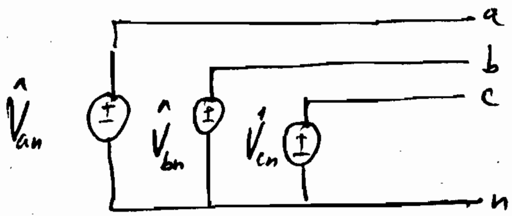
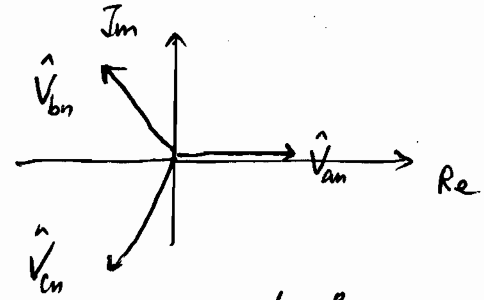
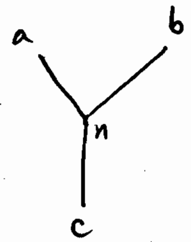
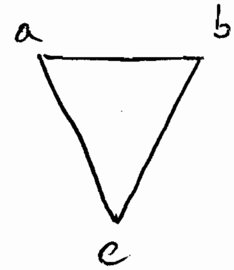
* **acb balanced three-phase circuit:**





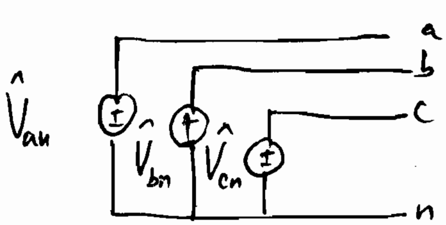


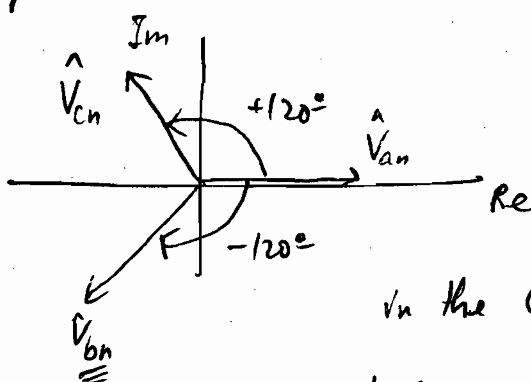
$$\underbrace{\begin{aligned}\hat{V}_{an} &= V_p\angle 0^\circ\\ \hat{V}_{bn} &= V_p\angle 120^\circ\\ \hat{V}_{cn} &= V_p\angle 240^\circ\end{aligned}}_{Y} \longrightarrow \underbrace{\begin{aligned}\hat{V}_{ab} &= V_p\sqrt{3}\angle -30^\circ\\ \hat{V}_{bc} &= V_p\sqrt{3}\angle 90^\circ\\ \hat{V}_{ca} &= V_p\sqrt{3}\angle 210^\circ\end{aligned}}_{\Delta}$$



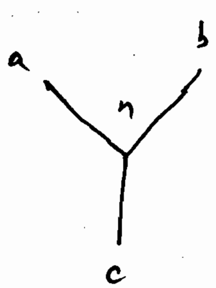
$$\begin{cases}\text{Magnitude}_\Delta = \text{magnitude}_Y \times \sqrt{3}\\ \text{Angle}_\Delta = \text{angle}_Y - 30^\circ.\end{cases}$$

Now: **abc balanced three-phase circuit:**

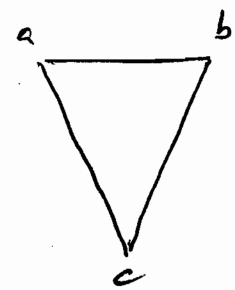




in the CW direction.



$$\underbrace{\begin{aligned}\hat{V}_{an} &= V_p\angle 0^\circ\\ \hat{V}_{bn} &= V_p\angle -120^\circ\\ \hat{V}_{cn} &= V_p\angle 120^\circ\end{aligned} \qquad \begin{aligned}\hat{V}_{ab} &= V_p\sqrt{3}\angle 30^\circ\\ \hat{V}_{bc} &= V_p\sqrt{3}\angle -90^\circ\\ \hat{V}_{ca} &= V_p\sqrt{3}\angle 150^\circ\end{aligned}}$$



$$\begin{cases}\text{magnitude}_\Delta = \text{magnitude}_Y\,\sqrt{3}\\ \text{angle}_\Delta = \text{angle}_Y + 30^\circ\end{cases}$$

$$\hat{V}_{ab} = V_p - V_p \angle{-120^\circ} = \left[\hat{V}_{ac}\right]_{acb} = -\left[\hat{V}_{ca}\right]_{acb}$$

$$= -V_p\sqrt{3}\angle{210^\circ} = V_p\sqrt{3}\angle{30^\circ}$$

$$\hat{V}_{bc} = V_p\angle{-120^\circ} - V_p\angle{120^\circ} = \left(\hat{V}_{cn} - \hat{V}_{bn}\right)_{acb} = \left(-\hat{V}_{bc}\right)_{acb}$$

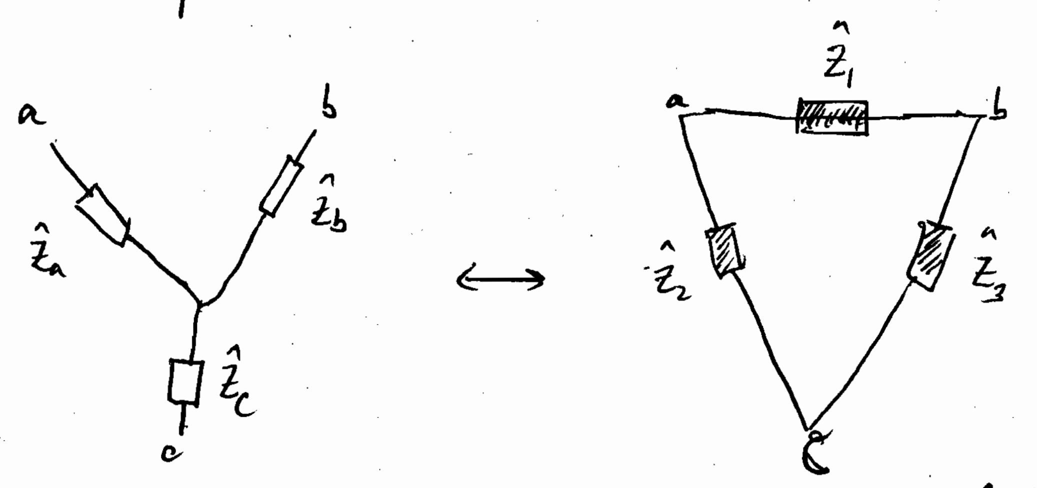
$$= -V_p\sqrt{3}\angle{90^\circ} = V_p\sqrt{3}\angle{-90^\circ}$$

$$\hat{V}_{ca} = V_p\angle{120^\circ} - V_p$$

$$= \left(\hat{V}_{bn} - \hat{V}_{an}\right)_{acb} = -\left(\hat{V}_{ab}\right)_{acb} = -V_p\sqrt{3}\angle{-30^\circ}$$

$$= V_p\sqrt{3}\angle{-210^\circ} = V_p\sqrt{3}\angle{150^\circ}$$

We have seen the source transformation b/w Y & Δ.
The load transformation:



① $\underbrace{\hat{Z}_a + \hat{Z}_b}_{Y} =$ total b/w a & b $= \underbrace{\hat{Z}_1 \parallel (\hat{Z}_2 + \hat{Z}_3)}_{\Delta}$

② $\hat{Z}_a + \hat{Z}_c$ = total b/w a & c $= \hat{Z}_2 \parallel (\hat{Z}_1 + \hat{Z}_3)$

③ $\hat{Z}_b + \hat{Z}_c$ = total b/w b & c $= \hat{Z}_3 \parallel (\hat{Z}_1 + \hat{Z}_2)$

① $$\hat{z}_a + \hat{z}_b = \frac{\hat{z}_1(\hat{z}_2 + \hat{z}_3)}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}$$

② $$\hat{z}_a + \hat{z}_c = \frac{\hat{z}_2 \cdot (\hat{z}_1 + \hat{z}_3)}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}$$

③ $$\hat{z}_b + \hat{z}_c = \frac{\hat{z}_3 \cdot (\hat{z}_1 + \hat{z}_2)}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}$$

② - ③ $$\hat{z}_a - \hat{z}_b = \frac{\hat{z}_1 \hat{z}_2 - \hat{z}_1 \hat{z}_3}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}$$

① + ② - ③ $$\underbrace{\cancel{2}\hat{z}_a}_{Y} = \frac{\cancel{2}\hat{z}_1 \hat{z}_2}{\underbrace{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}_{\Delta}}$$

$$\boxed{\hat{z}_a = \frac{\hat{z}_1 \hat{z}_2}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}}$$

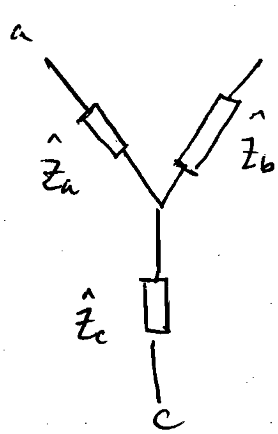
$$\boxed{\hat{z}_b = \frac{\hat{z}_1 \hat{z}_3}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}}$$

③ - ② + ① $$= 2\hat{z}_b = \frac{\hat{z}_1 \hat{z}_3 - \cancel{\hat{z}_1 \hat{z}_2} + \cancel{\hat{z}_1 \hat{z}_2} + \hat{z}_1 \hat{z}_3}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}$$

$$\boxed{\hat{z}_c = \frac{\hat{z}_2 \hat{z}_3}{\hat{z}_1 + \hat{z}_2 + \hat{z}_3}}$$

③ + ② - ①

Load transformation:



$$\hat{Z}_a = \frac{\hat{Z}_1 \hat{Z}_2}{\hat{Z}_1 + \hat{Z}_2 + \hat{Z}_3}$$

$$\hat{Z}_b = \frac{\hat{Z}_1 \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_2 + \hat{Z}_3}$$

$$\hat{Z}_c = \frac{\hat{Z}_2 \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_2 + \hat{Z}_3}$$

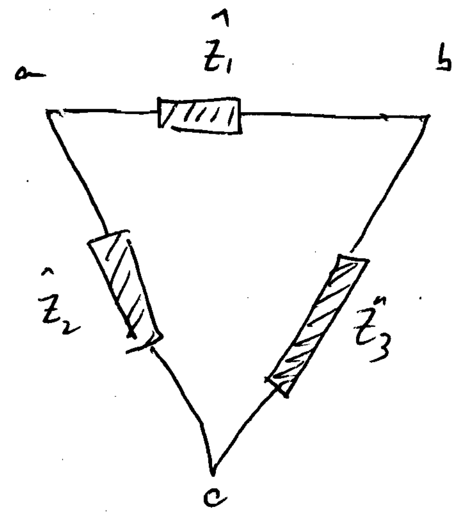
$$\longleftarrow$$

$$\longrightarrow$$

$$\hat{Z}_1 = \frac{\hat{Z}_a \hat{Z}_b + \hat{Z}_b \hat{Z}_c + \hat{Z}_a \hat{Z}_c}{\hat{Z}_c}$$

$$\hat{Z}_2 = \frac{\text{same as above}}{\hat{Z}_b}$$

$$\hat{Z}_3 = \frac{\text{same as above}}{\hat{Z}_a}$$

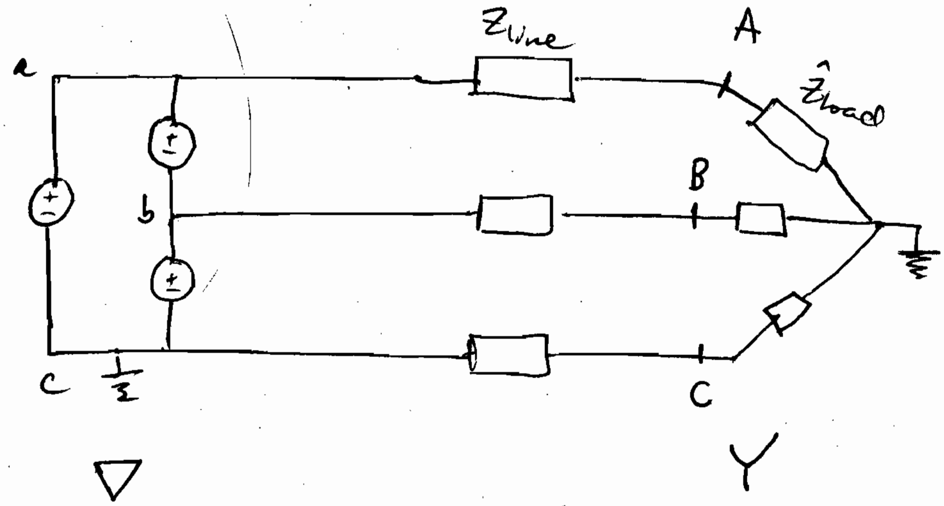


For balanced loads: $\hat{Z}_a = \hat{Z}_b = \hat{Z}_c = \hat{Z}_Y \rightarrow \boxed{\hat{Z}_\Delta = 3\hat{Z}_Y}$

or $\boxed{\hat{Z}_Y = \frac{1}{3}\hat{Z}_\Delta}$

5 exam 2 points :

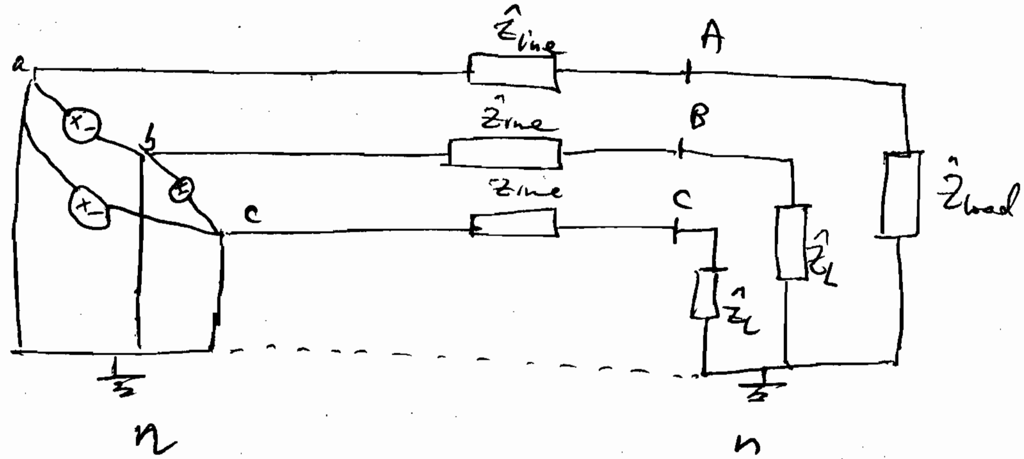
Balanced three phase Δ-Y system (abc sequence)



$\hat{Z}_{line} = 1 + 0.5j$ $\hat{Z}_{load} = 10 + 6j$

$\hat{I}_{aA} = 10 \angle -20° \; A_{RMS}$

$\hat{V}_{an}$? (Hint : 1st ~~change~~ Redraw source to Y)



9.49 (7ed) ~~or 9.57 (8ed)~~ :

$\hat{Z}_{line} = 0.08 + j0.25\,\Omega$

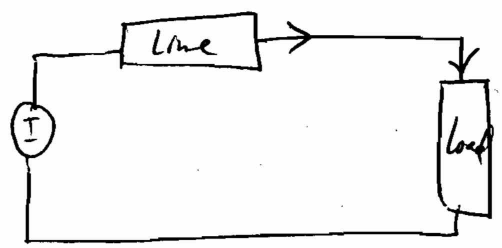
$\hat{V}_L = 220\angle 0° \; V_{RMS}$ 60 Hz.

$P_L = 12\,kW$

$P_{Line} = 560\,W$

Power factor angle at load? (inductive)

$$pf_L = \frac{P_L}{V_{RMS}\, I_{RMS}}$$



$$Re\left[\hat{S}_{Line} = \hat{I}^*_{RMS}\hat{V}_{Line} = I^2_{RMS}\hat{Z}_{Line}\right] \longrightarrow P_{Line} = I^2_{RMS}\, Re[\hat{Z}_{line}]$$

$$560 = I^2_{RMS} \cdot 0.08$$

$$pf_L = \frac{12000}{220\sqrt{\frac{560}{0.08}}} = \frac{12000}{220 \times 83.67} \approx 0.65 = \cos(\theta_v - \theta_i)$$

$$\theta_v - \theta_i = \cos^{-1} 0.65 = 49.3°$$

Inductive: $\left\{ \hat{Z} = j\omega L = \omega L\angle 90° \right.$, $\frac{\hat{V}}{\hat{I}} = \hat{Z} = \omega L\angle 90°$

$\theta_v - \theta_i = 90°$

current lagging voltage by

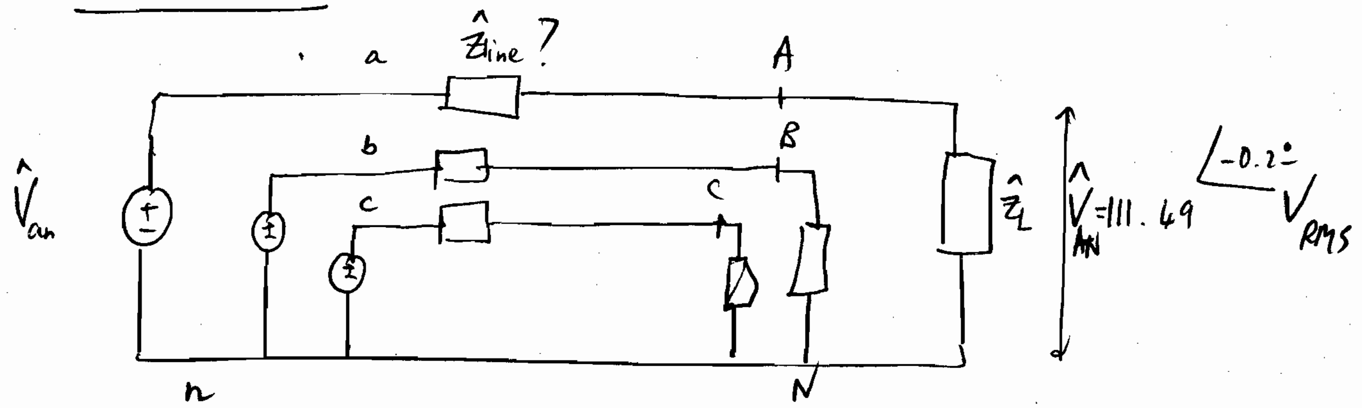
$\theta_v = 0 \rightarrow \theta_i = -49.3°$.

10.19 (7ed):

Balanced 3 phase Y−Y $\hat{Z}_L = 20 + j12\ \Omega$

Source abc: $\hat{V}_{an} = 120\angle 0°\ V_{RMS}$. If load voltage $\hat{V}_{AN} = 111.49\angle -0.2°\ V_{RMS}$

Magnitude of $\hat{I}_{Line}$ if load is suddenly short-circuited.



$$\hat{V}_{an} = 120\angle 0°\ V_{RMS} \xrightarrow[\text{balanced}]{\text{abc}} \hat{V}_{bn} = 120\angle 240°\ V_{RMS}\ ;\quad \hat{V}_{cn} = 120\angle 120°\ V_{RMS}.$$

line current: $$\hat{I}_{aA} = \frac{\hat{V}_{AN}}{\hat{Z}_L} = \frac{111.49\angle -0.2°}{20 + j12} = \frac{111.49\angle -0.2°}{\sqrt{544}\ \angle \tan^{-1}\frac{3}{5}}$$

$$= \frac{111.49\angle -0.2°}{23.32\angle 30.96°} = 4.78\angle -31.16°\ A_{RMS}$$

$$\hat{V}_{Line} = \hat{V}_{aA} = \hat{V}_{an} - \hat{V}_{AN} = 120\angle 0° - 111.49\angle -0.2°$$

$$= 120 - 111.49\cos(-0.2°) + j\,111.49\sin(0.2°)$$

$$= 8.51 + j\,0.389$$

$$Z_{line} = \frac{\hat{V}_{Line}}{\hat{I}_{Line}} = \frac{\sqrt{8.51^2 + 0.389^2}\ \angle \tan^{-1}\frac{0.389}{8.51}}{4.78\angle -31.16°} = \frac{8.52\angle 2.617°}{4.78\angle -31.16°} = 1.78\angle 33.78°\ \Omega$$

→ Current in the line if the load is short-circuited is

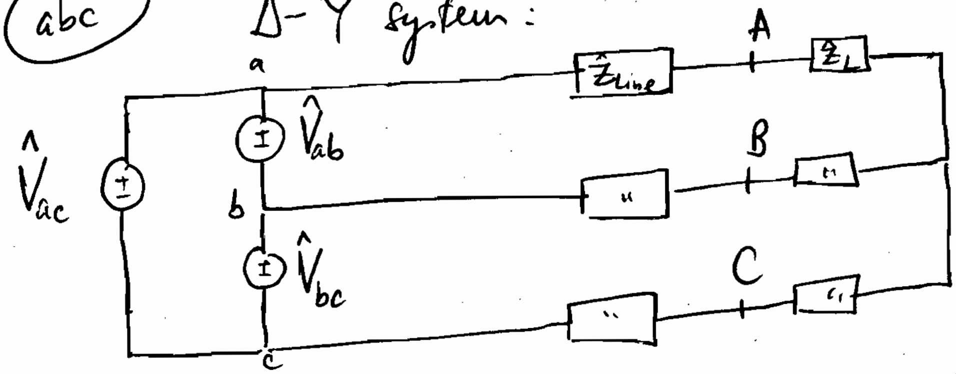
$$I_{line\ SC} = \frac{120}{1.78\angle 33.78°} = 67.3\angle -33.78°\ A_{RMS}$$

Revisit the extra credit question:

Balanced, three-phase,

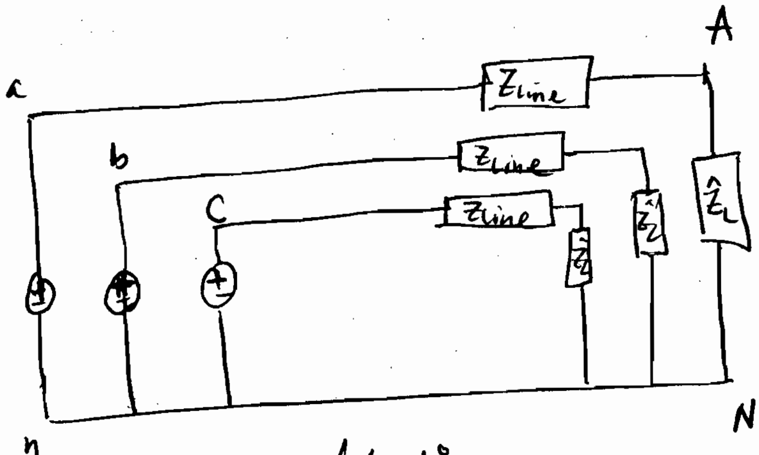
abc

Δ-Y system:



$\hat{Z}_{Line} = 1 + 0.5j$; $\hat{Z}_L = 10 + 6j$; $\hat{I}_{aA} = 10\angle -20°$ A_{RMS}

$\hat{V}_{ab} \stackrel{?}{=}$ Convert source to a Y connection, find $\hat{V}_{an}$, then transform to $\hat{V}_{ab}$:



$$\hat{V}_{an} = \hat{I}_{aA}(\hat{Z}_{line} + \hat{Z}_L)$$

$$= 10\angle -20° \left(\sqrt{11^2 + 6.5^2}\angle \tan^{-1}\frac{6.5}{11}\right)$$

$$= 127.7\angle 10.6° \ V_{RMS}$$

$$\hat{V}_{bn} = 127.7\angle 10.6° - 120°$$

$$\hat{V}_{cn} = 127.7\angle 10.6° - 240°$$

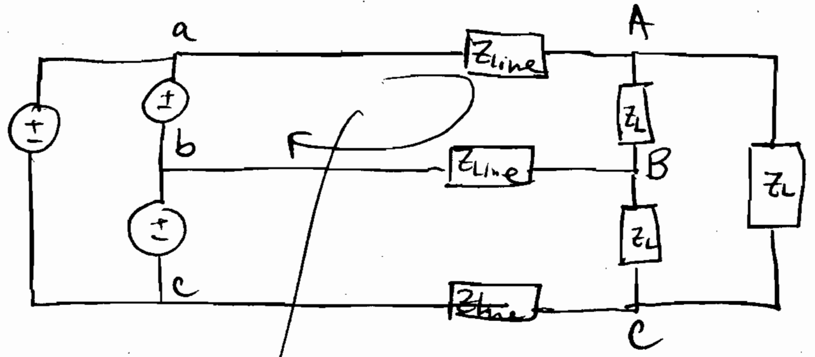
$$\hat{V}_{ab} = 127.7\sqrt{3}\angle 40.6° \ V_{RMS}$$

$$\hat{V}_{bc} = 127.7\sqrt{3}\angle 40.6° - 120°$$

$$\hat{V}_{ca} = 127.7\sqrt{3}\angle 40.6° - 240°$$

$$\hat{Z}_\Delta \overset{?}{=} 3\hat{Z}_Y = 30 + 18j\ \Omega .$$

$\Delta - \Delta$:



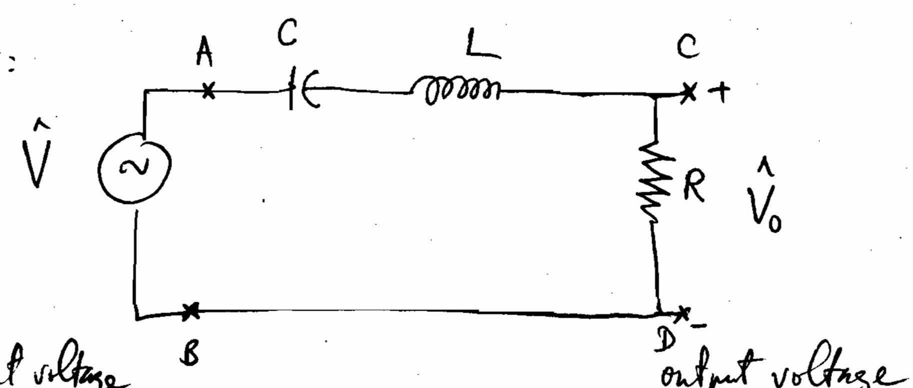
$\hat{V}_{ab} =$

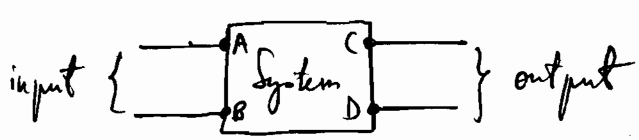
Variable Frequency Network:

$\hat{Z}_R = R$; $\hat{Z}_L = j\omega L$; $\hat{Z}_C = \dfrac{1}{j\omega C}$

frequency dependence

RLC circuit:





Transfer function $\equiv \dfrac{\hat{V}_0}{\hat{V}} =$

Voltage division: $\hat{V}_o = \hat{V} \dfrac{R}{\frac{1}{j\omega C} + j\omega L + R}$

→ Transfer function: $\dfrac{\hat{V}_o}{\hat{V}} = \dfrac{R}{\frac{1}{j\omega C} + j\omega L + R}$ a function of frequency

Conclusion: a circuit that contains some C or L in addition to R is a frequency-variable network

Characterization or classification of variable-frequency circuits:

Example (same circuit): $j\omega \longrightarrow s$

$$\frac{\hat{V}_o}{\hat{V}} = \frac{R}{\frac{1}{sC} + sL + R}$$

→ Classification based on zeroes of the denominator.

$$= \frac{R \cdot \frac{s}{L}}{\left(\frac{1}{sC} + sL + R\right)\frac{s}{L}} = \frac{\frac{R}{L}s}{\frac{1}{LC} + s^2 + \frac{R}{L}s}$$

] Numerator

] Denominator

Zeros of the denominator: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$\boxed{\begin{aligned} x^2 + bx + c &= (x - x_1)(x - x_2) \\ x_1 &= \frac{-b + \sqrt{b^2 - 4c}}{2} \\ x_2 &= \frac{-b - \sqrt{b^2 - 4c}}{2} \end{aligned}}$$

$$\rightarrow s = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$$\rightarrow s = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

Case 1) $R^2 - \frac{4L}{C} = 0$ → Our transfer function (or circuit) has a pole of 2nd order at $\left(-\frac{R}{2L}\right)$

$$\frac{\hat{V}_o}{\hat{V}} = \frac{\frac{R}{L}s}{\left(s + \frac{R}{2L}\right)^{2}}$$

2) $R^2 - \frac{4L}{C} > 0 \rightarrow S_\pm = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$

Our transfer function (or circuit) has two poles of 1st order at S_+ & S_-

$$\frac{\hat{V}_o}{\hat{V}} = \frac{\frac{R}{L}s}{(S-S_+)(s-s_-)} = \frac{\frac{R}{L}s}{\left(S+\frac{R}{2L}-\frac{1}{2L}\sqrt{R^2-\frac{4L}{C}}\right)\left(S+\frac{R}{2L}+\frac{1}{2L}\sqrt{R^2-\frac{4L}{C}}\right)}$$

3) $R^2 - \frac{4L}{C} < 0 \rightarrow S_\pm = -\frac{R}{2L} \pm \frac{j}{2L}\sqrt{\left|R^2 - \frac{4L}{C}\right|}$

Our transfer function (or circuit) has two complex conjugate poles at S_+ & S_-

$$\frac{\hat{V}_o}{\hat{V}} = \frac{\frac{R}{L}s}{\left(S+\frac{R}{2L}-\frac{j}{2L}\sqrt{\left|R^2-\frac{4L}{C}\right|}\right)\left(S+\frac{R}{2L}+\frac{j}{2L}\sqrt{\left|R^2-\frac{4L}{C}\right|}\right)}$$

Transfer function : $\hat{H}(s) = \frac{\hat{V}_o(s)}{\hat{V}(s)} \Rightarrow \hat{H}(j\omega) = \frac{\hat{V}_o(j\omega)}{\hat{V}(j\omega)}$

Graphing : Bode plots $\begin{cases} \text{Magnitude: } 20\log_{10}|\hat{H}(j\omega)| \text{ w.r.t. } \underbrace{\log_{10}\omega}_{\text{frequency } \omega \text{ in log. scale}} \\ \text{Phase: } \angle\hat{H}(j\omega) \text{ w.r.t. freq } \omega \text{ in log scale} \end{cases}$

We now learn how to sketch Bode plots for a given circuit (or transfer function).

First, write the transfer function in a standard form:

$$\hat{H}(j\omega) = \frac{K_0 (j\omega)^{\pm N} (1 + j\omega\tau_1)\left[1 + 2\xi_3 j\omega\tau_3 + (j\omega\tau_3)^2\right]\ldots}{(1+j\omega\tau_a)\left[1 + 2\xi_b j\omega\tau_b + (j\omega\tau_b)^2\right]\ldots}$$

τ_1 : "tau sub one"; ξ_3 "xi sub three"

Example: previous transfer function: $\hat{H}(j\omega) = \frac{\hat{V}_o}{\hat{V}} = \frac{\frac{R}{L} j\omega}{(j\omega)^2 + j\omega\frac{R}{L} + \frac{1}{LC}}$

$$= \frac{RCj\omega}{(j\omega)^2 LC + j\omega RC + 1}$$

$K_0 = RC$

$N = 1$

$\tau_1 = 0 = \tau_a$

$\tau_3 = 0$

$\xi_3 = 0$

$\tau_b = \sqrt{LC}$

$\xi_b \Rightarrow \quad 2\xi_b\tau_b = RC$

(xi sub b) $\quad \xi_b = \frac{RC}{2\sqrt{LC}} = \frac{R}{2}\sqrt{\frac{C}{L}}$

Review:

1) $\left|\frac{\hat{z}_1 \cdot \hat{z}_2}{\hat{z}_3}\right| = \frac{|\hat{z}_1| \cdot |\hat{z}_2|}{|\hat{z}_3|}$

2) $\log_{10} \frac{|\hat{z}_1| \cdot |\hat{z}_2|}{|\hat{z}_3|} = \log|\hat{z}_1| + \log|\hat{z}_2| - \log|\hat{z}_3|$

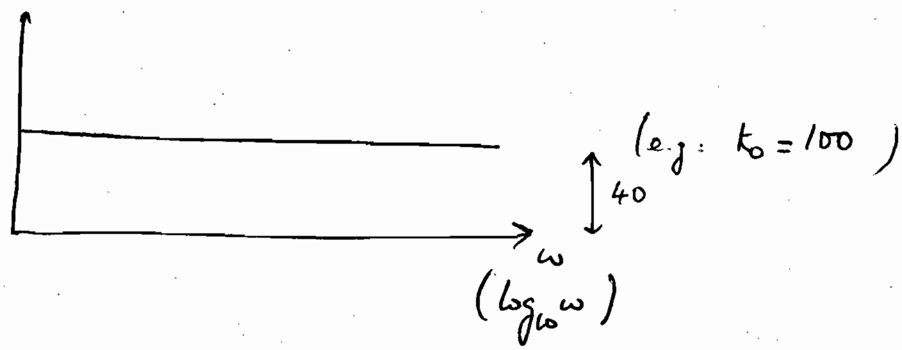
Magnitude Bode plot: $20\log_{10}|\hat{H}(j\omega)|$ versus ω in log scale

$$20\log_{10}|\hat{H}(j\omega)| = 20\log_{10}\frac{|k_0|\,|j\omega|^{\pm N}\,|1+j\omega\tau_1|\,|1+2\zeta_3 j\omega\tau_3+(j\omega\tau_3)^2|}{|1+j\omega\tau_a|\,|1+2\zeta_b j\omega\tau_b+(j\omega\tau_b)^2|}$$

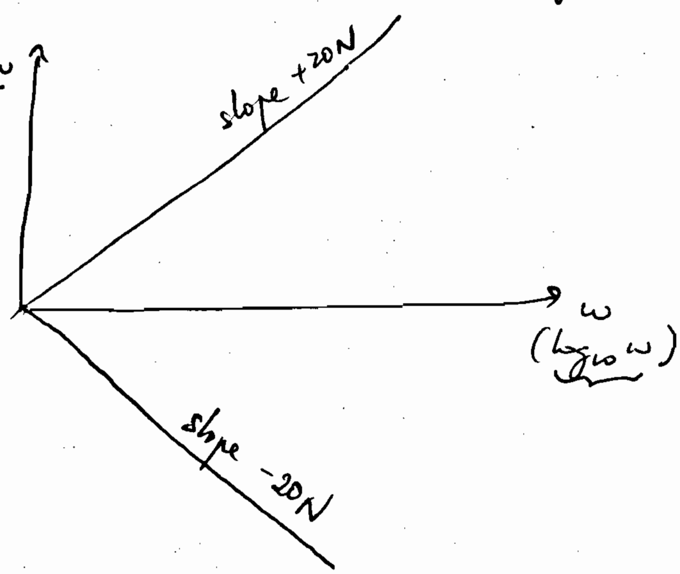
$|j\omega| = \omega$

$$= 20\log_{10}|k_0| + 20\log_{10}\omega^{\pm N} + 20\log_{10}|1+j\omega\tau_1| + 20\log_{10}|1+2\zeta_3 j\omega\tau_3+(j\omega\tau_3)^2|$$
$$-20\log_{10}|1+j\omega\tau_a| - 20\log_{10}|1+2\zeta_b j\omega\tau_b+(j\omega\tau_b)^2| \;\ldots$$

1) $20\log_{10}|k_0|$



2) $\pm 20N\log_{10}\omega$



3) $20\log_{10}|1+j\omega\tau_1|$ use asymptotic analysis.

$\omega \to 0$ (low freq) $\to j\omega\tau_1 \ll 1 \to 20\log_{10} 1 \simeq 0$

$\omega \to \infty$ (high frequency) $\to j\omega\tau_1 \gg 1 \to 20\log_{10}|j\omega\tau_1|$

$20\log_{10}(\omega\tau_1) = 20\log_{10}\omega + 20\log_{10}\tau_1$

$\omega\tau_1 \simeq 1$ or $\omega_c = \frac{1}{\tau_1}$ (critical frequency) $\to 1+j\omega\tau_1 \simeq 1+j$

$\to 20\log_{10}\underbrace{|1+j|}_{\sqrt{2}} = 10\log_{10}2 = 3\text{ dB}$