}{b} e^{j(\theta_a - \theta_b)} \\ &\rightarrow \theta_z = \theta_a - \theta_b \end{aligned}}$$

Analysis Techniques AC Circuits using $\boxed{\text{Phasors}}$;

Time-domain



$L = 2.65\ mH$

$C = 2.65\ mF$

$\omega = 2\pi f$; $f = 60Hz$

Solve for $i_o(t)$

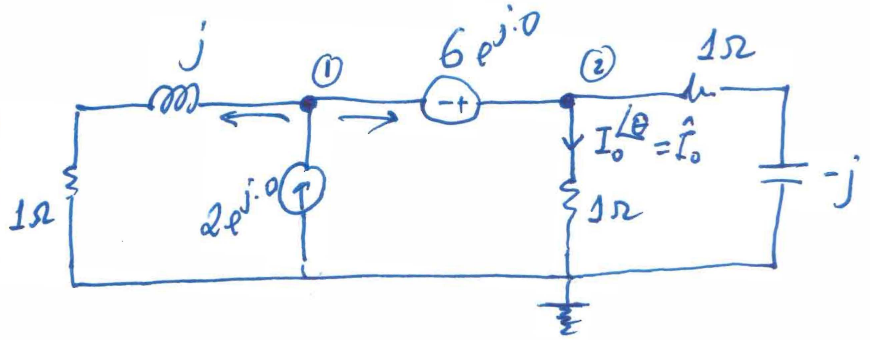
↓ Different Methods

Solve for I_o & θ

↓

$i_o(t) = I_o \cos(\omega t + \theta)$

Frequency domain or Phasor version



$$\hat{Z}_L = j\omega L = j\, 2\pi \cdot 60 \cdot 2.65 \cdot 10^{-3} = j \cdot 1 = j$$

$$\hat{Z}_C = \frac{1}{j\omega C} = \frac{1}{j\, 2\pi \cdot 60 \cdot 2.65 \cdot 10^{-3}} = \frac{1}{j} = -j$$

Note:
$$6e^{j0} = 6^{\angle 0}$$
$$Ze^{j\theta} = Z^{\angle\theta}$$

Node Analysis :

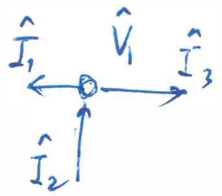
Step 1: identify the nodes : 2 → $\hat{V}_1$ & $\hat{V}_2$ $(\hat{V}_2 = \hat{V}_1 + 6)$

→ only one independent node, for example pick node #1

Step 2: write Kirchoff's Law for currents @ node #1

$$\sum_i I_i = 0 \begin{cases} \text{current into node : } + \\ \text{current leaving node : } - \end{cases}$$

Assuming



$$-\hat{I}_1 + \hat{I}_2 - \hat{I}_3 = 0$$

Ohm's Law:
$$-\frac{\hat{V}_1}{1+j} + 2 - \left(\hat{V}_2 + \frac{\hat{V}_2}{1-j}\right) = 0$$

Plug-in $\hat{V}_2 = \hat{V}_1 + 6$

$\hat{V}_1 = \hat{V}_2 - 6 \rightarrow$
$$-\frac{\hat{V}_2 - 6}{1+j} + 2 - \hat{V}_2 - \frac{\hat{V}_2}{1-j} = 0$$

$$-\hat{V}_2\left(\frac{1}{1+j} + 1 + \frac{1}{1-j}\right) + \frac{6}{1+j} + 2 = 0$$

$$\Rightarrow \hat{V}_2 = \frac{\frac{6}{1+j} + 2}{\frac{1}{1+j} + \frac{1}{1-j} + 1} = \frac{\frac{6+2+2j}{1+j}}{\frac{1-j+1+j}{2} + 1} = \frac{\frac{8+2j}{1+j}}{2} = \frac{4+j}{1+j}$$

Multiplying complex conjugates:

$$(a+jb)(a-jb) = a^2 + b^2 = \underbrace{|a+jb|^2}_{\text{Magnitude of.}}$$

$$(1+j)(1-j) = 1 + 1 = 2$$

$$\hat{I}_o = \frac{\hat{V}_2}{1\Omega} = \hat{V}_2 = \frac{4+j}{1+j}\,A = \frac{\sqrt{17}\, e^{j\tan^{-1}\left(\frac{1}{4}\right)}}{\sqrt{2}\, e^{j45^\circ}} = 2.91\, e^{j(-31^\circ)}$$

$\tan^{-1}\left(\frac{1}{4}\right) \sim 14^\circ$

Need convert into polar form

→ Magnitude & phase

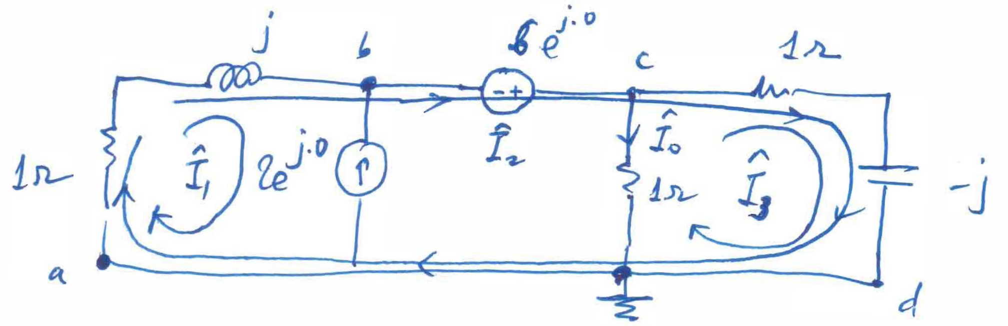
To go back to time-domain.

$$\boxed{i_o(t) = 2.91 \cdot \cos(377t - 31^\circ)\ A}$$

$\boxed{\text{Loop Analysis}}$ (using the same circuit):

Step 1: Identify the loops: 3 (3 small or 2 small + 1 big)

$\hat{I}_1$ & $\hat{I}_3$
CW CW



Step 2: Write Kirchoff's Law for voltage (KVL) for each of three loops

$$\begin{cases} 1) & \hat{I}_1 = -2A \quad (\hat{I}_1 \text{ was assumed to be CW}) \\ 2) & -(\hat{I}_1+\hat{I}_2)(1+j) + 6 - (\hat{I}_2+\hat{I}_3)(1-j) = 0 \\ 3) & -\hat{I}_3\cdot 1 - (\hat{I}_2+\hat{I}_3)\cdot(1-j) = 0 \end{cases}$$

Current x Impedance → Potential drop.

Sign convention $\begin{cases} \text{Potential drop: } - \\ \text{Potential gain: } + \end{cases}$

3 eqs & 3 unknowns: $\hat{I}_1, \hat{I}_2, \hat{I}_3$ Solve for $\hat{I}_3$ since $\hat{I}_0 = -\hat{I}_3$

3)
$$-\hat{I}_3 - \hat{I}_3(1-j) - \hat{I}_2(1-j) = 0$$

$$-\hat{I}_3\underbrace{(1+1-j)}_{(2-j)} - \hat{I}_2(1-j) = 0$$

$$\hat{I}_2 = -\hat{I}_3 \frac{(2-j)}{(1-j)}$$

Plugging this result back into 2):

$$-\left(-2 - \hat{I}_3 \frac{2-j}{1-j}\right)(1+j) + 6 - \underbrace{\left(\hat{I}_3 - \hat{I}_3\frac{2-j}{1-j}\right)}_{\hat{I}_3\left(1-\frac{2-j}{1-j}\right) = \hat{I}_3\left(\frac{-1}{1-j}\right)}(1-j) = 0$$

$$\hat{I}_3\left(\frac{2-j}{1-j}(1+j)+\frac{1-j}{1-j}\right)+2(1+j)+6=0$$

$$\hat{I}_3\left(\frac{3+j+1-j}{1-j}\right)+8+2j=0$$

$$\hat{I}_3\,\frac{4}{1-j}+8+2j=0$$

$$\hat{I}_3=-\frac{8+2j}{\frac{4}{1-j}}=\frac{(8+2j)(1-j)}{-4}$$

$$=-\frac{10-6j}{4}$$

4th quadrant

$$\hat{I}_0=-\hat{I}_3=\frac{10-6j}{4}=\frac{5-3j}{2}=\frac{\sqrt{34}\,e^{j\tan^{-1}\left(-\frac{3}{5}\right)}}{2}$$

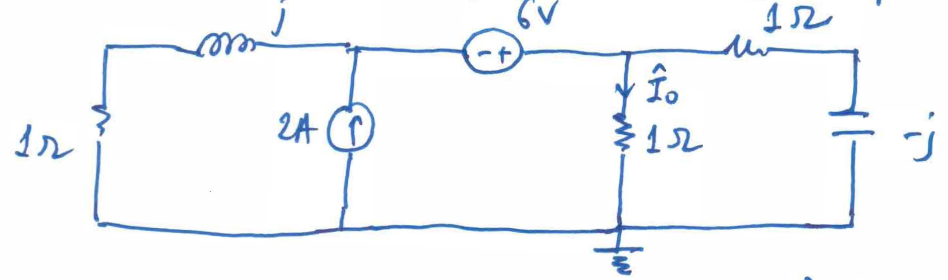
Need to convert to polar form.

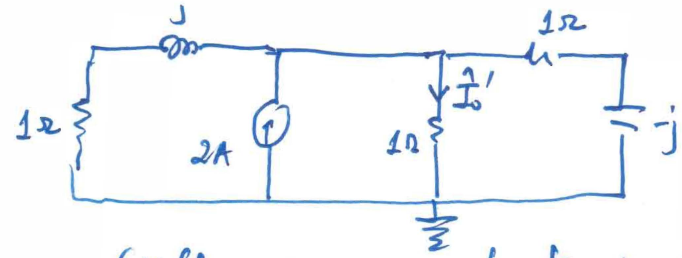
$$\boxed{\hat{I}_0=2.91\,e^{j(-31^\circ)}}$$

$$\boxed{i_0(t)=2.91\cos(377t-31^\circ)\ \text{A}}$$

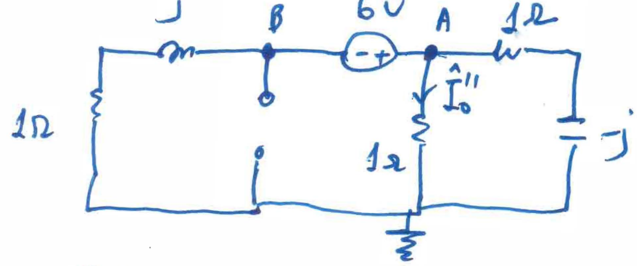
Superposition Analysis :

If we have 2 or more sources in a same circuit, can combine results from single source using series/parallel combinations of impedances.





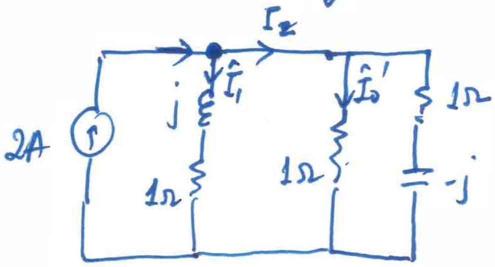
(Voltage source is short-circuited)



(Current source is open-circuited)

$$\boxed{\hat{I}_o = \hat{I}_o' + \hat{I}_o''}$$

Superposition Method.



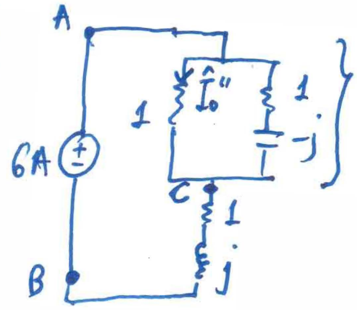
Current division :

$$\hat{I}_2 = 2A \frac{1+j}{1+j+\frac{1(1-j)}{2-j}}$$

$$\hat{I}_o' = \hat{I}_2 \frac{1-j}{2-j}$$

$$= 2A \frac{1+j}{1+j+\frac{1-j}{2-j}} \cdot \frac{1-j}{2-j}$$

$$= \frac{4A}{(1+j)(2-j)+1-j} = \frac{4}{3+j+1-j} = 1A$$



Voltage division:

$$V_{AC} = 6V \frac{\frac{1(1-j)}{2-j}}{\frac{1-j}{2-j}+1+j}$$

$$V_{AC} = 6V \frac{1-j}{1-j+(1+j)(2-j)} = 6V \frac{1-j}{1-j+3+j}$$

$$V_{AC} = \frac{6}{4}(1-j)$$

$$\hat{I}_o'' = \frac{\frac{6}{4}(1-j)}{1} A = \frac{3}{2}(1-j)\ A$$

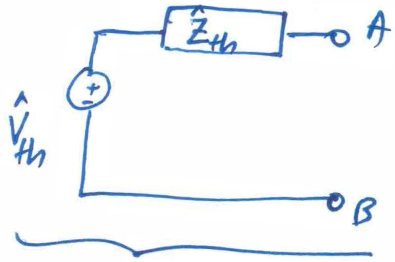
$$\hat{I}_o = \hat{I}_o' + \hat{I}_o'' = 1 + \frac{3}{2}(1-j) = \frac{5-3j}{2}$$

Convert into polar form:

$$\frac{1}{2}\sqrt{5^2+3^2}\ \angle \tan^{-1}\left(-\frac{3}{5}\right) = 2.91 \angle -31^\circ\ A\ ✓$$

AC Thevenin's Equivalent Analysis

To replace circuit b/w A & B by:
(without the element or load between A & B)



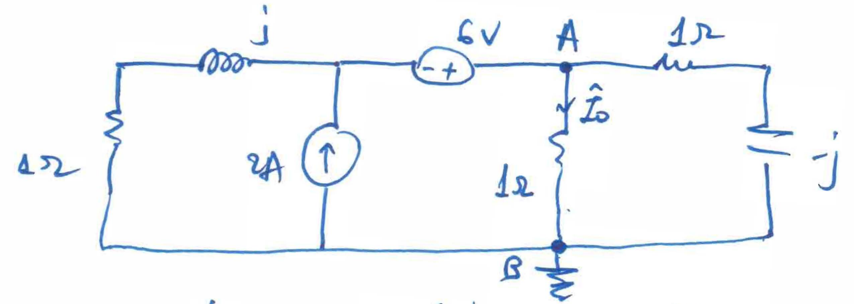
Thevenin's equivalent circuit b/w A & B

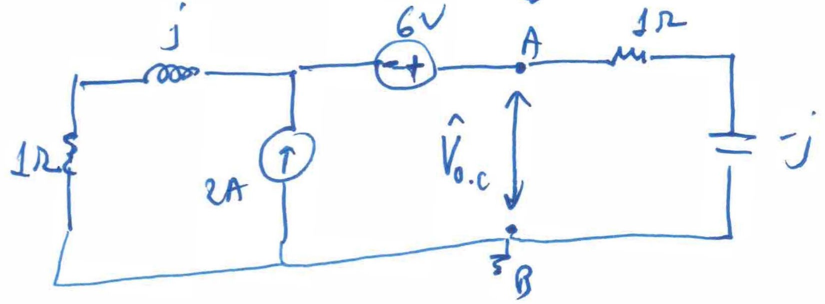
$\hat{V}_{th} \equiv \hat{V}_{o.c.}$ b/w A & B

open circuit voltage between A & B

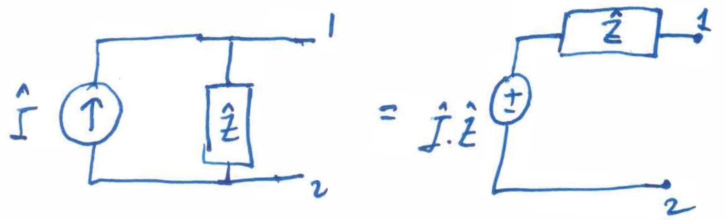
$\hat{Z}_{th} = \hat{Z}_{b/w\ A\&B}$ / w no sources.

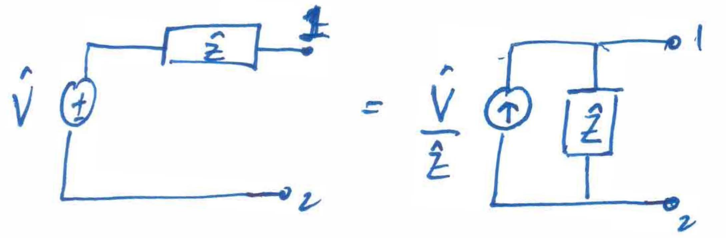
- Current source : open circuit
- Voltage source : short-circuit.



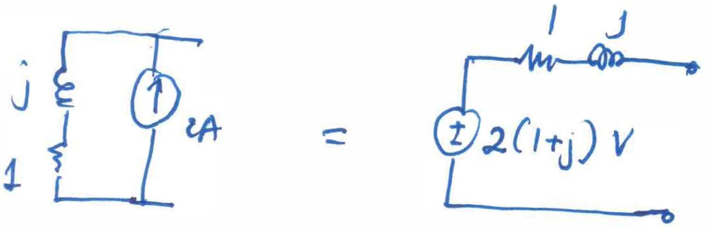


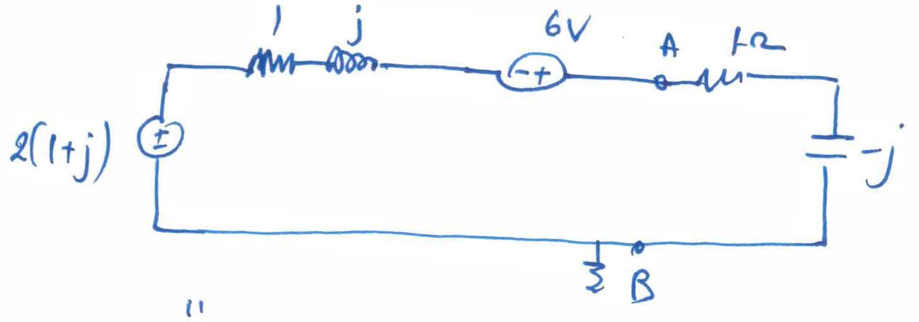
Also use Source Exchange:

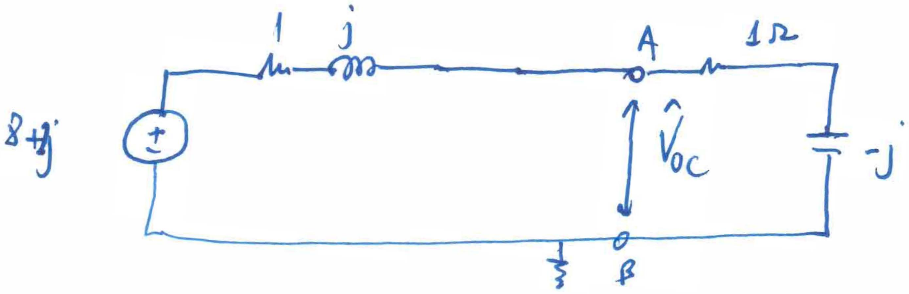




In our circuit:







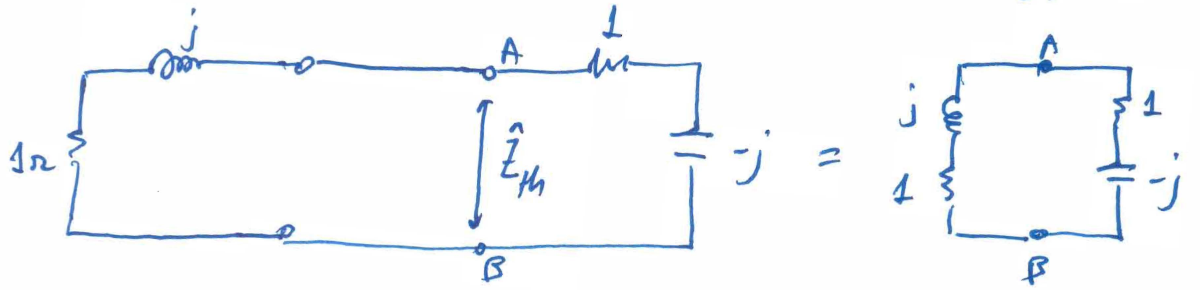
Voltage division:

$$\hat{V}_{oc} = (8+2j)\frac{1-j}{1+j+1-j}$$

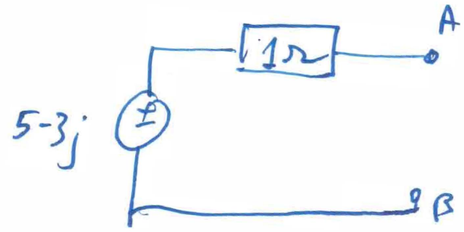
$$= (4+j)(1-j)$$

$$= (5-3j)\ V$$

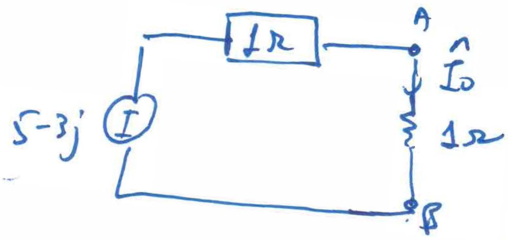
$\hat{Z}_{th}$:



$$\hat{Z}_{th} = (1+j)\,\|\,(1-j) = \frac{(1+j)(1-j)}{1+\cancel{j}+1-\cancel{j}} = \frac{2}{2} = 1\ \Omega$$



Now put back the load b/w A & B:



$$\Rightarrow \hat{I}_o = \frac{5-3j}{2}\ A$$

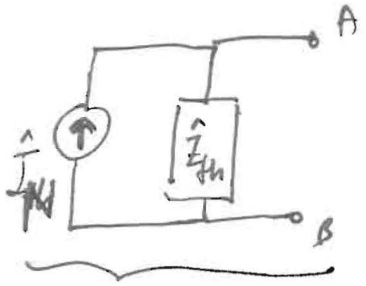
polar: $\frac{1}{2}\sqrt{34}\ \underline{/\tan^{-1}\left(\frac{-3}{5}\right)}$

$= 2.91\ \underline{/-31^\circ}\ A$

AC Norton's Equivalent Analysis

↓

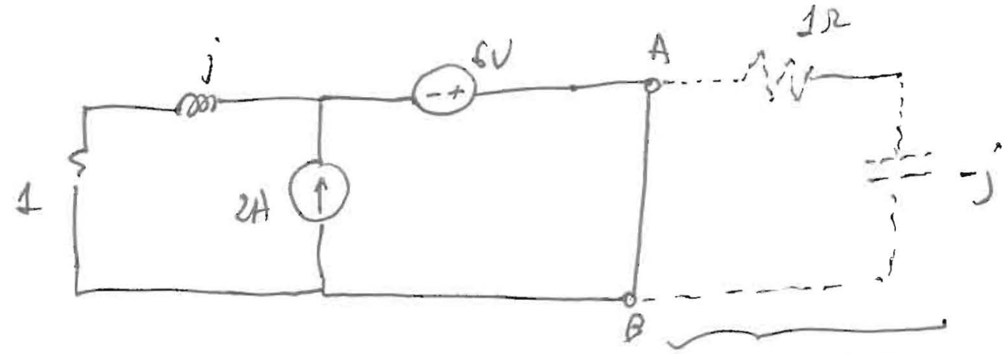
To replace the circuit b/w A & B by
(without the load between A & B)



Norton's equivalent circuit b/w A & B

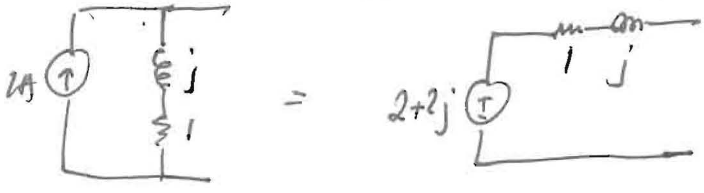
$\hat{I}_N = \hat{I}_{\boxed{S.C}}$ b/w A & B → <u>Short circuit</u>

$\hat{Z}_{Th} = 1\Omega$



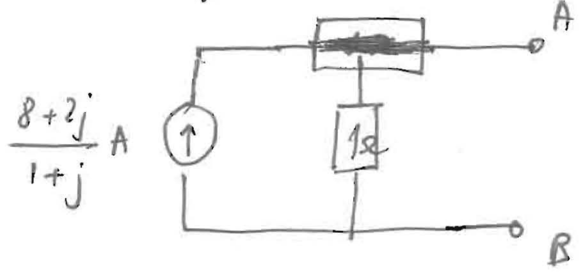
No current here if A & B are short-circuited!

<u>Source Exchange</u> on

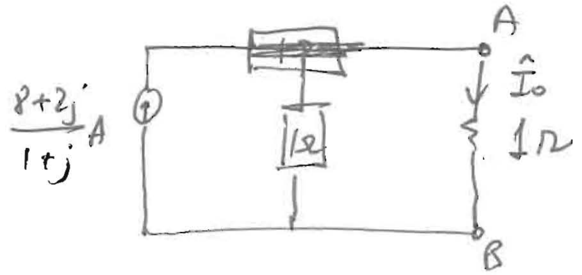


$$\hat{I}_N = \frac{8+2j}{1+j}$$

Norton's Equivalent :



Put back the load of 1Ω b/w A & B :



→ Current division :

$$\hat{I}_o = \frac{1}{2}\,\frac{8+2j}{1+j} = \frac{4+j}{1+j}$$

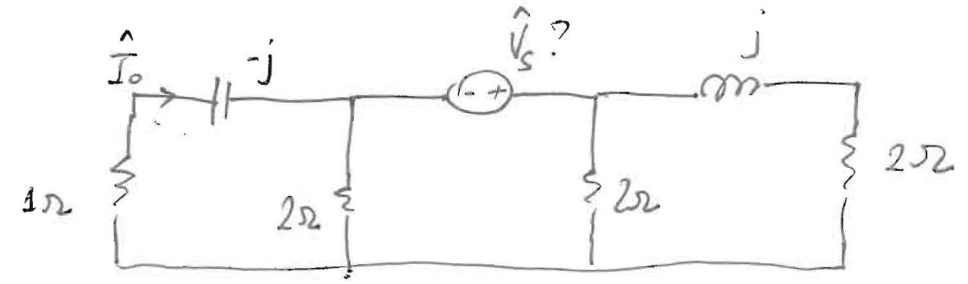
$$= \frac{(4+j)(1-j)}{2}$$

$$= \frac{5-3j}{2}$$

→ Polar form : $\hat{I}_o = \frac{1}{2}\sqrt{34}\,\angle -\tan^{-1}\frac{3}{5} = 2.91\,\angle -31^\circ$ A

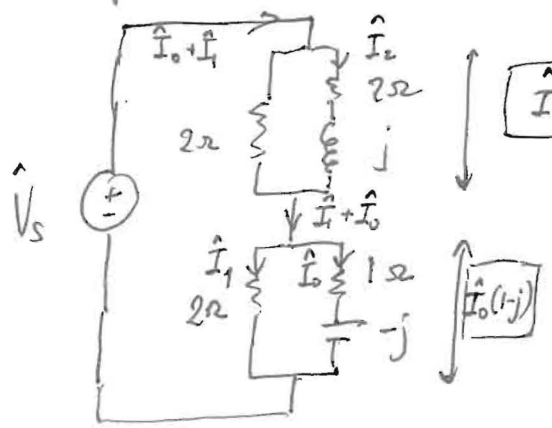
→ Back to time domain : $i_o(t) = 2.91\cos(377t - 31^\circ)$

7.38 $\quad \hat{V}_s = 5.9\angle{-23.5^\circ}\ V$



$\boxed{\hat{I}_0 = 2A} \rightarrow \hat{V}_s\ ?$

<u>Methods of Analysis:</u> multiple, but first let's try parallel & series combination since there is only one source:



$\boxed{\hat{I}_2\cdot(2+j)}$

$\boxed{\hat{I}_0(1-j)}$

$\hat{I}_0 = 2A$

$\underset{\text{Current div.}}{\hat{I}_1} = (\hat{I}_1 + 2)\cdot\dfrac{1-j}{3-j} \rightarrow \hat{I}_1\left(1-\dfrac{1-j}{3-j}\right) = \dfrac{2-2j}{3-j}$

$$\hat{I}_1 = \frac{\frac{2-2j}{3-j}}{1-\frac{1-j}{3-j}} = \frac{2-2j}{3-\cancel{j}-1+\cancel{j}} = 1-j$$

$\rightarrow$ Total current is $\hat{I}_0 + \hat{I}_1 = 2+1-j = 3-j$

Getting $\hat{I}_2$:

(Current div.): $\hat{I}_2 = (3-j)\cdot\dfrac{2}{4+j}$

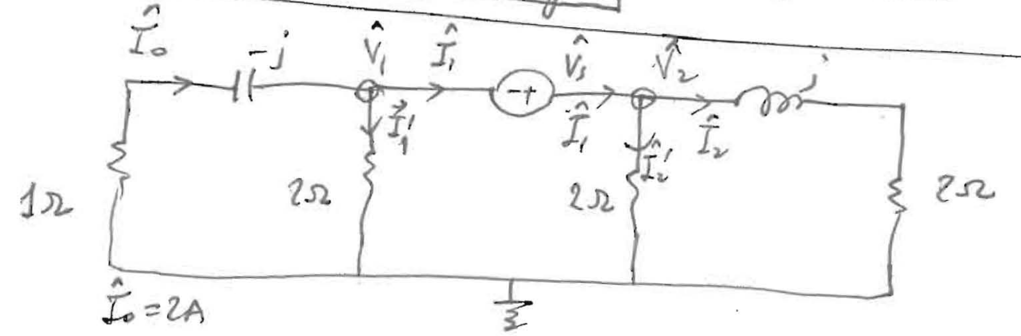
$$\rightarrow \hat{V}_s = \hat{I}_2(2+j) + \hat{I}_0(1-j) = \frac{2(3-j)}{4+j}(2+j) + 2(1-j)$$

$$= 2\,\frac{7+j+5-3j}{4+j} = 2\,\frac{12-2j}{4+j} = 4\,\frac{6-j}{4+j}$$

Polar form: $\hat{V}_s = 4\dfrac{\sqrt{37}}{\sqrt{17}}\angle{-\tan^{-1}\frac{1}{6}-\tan^{-1}\frac{1}{4}} = 5.9\angle{-9.46^\circ-14^\circ} =$

$= 5.9\angle{-23.5^\circ}\ V.$

Let's also use a $\boxed{\text{Node Analysis}}$ on this same circuit:



→ Two nodes: → $\hat{V}_1$ & $\hat{V}_2$

→ KCL @ each node:

Node 1: $\hat{I}_o - \hat{I}_1 - \hat{I}_1' = 0$

Write currents in terms of $\hat{V}_1$ & $\hat{V}_2$ using Ohm's Law:

$$\boxed{\frac{0-\hat{V}_1}{1-j} - \frac{\hat{V}_2 - 0}{2\|(2+j)} - \frac{\hat{V}_1 - 0}{2} = 0}$$

Node 2: $\hat{I}_1 - \hat{I}_2 - \hat{I}_2' = 0$

$$\hat{I}_1 = \hat{I}_2 + \hat{I}_2'$$

$$= \frac{\hat{V}_2}{2+j} + \frac{\hat{V}_2}{2} = \hat{V}_2\left(\frac{1}{2+j} + \frac{1}{2}\right)$$

$$= \hat{V}_2 \cdot \frac{2+2+j}{2(2+j)} = \frac{\hat{V}_2}{\frac{2(2+j)}{2+2+j}}$$

$$= \frac{\hat{V}_2}{2\|(2+j)}$$ (We already used this info in Node #1)

→ Only need to solve for Node 1 equation!

$$2A = \hat{I}_o = \frac{-\hat{V}_1}{1-j} \rightarrow \hat{V}_1 = -2(1-j)$$

$$\hookrightarrow -\hat{V}_1\left(\frac{1}{1-j} + \frac{1}{2}\right) - \hat{V}_2 \cdot \left(\frac{2+(2+j)}{2(2+j)}\right) = 0$$

$$\cancel{2}(1-j)\ \frac{2+1-j}{\cancel{2}(1-j)} = \hat{V}_2\ \frac{4+j}{2(2+j)}$$

$$\hat{V}_2 = \frac{(3-j)}{4+j}\,2(2+j)$$

$$\hat{V}_s = \hat{V}_2 - \hat{V}_1 = 2\,\frac{(3-j)(2+j)}{4+j} + 2(1-j)$$

$$= 2\,\frac{7+j \quad + (1-j)(4+j)}{4+j} = \cancel{2\,\frac{9-j}{4+j}}$$

$$= 2\,\frac{7+j \quad + 5-3j}{4+j} = 2\,\frac{12-2j}{4+j}$$

$$= 4\,\frac{6-j}{4+j} \xrightarrow[\text{Polar form}]{} 4\frac{\sqrt{37}}{\sqrt{17}}\ \angle -\tan^{-1}\tfrac{1}{6} - \tan^{-1}\tfrac{1}{4}$$

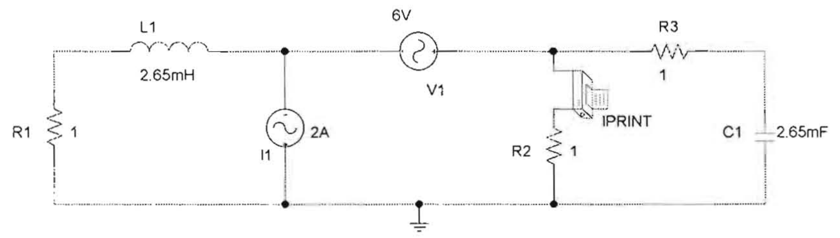
$$= 5.9\ \angle -23.5^{\circ}\ V$$

1.- Implement circuit using Get New Parts
In current sources IAC, current flows from
+ to -!!. For IPRINT, set AC=yes;
MAG=yes; PHASE=yes

2.- Analysis/Setup then select AC Sweep
with 1 points f=60Hz to 60Hz

3.- Analysis/Simulate

4.- Analysis/Examine Output, make sure current
agrees with pencil and paper methods



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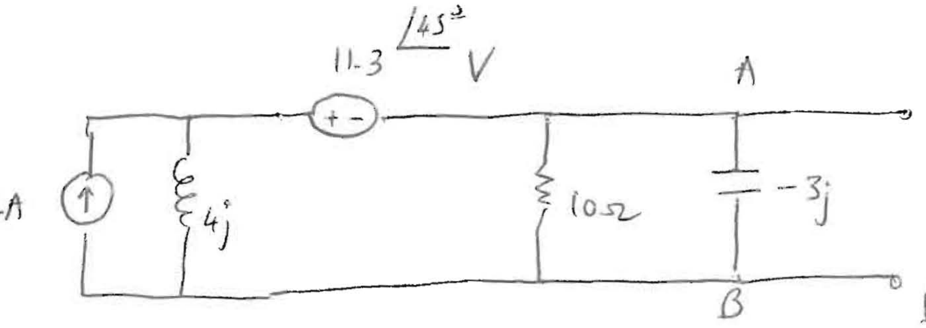
```
V_PRINT1            PRINT1(1=$N_0005 2=$N_0001 )

.ENDALIASES


**** RESUMING analysis_example.cir ****
.probe


.END

**** 02/11/10 13:57:46 *********** Evaluation PSpice (Nov 1999) **************

 * G:\CMSDec1908\e232\Sp10\analysis_example.sch


 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C

******************************************************************************



 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001)    0.0000                   ($N_0002)    0.0000

($N_0003)    0.0000                   ($N_0004)    0.0000

($N_0005)    0.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V_V1         0.000E+00
    V_PRINT1     0.000E+00

    TOTAL POWER DISSIPATION   0.00E+00  WATTS


**** 02/11/10 13:57:46 *********** Evaluation PSpice (Nov 1999) **************

 * G:\CMSDec1908\e232\Sp10\analysis_example.sch


 ****     OPERATING POINT INFORMATION      TEMPERATURE =   27.000 DEG C

******************************************************************************




**** 02/11/10 13:57:46 *********** Evaluation PSpice (Nov 1999) **************

 * G:\CMSDec1908\e232\Sp10\analysis_example.sch


 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C
```

****

```
  FREQ        IM(V_PRINT1)IP(V_PRINT1)

   6.000E+01   2.917E+00  -3.095E+01

          JOB CONCLUDED

          TOTAL JOB TIME             .20
```

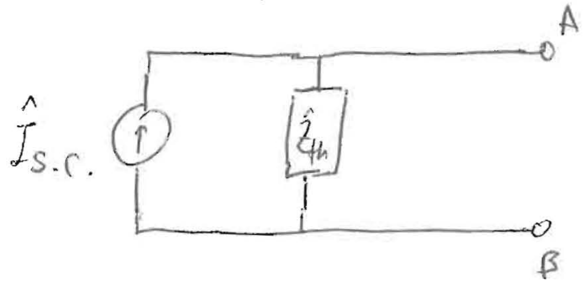
Agree with pencil + paper methods!

7.70

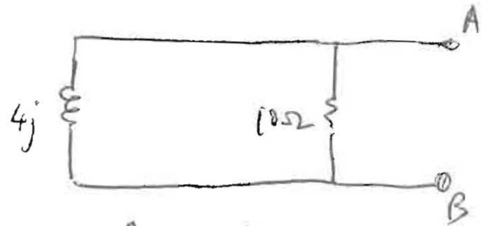


$\hat{V}_{AB}$? Use Norton's Theorem.

Norton's Equivalent b/w A & B without load $-3j$

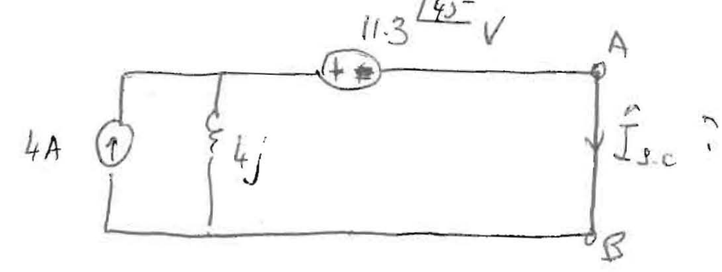


$\hat{Z}_{th}$: equiv. impedance b/w A & B when $\begin{cases} \text{Current source : Open circuit} \\ \text{Voltage source : Short-circuit} \end{cases}$

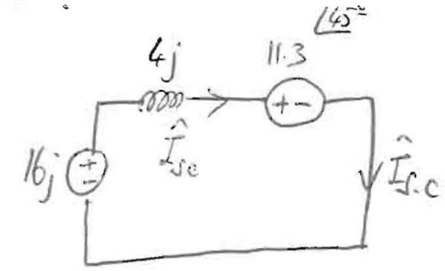


$\rightarrow \hat{Z}_{th} = 10 \parallel 4j = \dfrac{10 \cdot 4j}{10 + 4j}$

$\hat{I}_{S.C} \equiv \hat{I}_N = \hat{I}_{AB}$ when load is short-circuited.



$\longrightarrow$ Source Exchange



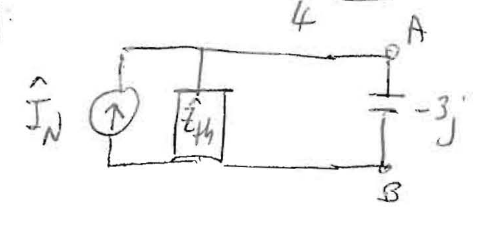
$$\hat{I}_{SC} = \frac{16j - 11.3(\cos 45^\circ + j\sin 45^\circ)}{4j} = \frac{16j - 7.99 - j7.99}{4j}$$

↓ Ohm's law

i) ×2

$$= \frac{-7.99 + 8.01j}{4j} = \frac{11.3\angle 135^\circ}{4\angle 90^\circ} = 2.825\angle 45^\circ \text{ A}$$

$\hat{V}_{AB} = \hat{I}_N (\hat{Z}_{th} \parallel (-3j))$



29

$$\hat{V}_{AB} = 2.825\angle 45^\circ \cdot \frac{\frac{40j(-3j)}{10+4j}}{\underbrace{\frac{40j}{10+4j} - 3j}} = \boxed{21.7\angle 5.19^\circ \ V}$$

$$\frac{120}{40j - 30j + 12} = \frac{120}{12 + 10j} = 60\frac{1}{\sqrt{36+25}}\angle \tan^{-1}\frac{5}{6}$$

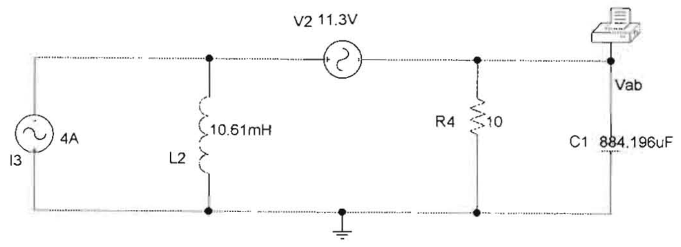
$$= 7.68\angle -39.8^\circ$$

1.- Implement circuit using Get New Parts
In current sources IAC, current flows from + to -!!. For VAC, set ACPHASE=45.
L=4/377; C=1/(3*377)=884uF
For VPRINT1, set AC=yes; MAG=yes; PHASE=yes

2.- Analysis/Setup then select AC Sweep with 1 points f=60Hz to 60Hz

3.- Analysis/Simulate

4.- Analysis/Examine Output, make sure current agrees with pencil and paper methods



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```
**** 02/16/10 13:52:11 *********** Evaluation PSpice (Nov 1999) **************

 * G:\CMSDec1908\e232\Sp10\7p70.sch


 ****     CIRCUIT DESCRIPTION


******************************************************************************




* Schematics Version 9.1 - Web Update 1
* Tue Feb 16 13:52:04 2010



** Analysis setup **
.ac LIN 1 60 60
.OP


* From [PSPICE NETLIST] section of pspiceev.ini:
.lib "nom.lib"

.INC "7p70.net"

**** INCLUDING 7p70.net ****
* Schematics Netlist *



I_I3         0 $N_0001 DC 0A AC 4A
V_V2         $N_0001 Vab DC 0V AC 11.3V 45
R_R4         0 Vab  10

.PRINT         AC
+ VM([Vab])
+ VP([Vab])
L_L2         0 $N_0001  10.61mH
C_C1         0 Vab  884.196uF

**** RESUMING 7p70.cir ****
.INC "7p70.als"



**** INCLUDING 7p70.als ****
* Schematics Aliases *

.ALIASES
I_I3             I3(+=0 -=$N_0001 )
V_V2             V2(+=$N_0001 -=Vab )
R_R4             R4(1=0 2=Vab )
L_L2             L2(1=0 2=$N_0001 )
C_C1             C1(1=0 2=Vab )
_     _(Vab=Vab)
.ENDALIASES


**** RESUMING 7p70.cir ****
```

```
.probe

.END
□
**** 02/16/10 13:52:11 *********** Evaluation PSpice (Nov 1999) **************
 * G:\CMSDec1908\e232\Sp10\7p70.sch

 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C

******************************************************************************


 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


(  Vab)    0.0000  ($N_0001)    0.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V_V2         0.000E+00

    TOTAL POWER DISSIPATION   0.00E+00  WATTS

□
**** 02/16/10 13:52:11 *********** Evaluation PSpice (Nov 1999) **************
 * G:\CMSDec1908\e232\Sp10\7p70.sch

 ****     OPERATING POINT INFORMATION      TEMPERATURE =   27.000 DEG C

******************************************************************************



□
**** 02/16/10 13:52:11 *********** Evaluation PSpice (Nov 1999) **************
 * G:\CMSDec1908\e232\Sp10\7p70.sch

 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C

******************************************************************************


  FREQ        VM(Vab)     VP(Vab)

  6.000E+01   2.173E+01   5.129E+00
```

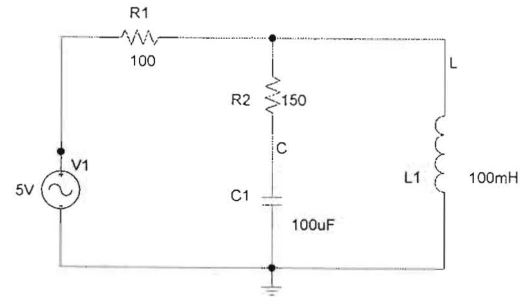
Agree with pencil & paper method.

1.- Implement circuit using Get New Parts Label the middle and right branches by double clicking then type in C and L. respectively

2.- Analysis/Setup then select AC Sweep with 1000 points f=1Hz to 10000Hz

3.- Analysis/Simulate

4.- Click on Graph window, Add Trace, put in M (I(C1)), and M(I(L1)). Zoom in and use Toggle Cursor to get information on the frequency at which these two currents are equal up to two decimal digits



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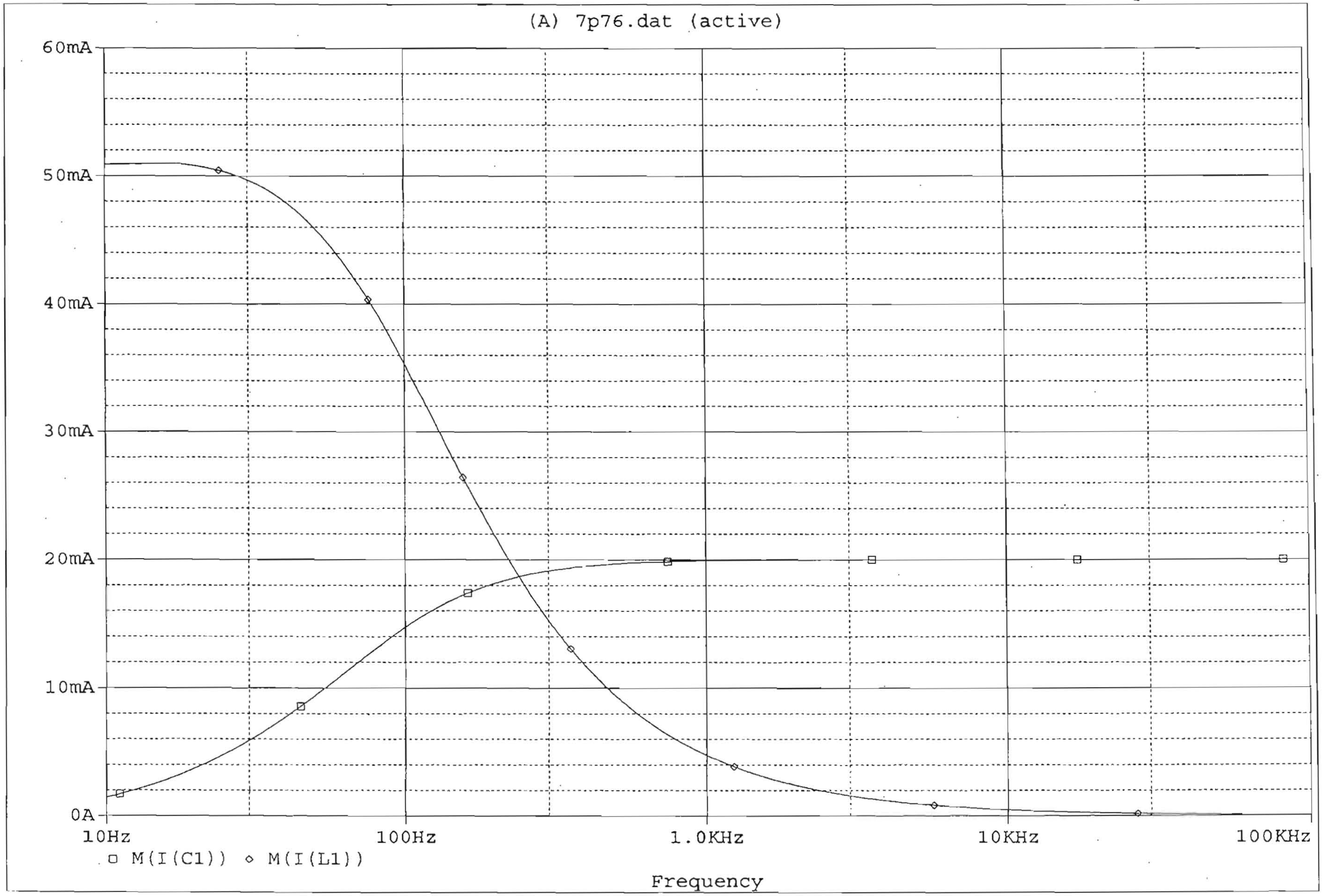
* C:\Temp\232s07\7p76.sch

Date/Time run: 02/18/08 14:27:30 Temperature: 27.0

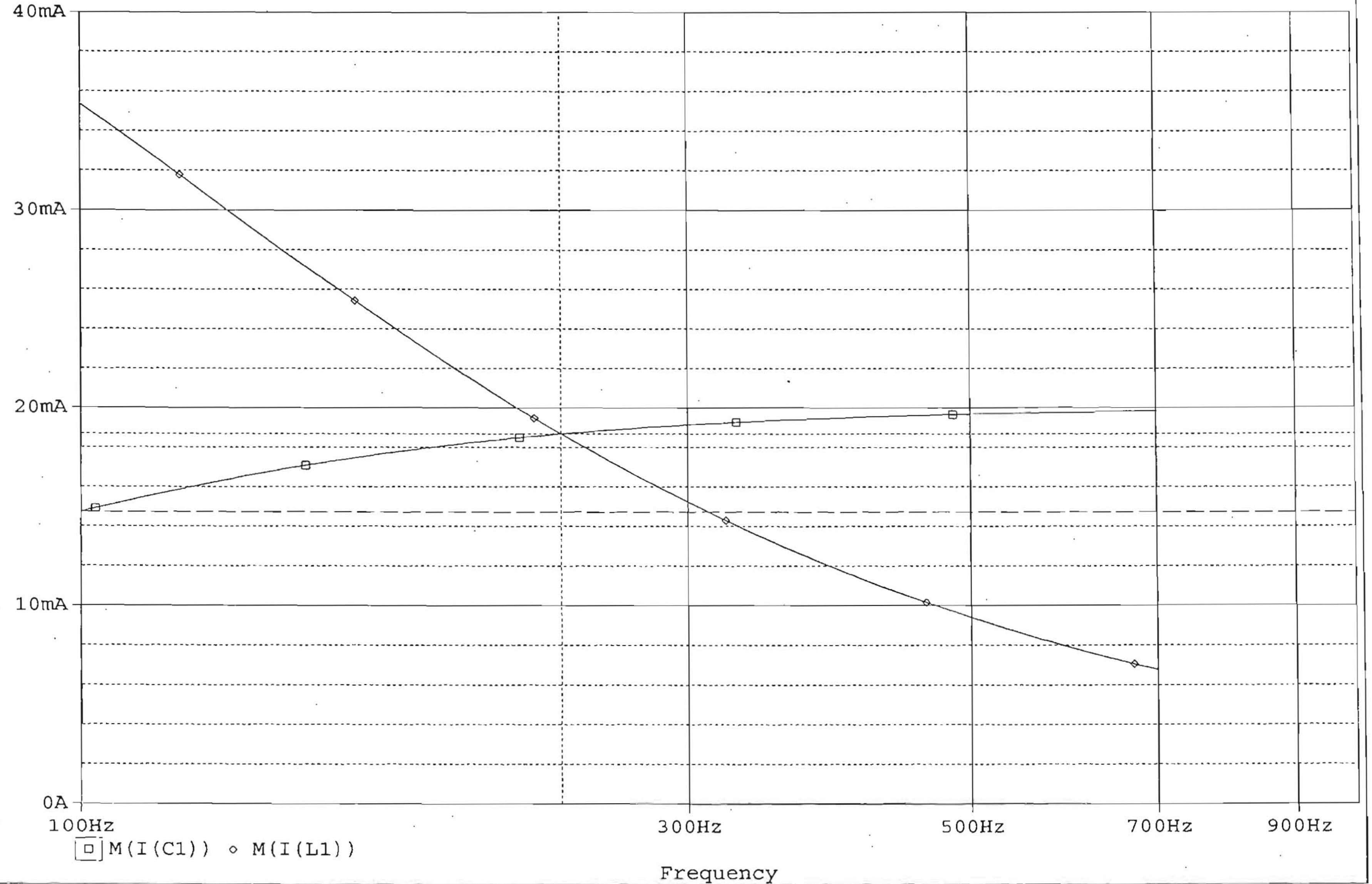


Date: February 18, 2008 Page 1 Time: 14:28:40

* C:\Temp\232s07\7p76.sch

Date/Time run: 02/18/08 14:33:26 Temperature: 27.0

(A) 7p76.dat (active)



A1:(238.841,18.677m) A2:(100.000,14.724m) DIFF(A):(138.841,3.9536m)

Date: February 18, 2008 Page 1 Time: 14:35:33

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