3) $20\log_{10}|1+j\omega\tau_1|$ use asymptotic analysis.

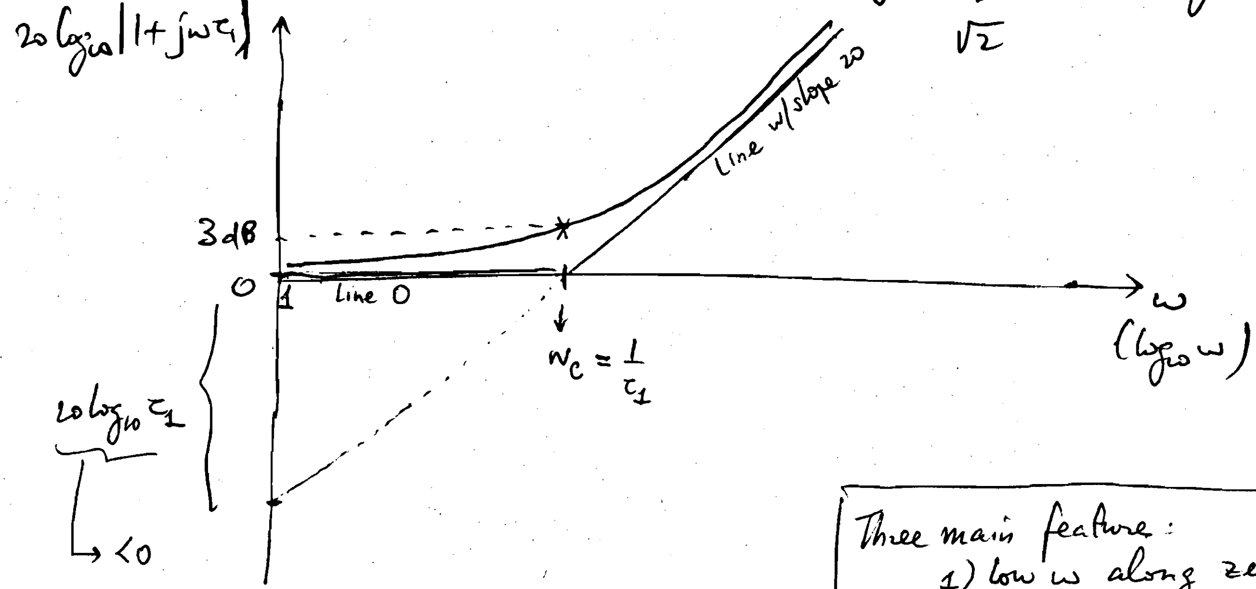
$\omega \to 0$ (low freq) $\to$ $j\omega\tau_1 \ll 1$ $\to$ $20\log_{10} 1 \simeq 0$

$\omega \to \infty$ (high frequency) $\to$ $j\omega\tau_1 \gg 1$ $\to$ $20\log_{10}|j\omega\tau_1|$

$20\log_{10}(\omega\tau_1) = 20\log_{10}\omega + 20\log_{10}\tau_1$

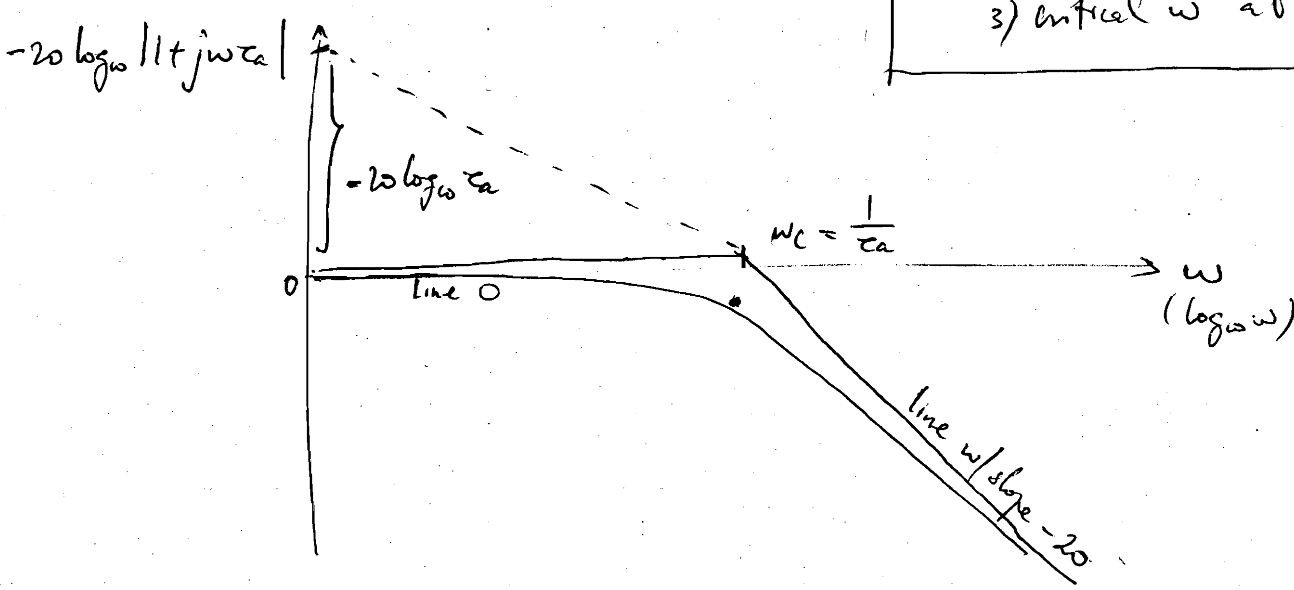
$\omega\tau_1 \simeq 1$ or $\omega_c = \frac{1}{\tau_1}$ (critical frequency) $\to$ $1+j\omega\tau_1 \approx 1+j$

$\to 20\log_{10}\underbrace{|1+j|}_{\sqrt{2}} = 10\log_{10}2 = 3\text{ dB}$



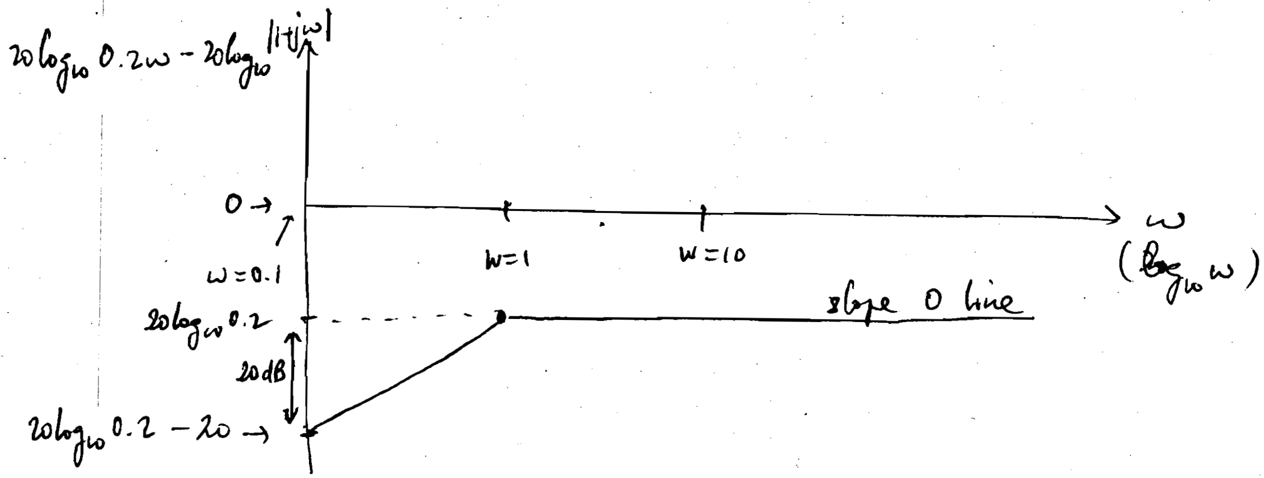
$\tau_1 = \frac{1}{\omega_c} < 1$

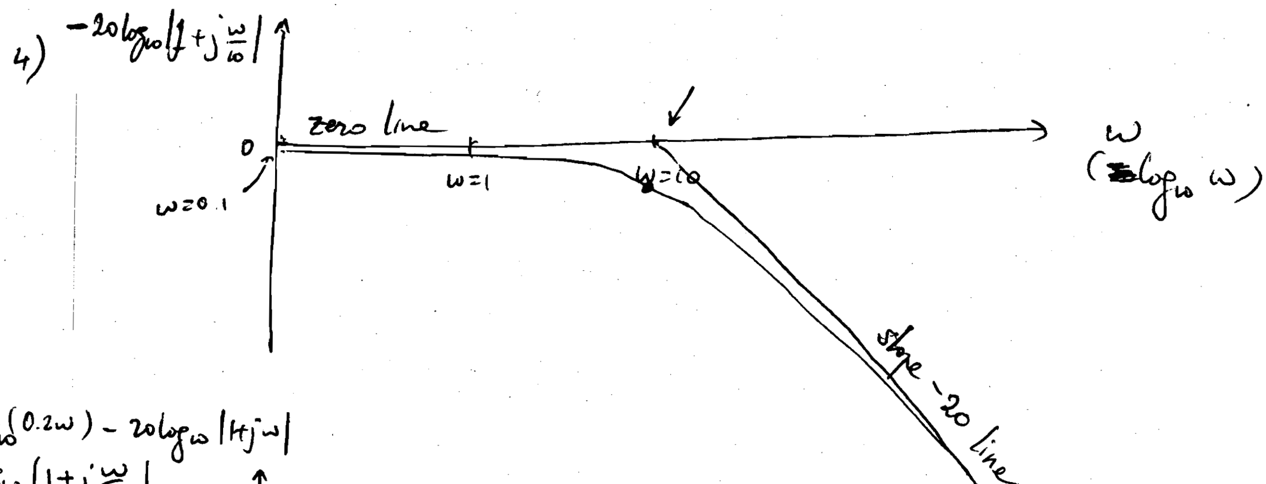
Three main feature:
1) low ω along zero
2) high freq along ± 20 slope
3) critical ω at ± 3 dB

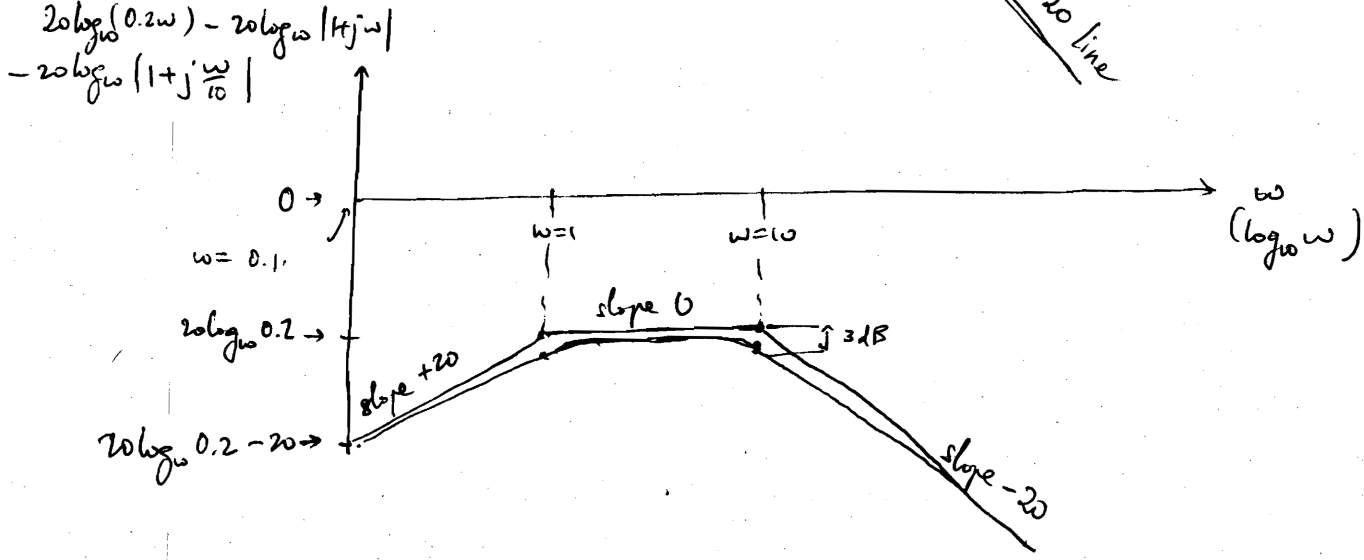


Let's add these two plots together: best way to describe the result is to use pieces separated by the critical frequency w_c

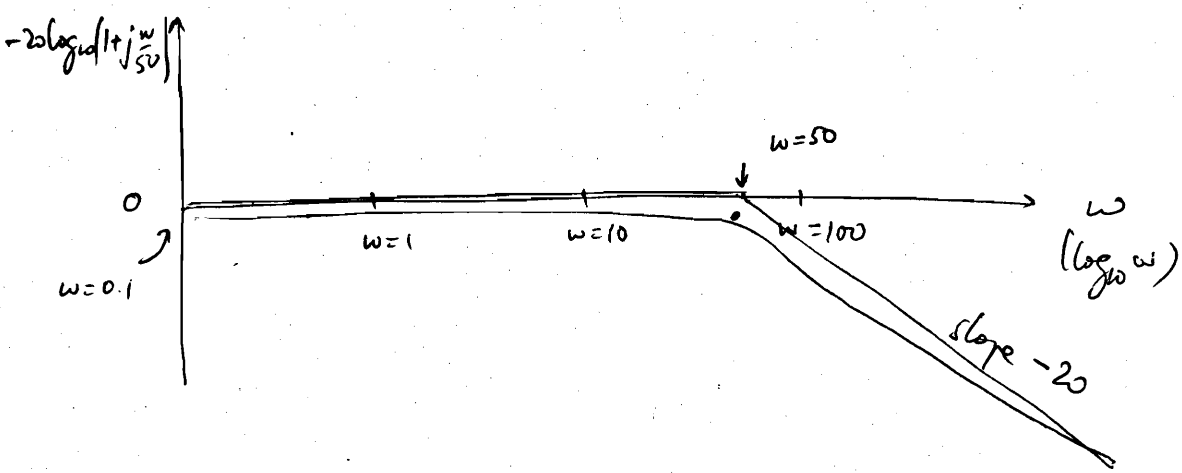
In this case: $w_c = 1$.

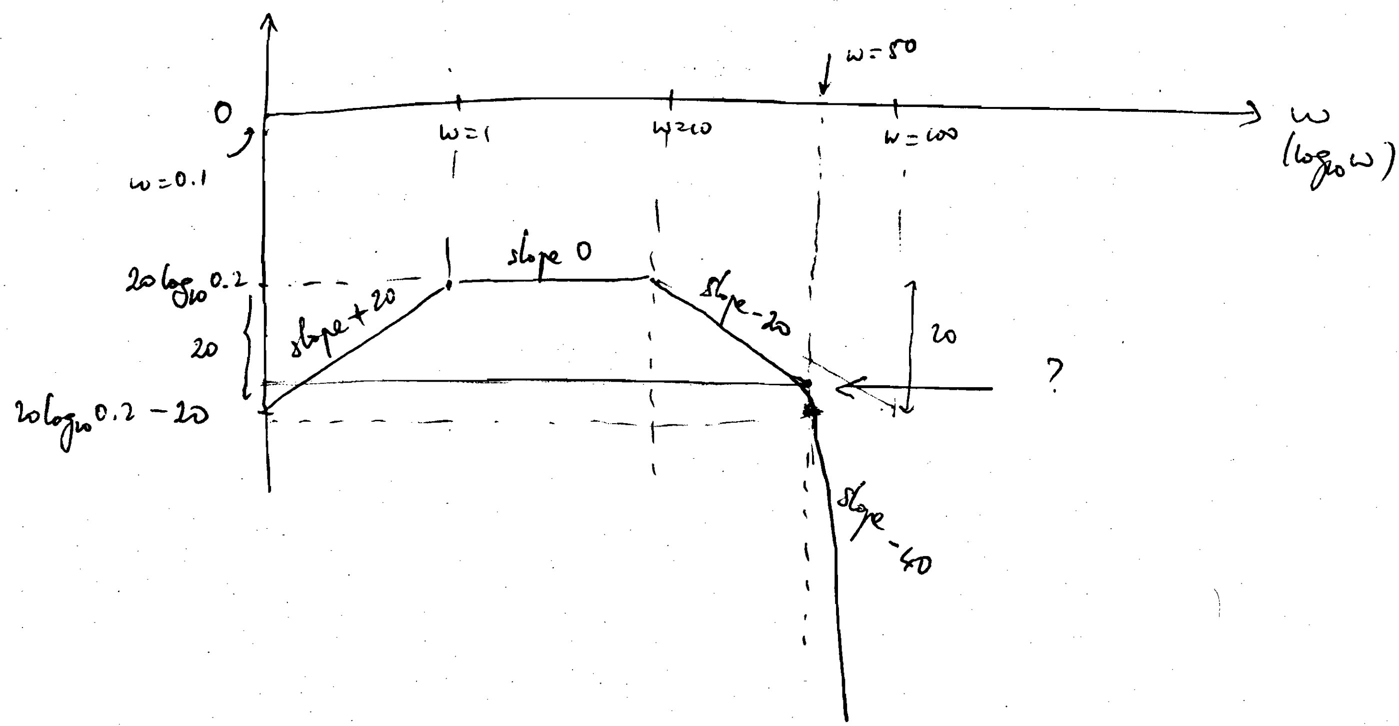






3)


$-20\log_{10}\left|1+j\frac{w}{50}\right|$
0
$w=0.1$
$w=1$
$w=10$
$w=50$
$w=100$
w ($\log_{10} w$)
slope -20


0
$w=0.1$
$w=1$
$w=10$
$w=50$
$w=100$
w ($\log_{10} w$)
$20\log_{10} 0.2$
20
$20\log_{10} 0.2 - 20$
slope $+20$
slope 0
slope -20
20
?
slope -40

Example: plot this transfer function:

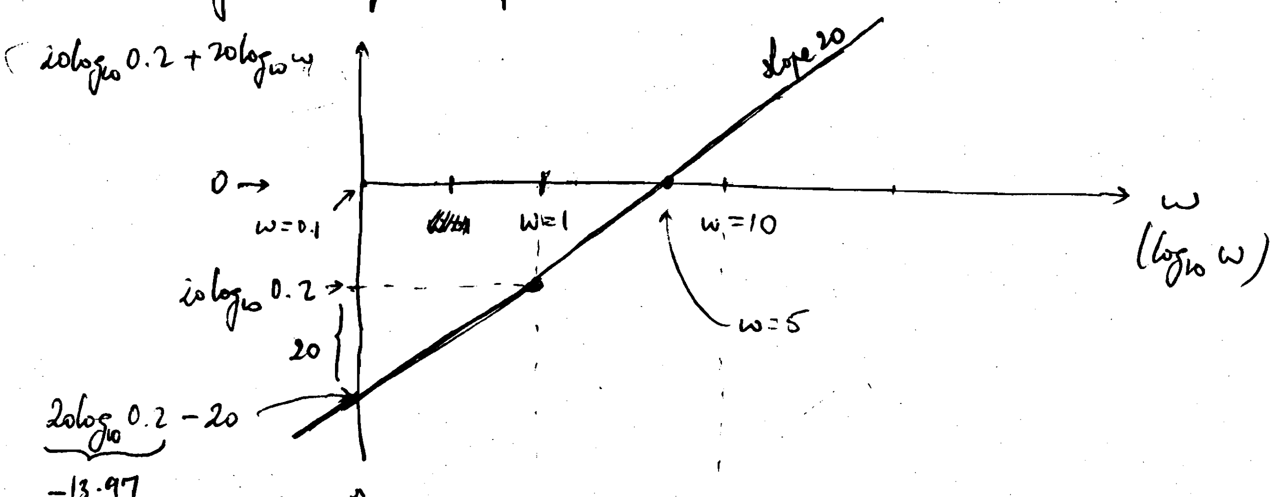
$$\hat{H}(j\omega) = \frac{100 j\omega}{(1+j\omega)(10+j\omega)(50+j\omega)}$$ Magnitude Bode Plot

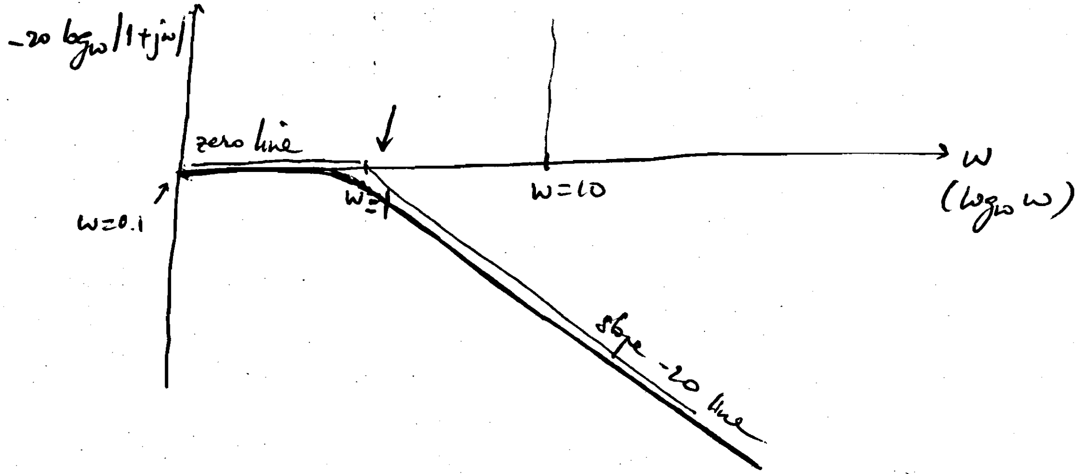
1) Write $\hat{H}$ in the standard format (always $(1 + \dots)$)

$$\hat{H}(j\omega) = \frac{\frac{100}{500} j\omega}{(1+j\omega)(1+j\frac{\omega}{10})(1+j\frac{\omega}{50})} \rightarrow |\hat{H}(j\omega)| = \frac{0.2\,\omega}{|1+j\omega|\,|1+j\frac{\omega}{10}|\,|1+j\frac{\omega}{50}|}$$

2) $$20\log_{10}|\hat{H}| = \underbrace{20\log_{10}0.2}_{\text{constant}} + \underbrace{20\log_{10}\omega}_{\text{linear}} \underbrace{- 20\log_{10}|1+j\omega|}_{1^{st}\text{ order}} \underbrace{- 20\log_{10}|1+j\tfrac{\omega}{10}|}_{1^{st}\text{ order}}$$
$$\underbrace{- 20\log_{10}|1+j\tfrac{\omega}{50}|}_{1^{st}\text{ order}}$$

3) The plot of LHS (Magnitude Bode Plot) would be the sum of the plots for each term in RHS:





Xcredit: 5 exam 3 points (20 min).

Sketch Bode plot for the following transfer function:

$$\hat{H}(j\omega) = \frac{10^5\, j\omega\left(1 + j\frac{\omega}{10}\right)}{(100 + j\omega)(1000 + j\omega)}$$

Bode Plot for quadratic poles and zeroes:

Standard form: $\pm 20\log_{10}\left|1 + 2\xi_b\underbrace{(j\omega\tau_b)} + \underbrace{(j\omega\tau_b)^2}\right|$

Asymptotic analysis:

1) Small ω or $\omega\tau_b \ll 1$

$$\pm 20\log_{10}|\quad| \longrightarrow \pm 20\log_{10} 1 = 0$$

2) large ω or $\omega\tau_b \gg 1$

$$\pm 20\log_{10}|\quad| \longrightarrow \pm 20\log_{10}\left|(j\omega\tau_b)^2\right|$$

$$= \pm 40\log_{10}(\omega\tau_b)$$

$$= \underbrace{\pm 40\log_{10}\omega}_{\text{line of slope } \pm 40} \pm 40\log_{10}\tau_b$$

3) Critical frequency: $\omega_c = \frac{1}{\tau_b}$ $(\omega_c\tau_b = 1)$

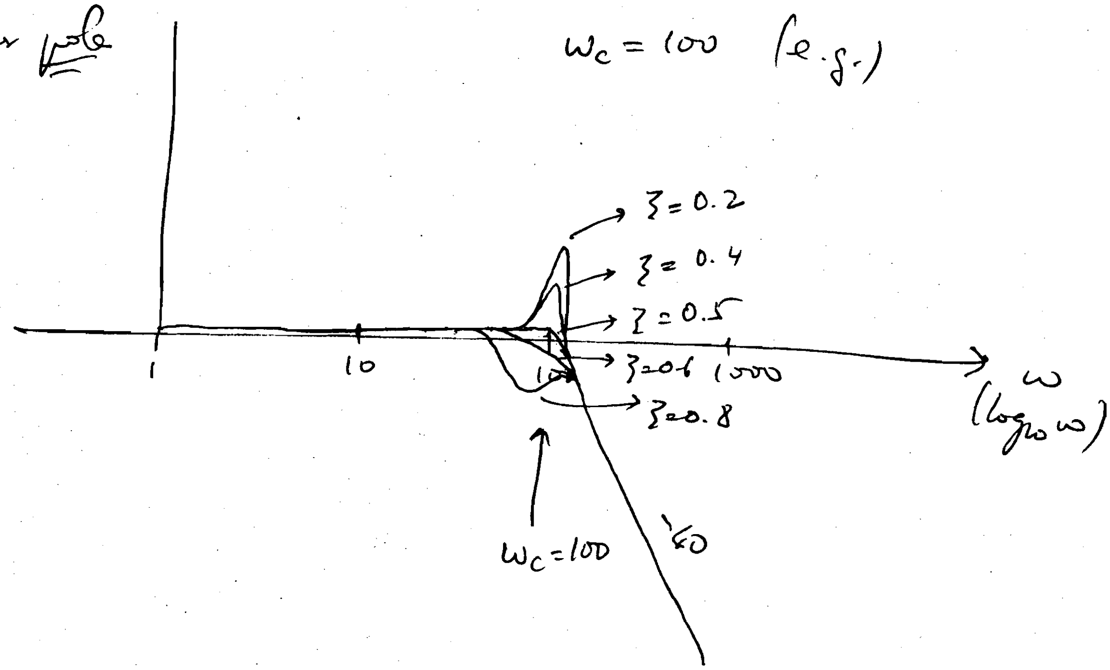
$$\pm 20\log_{10}|\quad| \longrightarrow \pm 20\log_{10}\left|1 + 2\xi_b j - 1\right|$$

$$= \pm 20\log_{10}(2\xi_b)$$

ζ_b	$-20\log_{10}(2\zeta_b)$
0.2	7.96
0.4	1.94
0.5	0
0.6	−1.58
0.8	−4.1

This controls the change in slope
(this was −3dB for the 1st order pole)

2nd order pole



Example: $$\hat{H}(j\omega) = \frac{25 j\omega}{(j\omega + 0.5)\left[(j\omega)^2 + 4j\omega + 100\right]}$$

standard form
$(1 + \cdots)$

$$= \frac{0.5 j\omega}{\left(1 + j\frac{\omega}{0.5}\right)\underbrace{\left(1 + 4j\frac{\omega}{100} + \left(j\frac{\omega}{10}\right)^2\right)}_{\omega_c = 10}}$$

$\omega_c = 0.5$

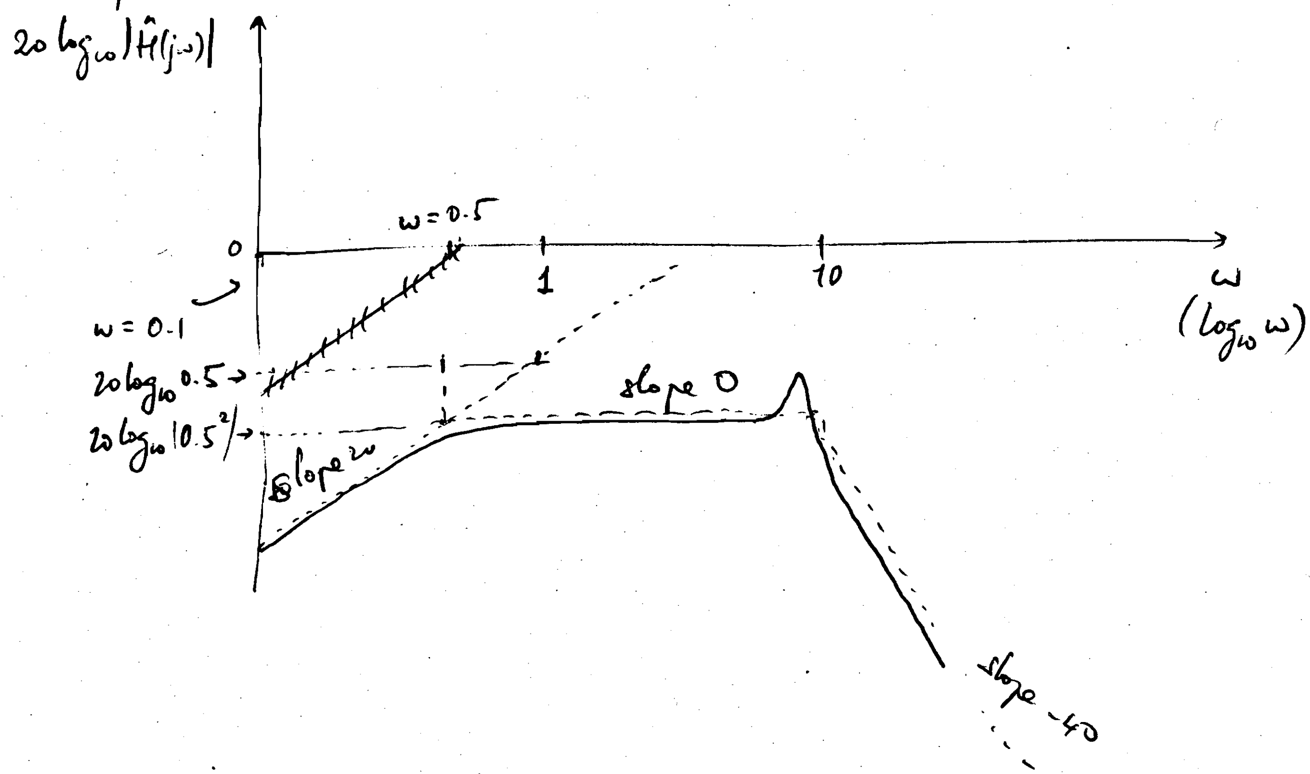
$2\zeta = \frac{4}{10} \rightarrow \zeta = 0.2$ (big bump).

Two critical frequencies at 0.5 & 10 → 3 intervals on the freq. axis to look at:

→ Before $\omega = 0.5$: $\underbrace{20\log_{10}\omega}_{\text{slope } 20} + \underbrace{20\log_{10}(0.5)}_{\text{intercept w/ vertical axis (negative)}}$

→ Between $\omega=0.5$ & $\omega=10$: due to a pole at $\omega=0.5$ with slope -20. → horizontal line.

→ After $\omega=10$: due to a quadratic pole at $\omega=10$ with slope -40 → line with -40 slope.



Phase Bode Plot:

$$\hat{H}(jw) = \frac{0.5jw}{\left(1 + j\frac{w}{0.5}\right)\left(1 + \frac{4}{10}j\frac{w}{10} + \left(j\frac{w}{10}\right)^2\right)}$$

$$\angle\hat{H}(jw) = \angle 0.5jw \;-\; \angle 1 + j\frac{w}{0.5} \;-\; \angle 1 + \frac{4}{10}j\frac{w}{10} + \left(j\frac{w}{10}\right)^2$$

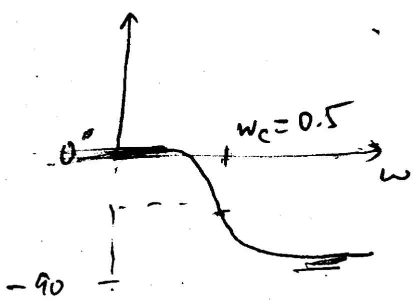
$$= 90^\circ$$

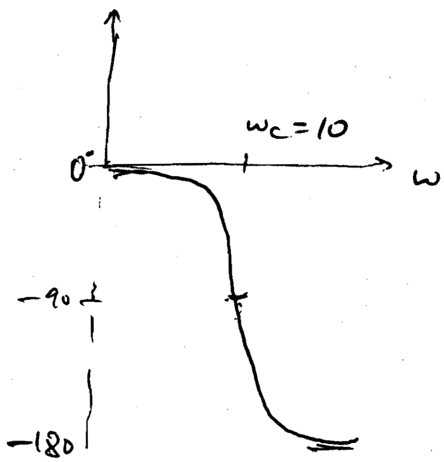
asymptotic analysis

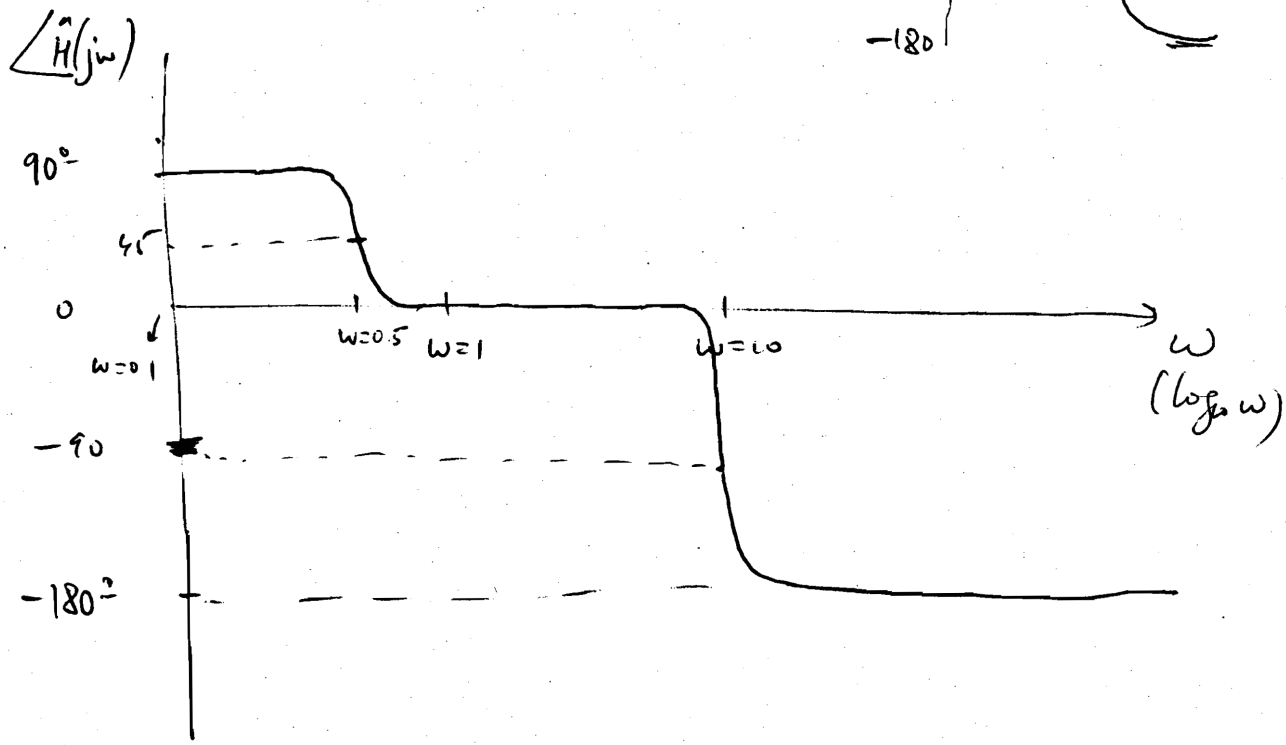
- small ω : $\to 0^\circ$
- large ω $\to 90^\circ$
- critical ω $\to 45^\circ$

asymptotic analysis

- small $\omega \to 0^\circ$
- large $\omega \to 180^\circ$
- critical $\omega \to \angle 0.4j = 90^\circ$



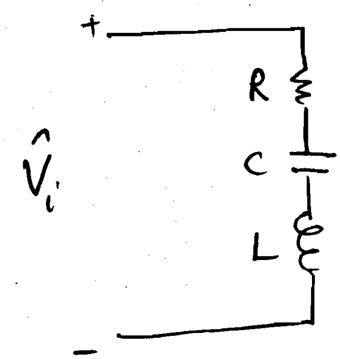




Resonant Frequency:

Highest consumption: $\hat{Z}$ is pure resistive or $p.f = \cos\theta$

$pf = \cos 0° = 1$ (or zero angle).



$$\hat{Z}(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

This is pure-resistive when
$$\omega L - \frac{1}{\omega C} = 0 \quad (\text{angle } 0°)$$

$$\rightarrow \omega^2 = \frac{1}{LC} \text{ or } \underbrace{w = \frac{1}{\sqrt{LC}}}_{\text{resonant freq. } \omega_0}$$

Quality factor $Q \equiv \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

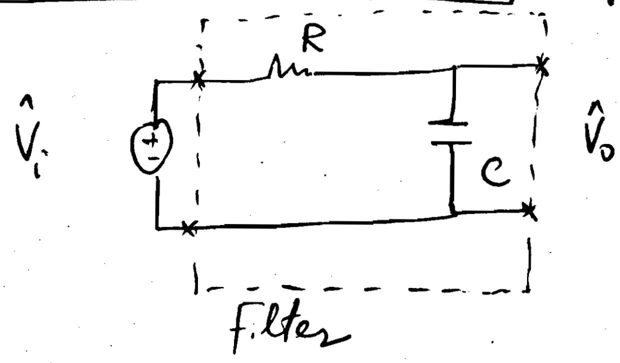
Practical applications: filter networks

What are we filtering? certain frequencies (e.g. those carried by noises)

→ Passive filters: using exclusively R, L, C (no gain)

→ Active filters: also use Op-amps (gain > 1)

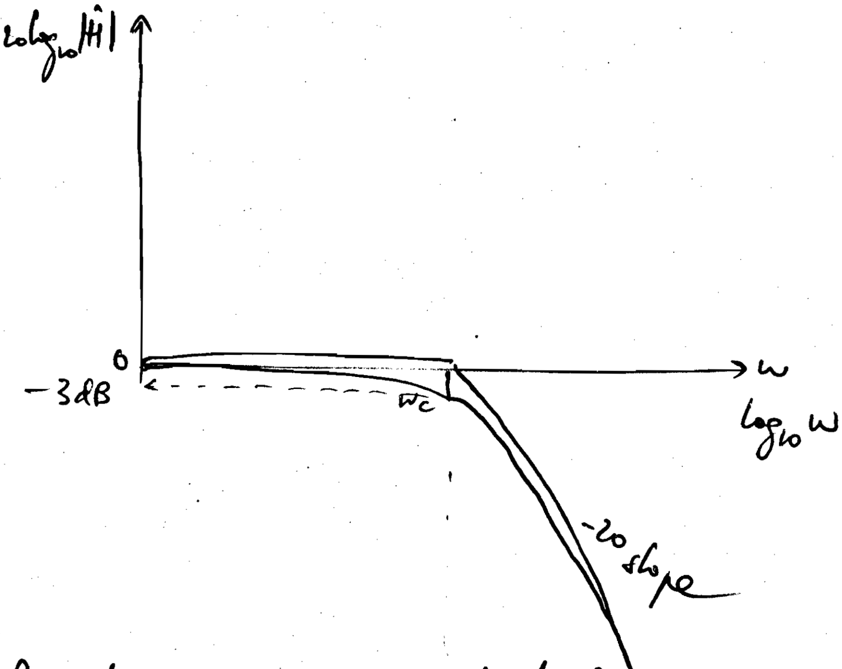
Passive - Low Pass filter: (passing low frequencies)



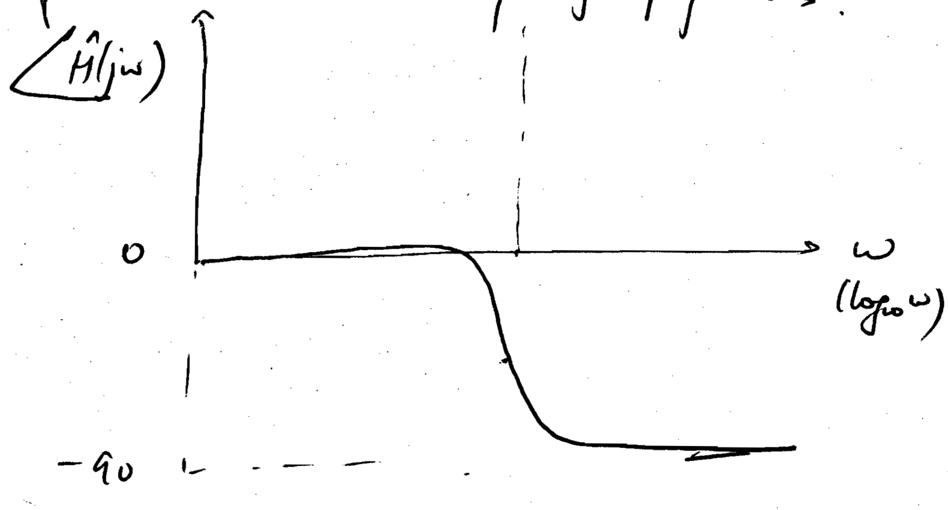
$$\hat{H} = \frac{\hat{V}_o}{\hat{V}_i} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}$$

$$= \frac{1}{1 + j\omega RC}$$

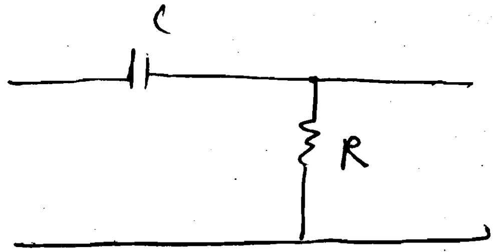
$$\omega_c = \frac{1}{RC}$$



This is useful when noises carry high frequencies.

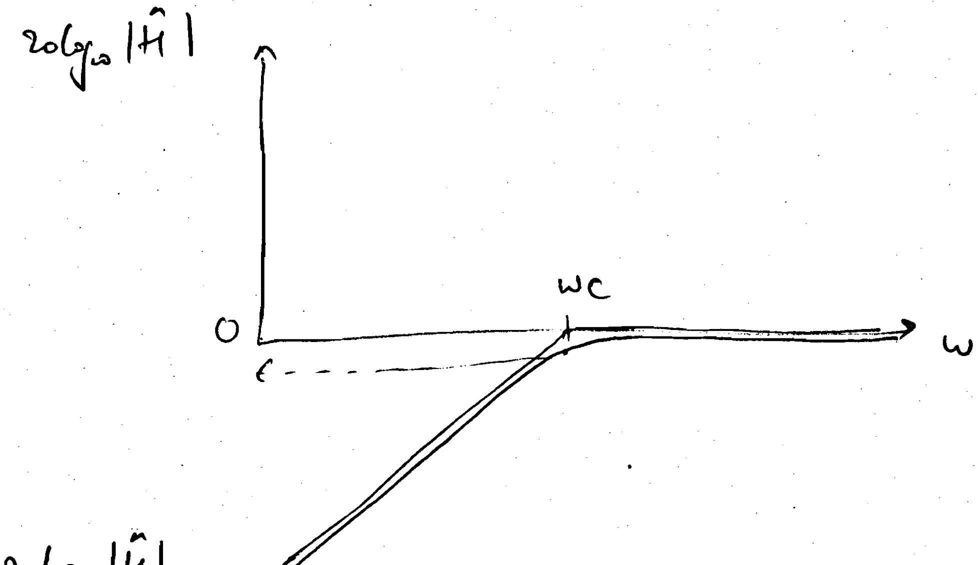


Passive High-Pass Filter :



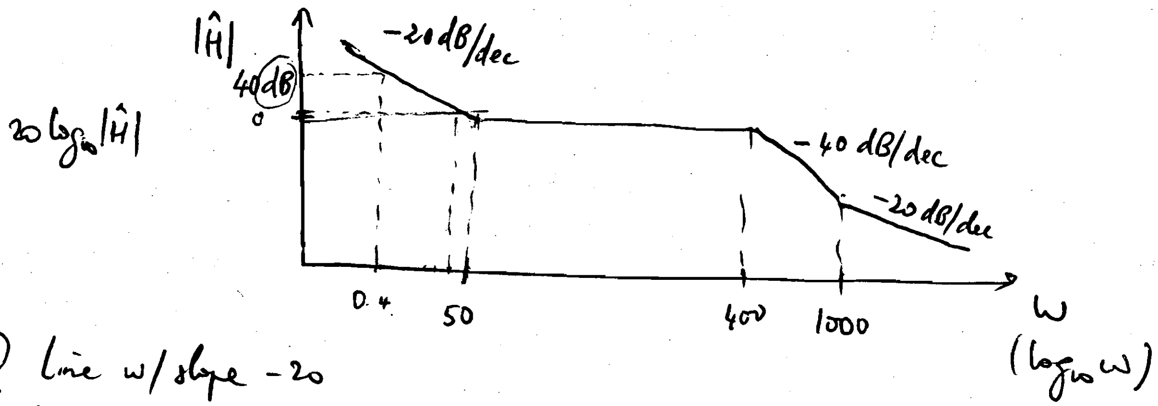
$$\hat{H}(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}.$$

$$20\log_{10}|\hat{H}(j\omega)| = 20\log_{10} RC + 20\log_{10}\omega - \underbrace{20\log_{10}|1 + j\omega RC|}_{\omega_c = \frac{1}{RC}}$$



This is useful when noise carry low frequency.

11-29 (7ed) or 12-30 (8ed).



① Line w/ slope −20

② Line w/ slope +20, $\omega_c = 50$.

③ line w/ slope −40, $\omega_c = 400$

↳ Quadratic pole: $\frac{1}{(1+2\zeta j\omega\tau + (j\omega\tau)^2)}$

↳ Double pole: $\frac{1}{(1+j\omega\tau)^2}$

④ line w/ slope +20, $\omega_c = 1000$

$\omega = 0.4$ These terms are 0

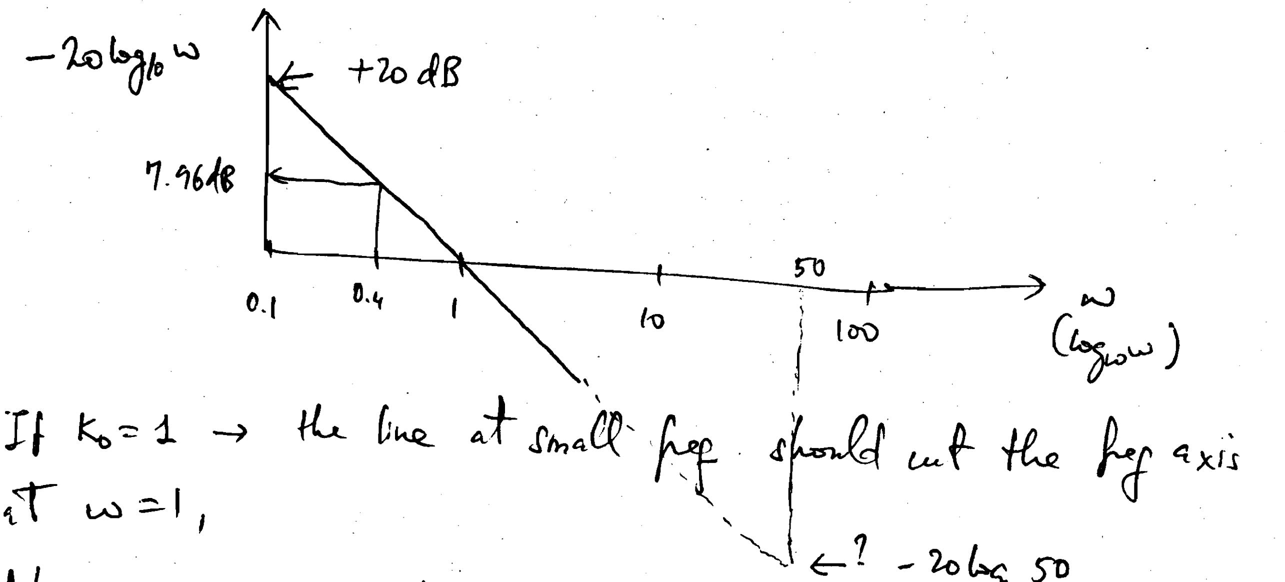
$$\hat{H} = \frac{\overset{②}{\left(1+j\frac{\omega}{50}\right)}\cdot\overset{④}{\left(1+j\frac{\omega}{1000}\right)}}{\overset{①}{j\omega}\ \overset{③}{\left(1+j\frac{\omega}{400}\right)^2}}$$

$$20\log_{10}|\hat{H}| = \overset{②}{20\log_{10}\left|1+j\frac{\omega}{50}\right|} \overset{①}{- 20\log_{10}\omega} \overset{③}{- 40\log_{10}\left|1+j\frac{\omega}{400}\right|} \overset{④}{+ 20\log_{10}\left|1+j\frac{\omega}{1000}\right|}$$

$$40 = \underbrace{-20\log_{10} 0.4}_{+7.96} + 20\log_{10} K_o$$

$$K_o = 10^{\left(\frac{40-7.96}{20}\right)} = 40$$

$$\boxed{\hat{H} = \frac{40\left(1+j\frac{\omega}{50}\right)\left(1+j\frac{\omega}{1000}\right)}{j\omega\left(1+j\frac{\omega}{400}\right)^2}}$$



If $K_o = 1 \rightarrow$ the line at small freq should cut the freq axis at $\omega = 1$,

At $\omega = 0.4 \rightarrow \quad -20\log_{10} 0.4 = 7.96$dB (but in the given Bode plot, at $\omega = 0.4$ we had 40 dB $\rightarrow$ there should a K_o such that

$$20\log_{10} K_o = 40 - 7.96 \quad \text{or} \quad K_o = 10^{\left(\frac{40-7.96}{20}\right)}$$

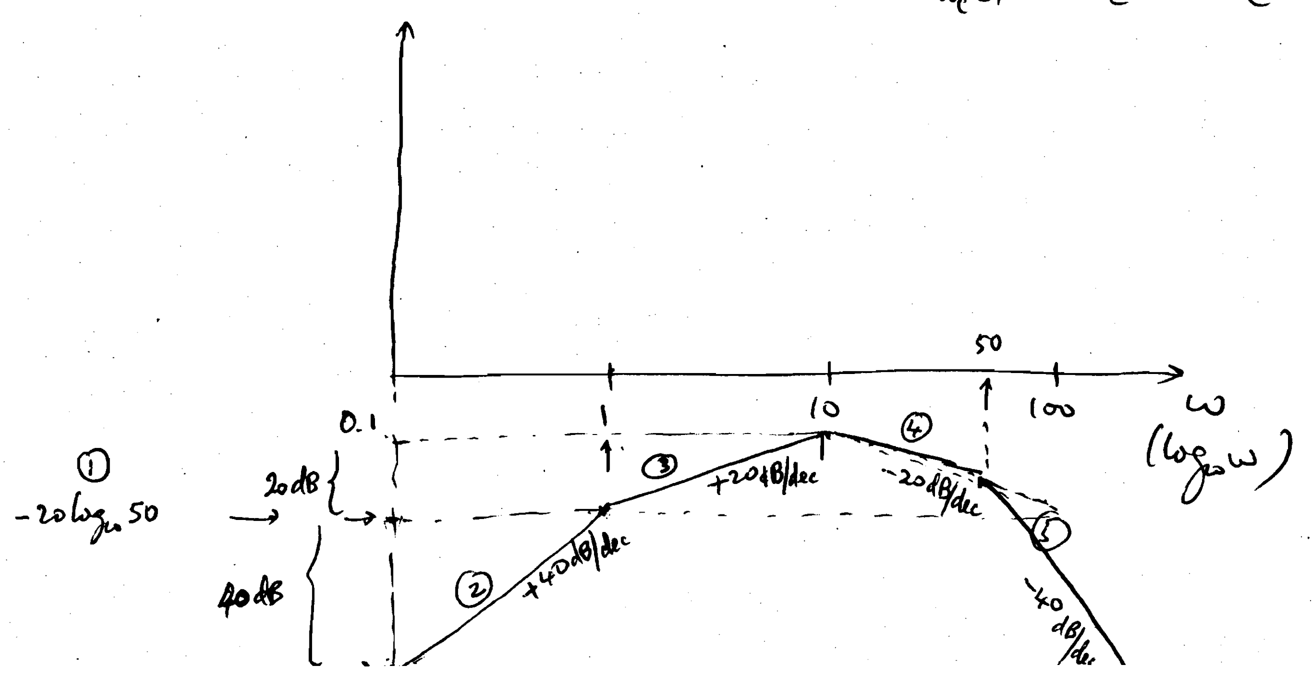
$$= 40.$$

11.18 (7^{ed})

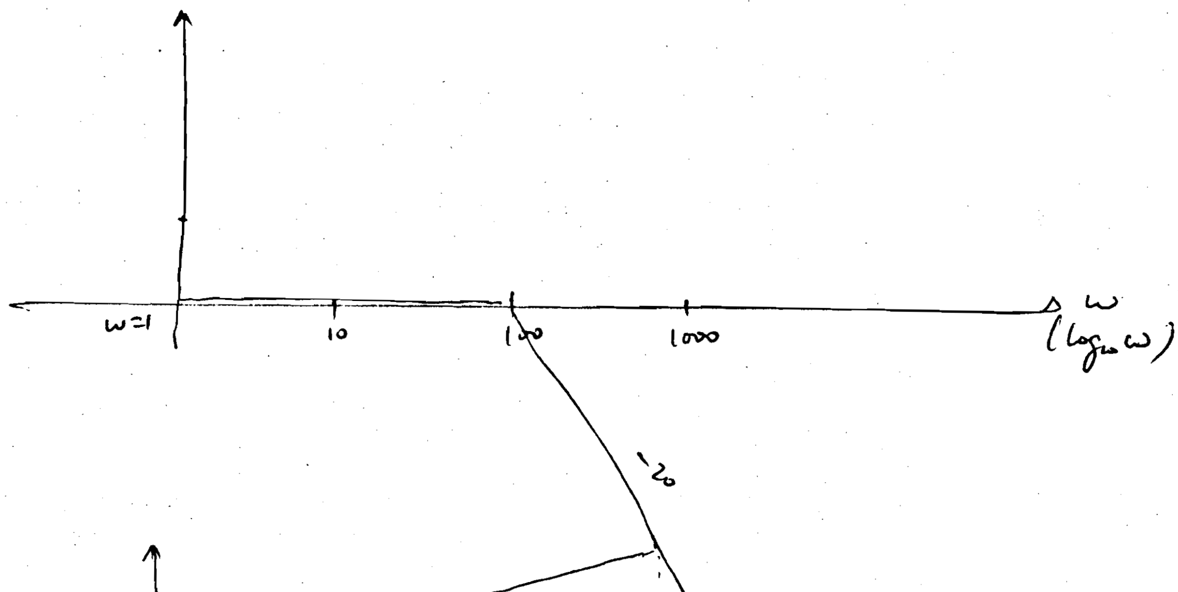
$$\hat{H}(j\omega) = \frac{100\,(j\omega)^2}{(1+j\omega)\,(10+j\omega)^2\,(50+j\omega)} = \frac{\overset{①}{\frac{1}{50}}\cdot \overset{②}{(j\omega)^2}}{\underset{③}{(1+j\omega)}\,\underset{④}{\left(1+j\frac{\omega}{10}\right)^2}\,\underset{⑤}{\left(1+j\frac{\omega}{50}\right)}}$$

Standard form of $1+j\omega\tau$, etc.

$\omega_c = 1 \qquad \omega_c = 10 \qquad \omega_c = 50$

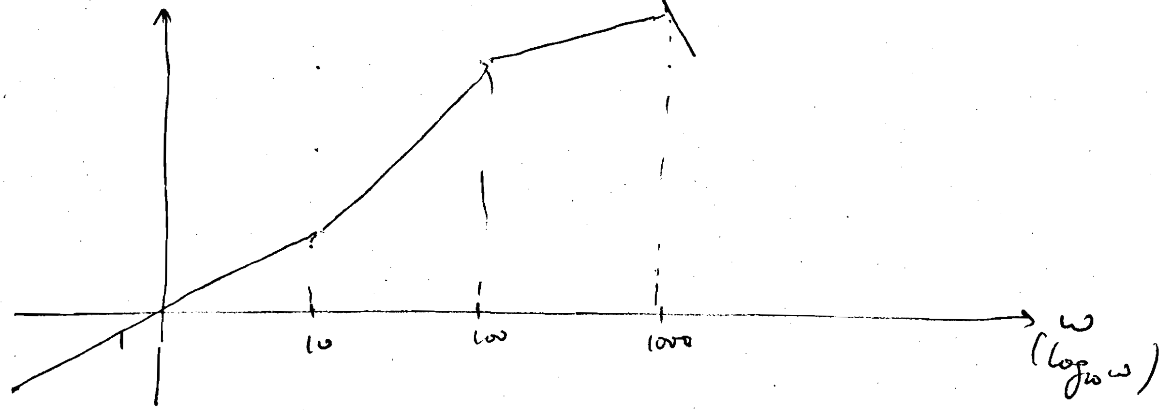


3



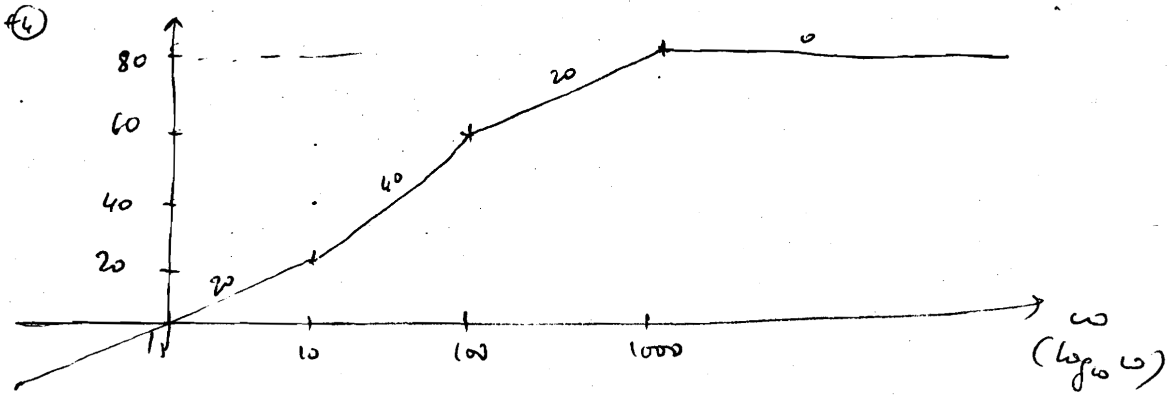
ω=1
10
100
1000
ω
(log10 ω)
-20

1 + 2 + 3



1
10
100
1000
ω
(log10 ω)

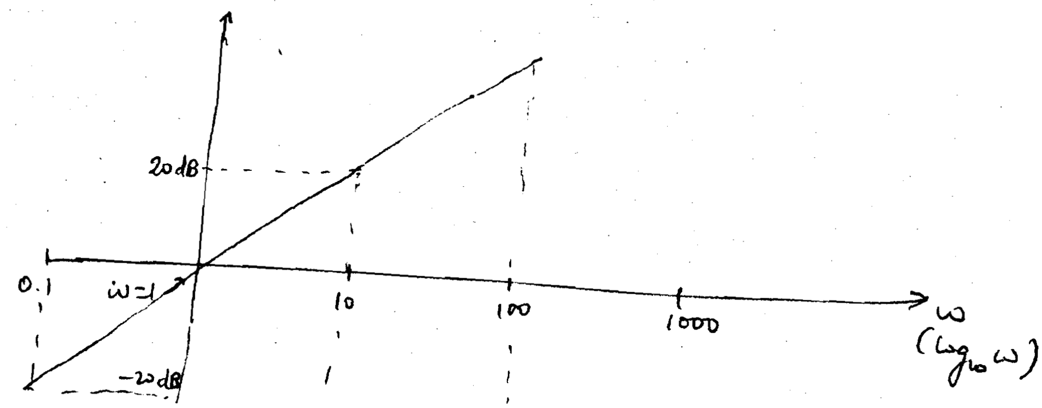
1 + 2 + 3 + 4



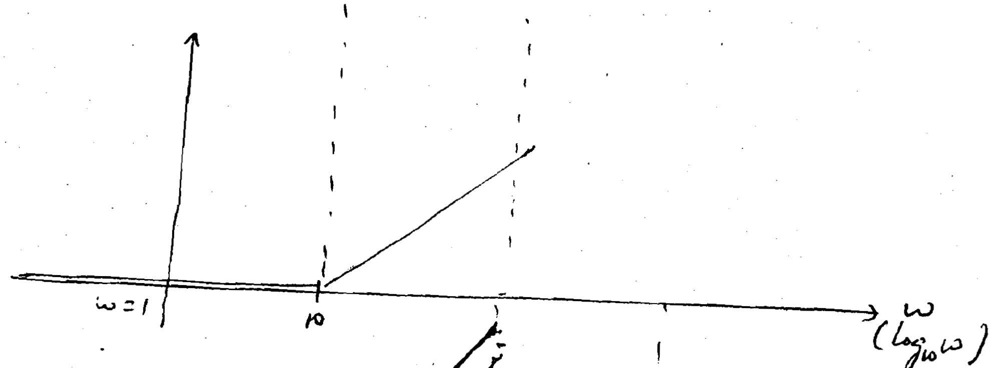
80
60
40
20
20
40
20
0
1
10
100
1000
ω
(log10 ω)

$$20\log_{10}|\hat{H}(j\omega)| = \overset{①}{20\log_{10}\omega} + \overset{②}{20\log_{10}\left|1+j\frac{\omega}{10}\right|} - \overset{③}{20\log_{10}\left|1+j\frac{\omega}{100}\right|}$$
$$\overset{④}{-20\log_{10}\left|1+j\frac{\omega}{1000}\right|}$$

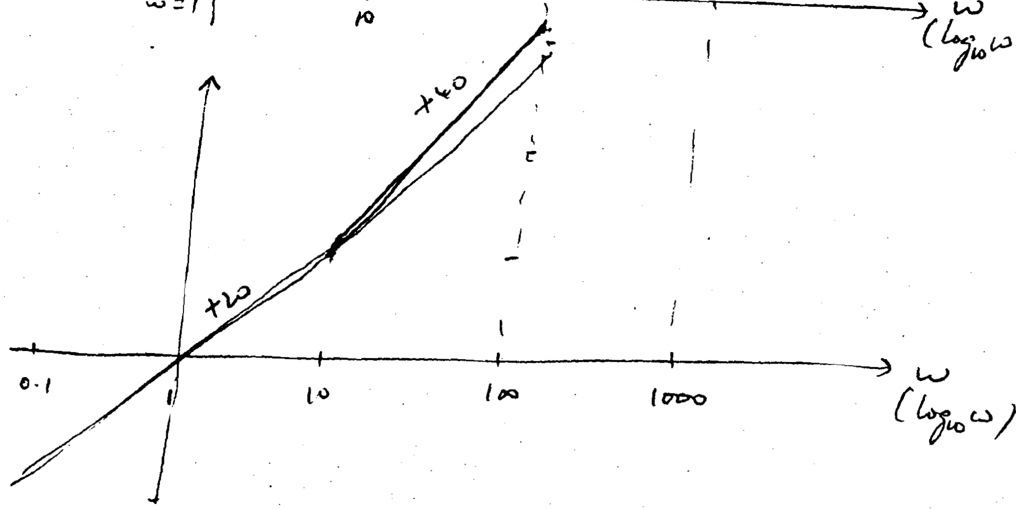
①



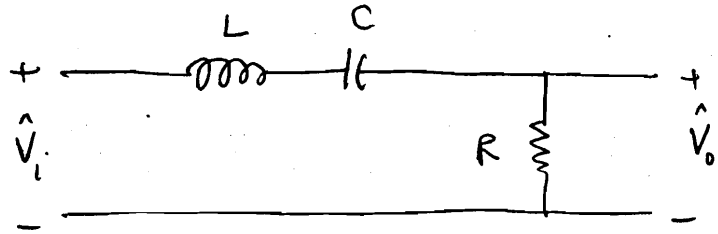
②



①+②



Passive Band-Pass Filter:



$$\hat{H} = \frac{\hat{V}_o}{\hat{V}_i} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\hat{H} = \frac{RC\omega}{RC\omega + j\left(LC\omega^2 - 1\right)}$$

Magnitude: $|\hat{H}| = \dfrac{RC\omega}{\sqrt{(RC\omega)^2 + (LC\omega^2 - 1)^2}}$

Phase: $\angle\hat{H} = -\tan^{-1}\left(\dfrac{LC\omega^2 - 1}{RC\omega}\right)$

Asymptotic analysis:

Small ω: $\omega^2 \ll \dfrac{1}{LC}$ (or $\omega \ll \dfrac{1}{\sqrt{LC}}$) $\rightarrow$

$|\hat{H}| = \dfrac{RC\omega}{\sqrt{(RC\omega)^2 + 1}}$

$\omega \ll \dfrac{1}{RC} \rightarrow |\hat{H}| \simeq RC\omega \approx 0$

$\angle\hat{H} = -\tan^{-1}\left(\dfrac{-1}{RC\omega}\right)$

$\omega \ll \dfrac{1}{RC} \rightarrow \angle\hat{H} = \tan^{-1}(\infty)$

$= 90°$

Large ω: $\omega^2 \gg \dfrac{1}{LC}$ (or $\omega \gg \dfrac{1}{\sqrt{LC}}$)

$|\hat{H}| = \dfrac{R\cancel{C}\cancel{\omega}}{L\cancel{C}\omega^{\cancel{2}}} = \dfrac{R}{L}\dfrac{1}{\omega}$

≈ 0

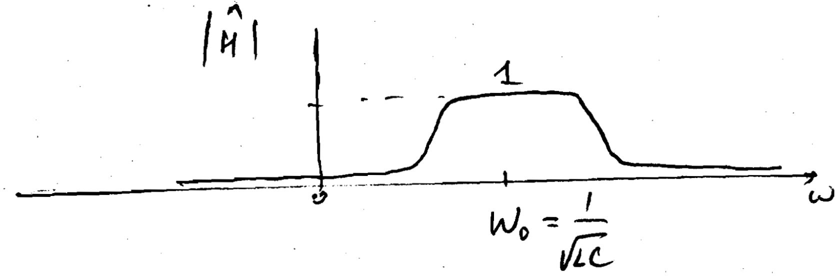
$\angle\hat{H} = -\tan^{-1} \dfrac{L\cancel{C}\omega^{\cancel{2}}}{R\cancel{C}\cancel{\omega}} = -\tan^{-1}\dfrac{L\omega}{R}$

$= -\tan^{-1}\infty = -90°$

Resonant freq: $\omega = \dfrac{1}{\sqrt{LC}}$

$$\begin{cases} |\hat{H}| = \dfrac{RC\omega}{\sqrt{(RC\omega)^2}} = 1 \\ \angle \hat{H} = -\tan^{-1} 0 = 0^\circ \end{cases}$$

Band-pass filter:

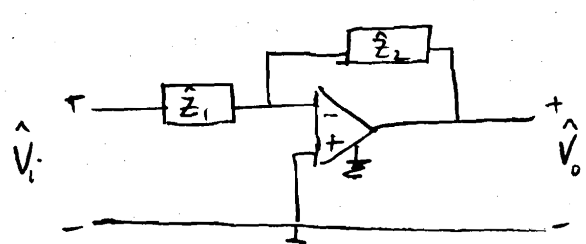


Xcredit (due 5/8): (5 exam points)

Design a band-pass filter using only R, C elements. Provide concrete values for R and C to make this happen. (Simplest design please)

Active Filters using Op-Amp:

Inverting



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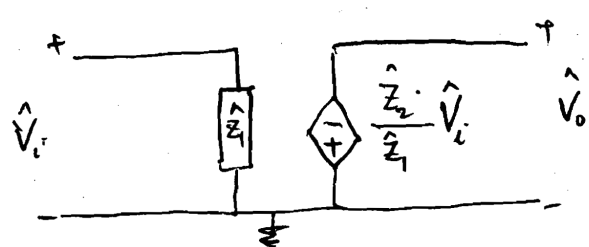
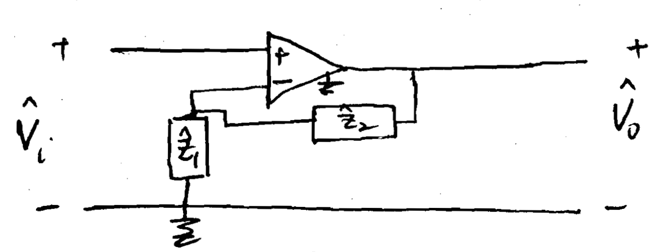


Non-inverting



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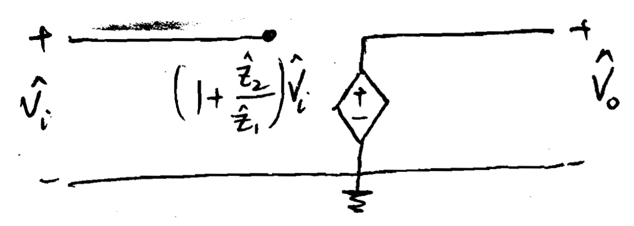
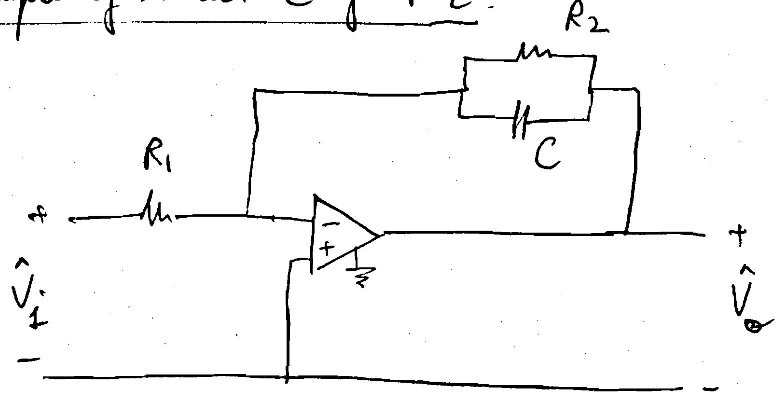


Example of an active filter:

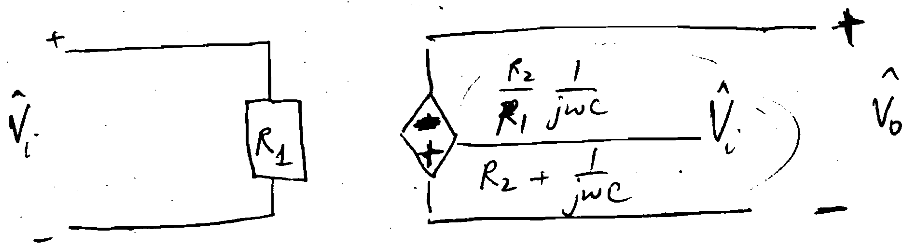


→ What type of filter?

→ What is the gain or amplification?

Make use of the equivalent circuit for the inverting configuration; find $\hat{H}$, answer questions.

$$\hat{Z}_1 = R_1 ; \quad \hat{Z}_2 = R_2 \parallel C = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}$$



$$\hat{H} = \frac{\hat{V}_o}{\hat{V}_i} = \frac{-\frac{R_2}{R_1} \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{-\frac{R_2}{R_1}}{1 + j\omega R_2 C}$$

→ Type of filter: active low-pass filter

→ Gain is $\frac{R_2}{R_1}$ (related to the band width: higher gain ~ smaller bandwidth)

Laplace Transform : (L.T.)

$$f(t) \xrightarrow{\text{direct L.T.}} \hat{F}(s) \equiv \int_0^\infty dt\ f(t)\, e^{-st}$$

$s \equiv \alpha + j\omega\ ;\ \alpha > 0$

(when $\alpha = 0$ → related to Fourier Transform)

$\underbrace{\quad}_{\text{in time-domain}}$ $\underbrace{\quad}_{\text{in frequency domain}}$ (s related to ω)

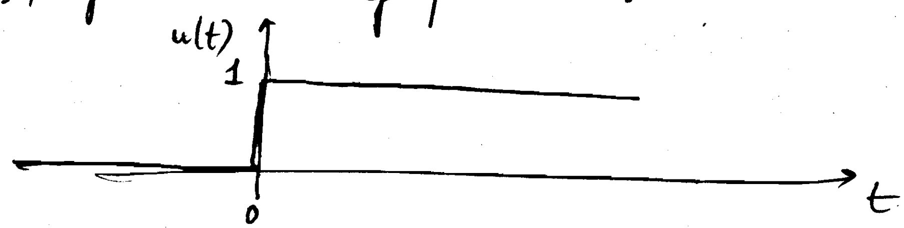
$$f(t) \xleftarrow[\text{inverse}]{L.T.^{-1}} \hat{F}(s)$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} ds\ \hat{F}(s)\, e^{st}$$ (integral in the complex plane)

↓

We will use Partial Fraction Expansion & Properties of L.T.

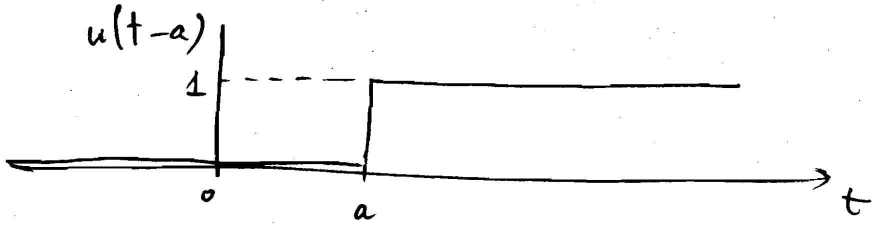
- L.T. for a unit step function :



$$L.T.[u(t)] = \int_0^\infty dt\ u(t)\, e^{-st} = \int_0^\infty dt\ e^{-st} = \left[\frac{e^{-st}}{-s}\right]_{t=0}^{t=\infty}$$

$$\underset{(\alpha > 0)}{=}\ 0 - \frac{1}{-s} = \frac{1}{s}$$

- L.T. of a time-shifted unit step function:

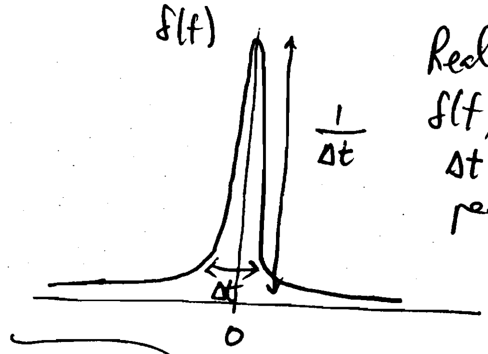


$$L.T.[u(t-a)] = \int_0^\infty dt \; u(t-a)\, e^{-st} = \int_a^\infty dt \; e^{-st} = \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$\underset{\alpha>0}{\overset{\downarrow}{=}} \; 0 - \frac{e^{-sa}}{-s} = \underbrace{e^{-sa}}\,\frac{1}{s}$$

$$\boxed{\text{time shift of } a \longrightarrow \text{multiplication by } e^{-sa} \text{ in frequency}}$$

- L.T. of a delta function $\delta(t)$



Real or actual $\delta(t)$ is when $\Delta t \to 0$ and peak is ∞

Its effect is to pick out the value of a function at its center

$\delta(t) \to$ center at 0

$\delta(t-a) \longrightarrow$ center at a

$$LT[\delta(t)] = \int_0^\infty dt \; \delta(t)\, e^{-st} = e^{-s0} = 1$$

$$LT[\delta(t-a)] = e^{-sa} \cdot 1 = e^{-sa}$$

- LT. of $f(t) = t$?

$$LT[t] = \int_0^\infty dt\, t e^{-st} = -\frac{d}{ds}\underbrace{\left[\int_0^\infty dt\, e^{-st}\right]}_{\frac{1}{s}} = \frac{1}{s^2}$$

- LT of $f(t) = \cos\omega t$? $\rightarrow \cos\omega t = \dfrac{e^{j\omega t} + e^{-j\omega t}}{2}$

$$LT[\cos\omega t] = \frac{1}{2}\int_0^\infty dt\, \left[e^{j\omega t}e^{-st} + e^{-j\omega t}e^{-st}\right]$$

$$= \frac{1}{2}\int_0^\infty dt\, \left[e^{-(s-j\omega)t} + e^{-(s+j\omega)t}\right]$$

$$= \frac{1}{2}\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)} + \frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\right]_0^\infty$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right]$$

$$= \frac{s}{s^2+\omega^2}$$

$$LT[\sin\omega t] = \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{\cancel{2j}}\,\frac{\cancel{2j}\omega}{s^2+\omega^2} = \frac{\omega}{s^2+\omega^2}$$