Power Analysis:

Instantaneous power: $p(t) = i(t)\,v(t) = I_M \cos(\omega t + \theta_i)\, V_M \cos(\omega t + \theta_v)$

(lower case) AC circuits

time-independent parameters: $\left\{ \begin{array}{l} I_M \;\&\; V_M \text{ are magnitudes of alternating current and voltage, respectively} \\ \theta_i \;\&\; \theta_v \text{ are the respective phase angles.} \end{array} \right.$

Average power: $\underset{\text{(upper case)}}{P} = \dfrac{1}{T}\displaystyle\int_{t_0}^{t_0+T} p(t)\,dt$

(our current & voltage are periodic w/ period $T = \frac{2\pi}{\omega}$)

$$P = \frac{1}{T}\int_T \frac{1}{2} I_M V_M \left\{ \cos(\cancel{\omega t} + \theta_i - \cancel{\omega t} - \theta_v) + \cos(\omega t + \theta_i + \omega t + \theta_v) \right\} dt$$

Trig. identity: $\cos\alpha \cos\beta = \frac{1}{2}\left\{ \cos(\alpha - \beta) + \cos(\alpha + \beta) \right\}$

$$P = \frac{1}{T}\int_T dt\, \underbrace{\frac{1}{2} I_M V_M \cos(\theta_i - \theta_v)}_{\text{time-independent}} + \frac{1}{T}\int_T dt\, \frac{1}{2} I_M V_M \cos(2\omega t + \theta_i + \theta_v)$$

$$= \underbrace{\frac{1}{2} I_M V_M \cos(\theta_i - \theta_v)} + \underbrace{\frac{1}{2} I_M V_M \frac{1}{T}\int_T dt \cos(\underset{\substack{\downarrow \\ \text{period: is } \frac{T}{2}}}{2\omega} t + \theta_i + \theta_v)}_{\substack{\text{average over 2 periods} \\ \text{of a sinusoid} \to 0}}$$

$$\rightarrow \boxed{P = \frac{1}{2} I_M V_M \cos(\theta_i - \theta_v)}$$

Makes sense!

$\rightarrow \left\{ \begin{array}{l} \text{At a pure-reactive element} = P = 0 \\ \text{(inductor or capacitor)} \\ \\ \text{At a pure-resistive element: } P = \frac{1}{2} I_M V_M \end{array} \right.$

at an inductor: (~~electric~~ magnetic energy storage)

$$\hat{Z}_L = \underbrace{j\omega L}_{\uparrow} = \frac{\hat{V}}{\hat{I}} = \underbrace{\frac{V_m}{I_m} \angle \theta_v - \theta_i}_{\uparrow}$$

$$90° = \theta_v - \theta_i \rightarrow \cos(\theta_v - \theta_i) = 0$$

$$\underline{P} = 0$$

at a capacitor: (electric energy storage)

$$\hat{Z}_C = \underbrace{\frac{1}{j\omega C}}_{\uparrow} = \frac{\hat{V}}{\hat{I}} = \underbrace{\frac{V_m}{I_m} \angle \theta_v - \theta_i}_{\uparrow}$$

$$-90° = \theta_v - \theta_i \rightarrow \cos(\theta_v - \theta_i) = 0$$

$$\underline{P} = 0$$

RMS (root-mean-square) or effective values:

Current: $i(t) = I_m \cos(\omega t + \theta_i)$

RMS value or effective value is $I_{RMS} = \sqrt{\frac{1}{T}\int_T i^2(t)\, dt}$

$$= I_m \sqrt{\frac{1}{T}\int_T dt \cos^2(\omega t + \theta_i)} = \frac{I_m}{\sqrt{2}} \sqrt{\frac{1}{T}\int_T dt\left(1 + \cos(2\omega t + 2\theta_i)\right)}$$

Trig. Id: $\cos^2\alpha = \frac{1}{2}(1 + \cos 2\alpha)$

$$= \frac{I_m}{\sqrt{2}}\left(\sqrt{1 + \underbrace{\frac{1}{T}\int_T \cos(2\omega t + 2\theta_i)}_{\text{average over 2 periods of a sinusoid} \Rightarrow 0}}\right) \Rightarrow \boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}}$$

Voltage: $v(t) = V_m \cos(\omega t + \theta_v) \rightarrow \boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

$$\rightarrow \underline{P} = \frac{1}{2} I_m V_m \cos(\theta_i - \theta_v) = I_{RMS} V_{RMS} \underbrace{\cos(\theta_i - \theta_v)}_{\equiv \text{ power factor or p.f.}}$$

$$\boxed{\underline{P} = I_{RMS} V_{RMS}\, pf} \left\{ \begin{array}{l} \text{at pure-reactive: } 0 \\ \text{at pure-resistive: } I_{RMS} V_{RMS} \end{array} \right.$$

<u>Complex Power</u>: (not measurable, just a definition)

$$\hat{S} = \hat{V}_{RMS}\,\hat{I}^*_{RMS} = V_{RMS}\angle\theta_v \cdot I_{RMS}\angle{-\theta_i}$$

(upper case)

$$= V_{RMS} I_{RMS}\angle\theta_v-\theta_i = V_{RMS} I_{RMS}\cos(\theta_v-\theta_i)$$

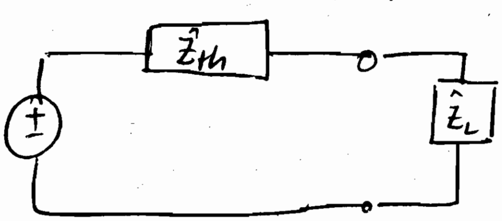
polar form   rectangular form

$$+j\,V_{RMS} V_{RMS}\sin(\theta_v-\theta_i)$$

$$= \underline{P} + j\,V_{RMS} I_{RMS}\sin(\theta_v-\theta_i)$$

The real part of the complex power $\hat{S}$ is the average power $\underline{P}$.

→ Given a circuit, what load would draw the maximum power?
To find out, let's replace our circuit by a Thevenin equivalent



What $\hat{Z}_L$ would extract the max. power? It's $\hat{Z}_L = \hat{Z}^*_{th}$

<u>Proof</u>: Ave. power at load is $\underline{P}_L = \frac{1}{2} I_L V_L \cos(\theta_{v_L} - \theta_{i_L})$

$$= \frac{1}{2}\,\frac{V_{oc}^2\,(R_L^2+X_L^2)^{1/2}}{(R_{th}+R_L)^2+(X_{th}+X_L)^2}\,\frac{R_L}{(R_L^2+X_L^2)^{1/2}}$$

To get max. P_L: 1) Get smaller denominator: $\boxed{X_L = -X_{th}}$

$$\Rightarrow P_L = \frac{1}{2}\,\frac{V_{oc}^2\,R_L}{(R_{th}+R_L)^2}$$

2) Max P_L w.r.t. R_L :

$$\frac{\partial P_L}{\partial R_L} = 0$$

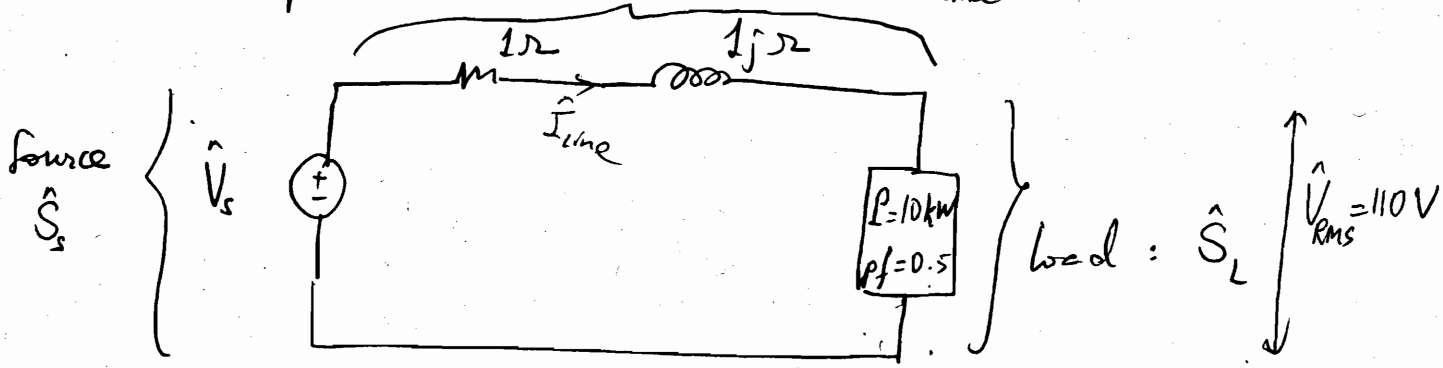
$$\frac{1}{2}\,\frac{\cancel{V_{oc}^2}}{\cancel{(R_{th}+R_L)^2}} - \frac{\cancel{V_{oc}^2}\, R_L}{(R_{th}+R_L)^{\cancel{3}\,1}} = 0$$

$$\frac{1}{2} - \frac{R_L}{R_{th}+R_L} = 0 \rightarrow 2R_L = R_{th} + R_L$$

$$\Rightarrow \boxed{R_L = R_{th}}$$

$$\rightarrow \boxed{\hat{Z}_L = \hat{Z}_{th}^{*}}$$

<u>Example problem</u> :

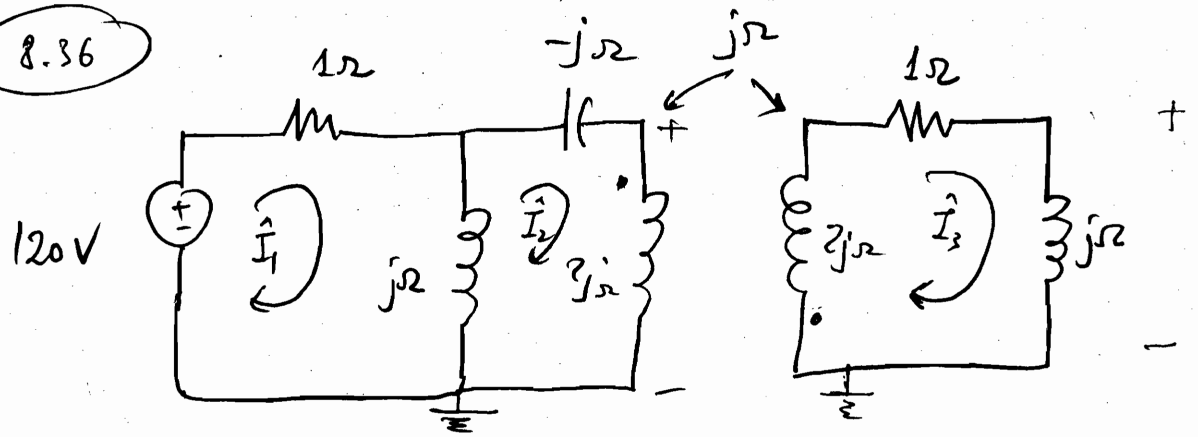


Find $\hat{S}_s$ and p.f at the source. (To design a power supply)

$$\hat{S}_s = \hat{S}_{Line} + \hat{S}_L$$

4) $\hat{S}_L$: $\longrightarrow$ cont. on page 48

8.36



<u>Loop analysis:</u>

1) $$120 = I_1 + j(\hat{I}_1 - \hat{I}_2) \checkmark = \hat{I}_1(1+j) - j\hat{I}_2$$

2) $$0 = (\hat{I}_2 - \hat{I}_1)j - j\hat{I}_2 + 2j\hat{I}_2 \pm j\hat{I}_3$$

$$= -j\hat{I}_1 + 2j\hat{I}_2 \pm j\hat{I}_3$$

3) $$0 = 2j\hat{I}_3 + j\hat{I}_2 + (1+j)\hat{I}_3$$

$$= (1+3j)\hat{I}_3 + j\hat{I}_2$$

$$\hat{I}_3 = \frac{-j}{1+3j}\hat{I}_2$$

$$0 = -j\hat{I}_1 + \hat{I}_2\left(2j - \frac{j^2}{1+3j}\right)$$

$$\hat{I}_2 = \frac{j\hat{I}_1}{2j - \frac{j^2}{1+3j}}$$

$$120 = \hat{I}_1(1+j) + \hat{I}_1\frac{1}{2j + \frac{1}{1+3j}}$$

$$= \hat{I}_1\left[1+j + \frac{1}{2j + \frac{1}{1+3j}}\right]$$

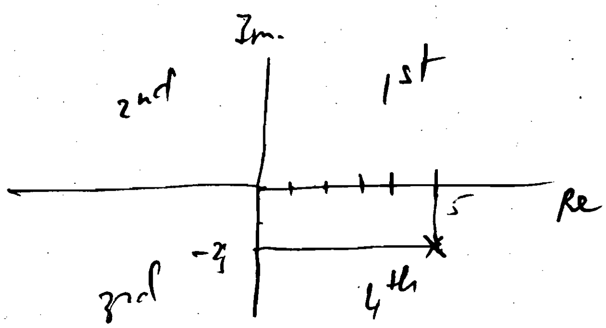
$$120 = \hat{I}_1\left[1+j + \frac{1+3j}{2j-6+1} \right] = \hat{I}_1 \frac{(1+j)(-5+2j) + 1+3j}{-5+2j}$$

$$= \hat{I}_1 \frac{-7 - \cancel{3j} + 1 + \cancel{3j}}{-5+2j} = \hat{I}_1 \frac{6}{5-2j}$$

$$\hat{I}_1 = 20(5-2j)$$

$$\rightarrow \hat{Z}_0 = \frac{120}{\hat{I}_1} = \frac{120}{20(5-2j)} = \frac{6}{5-2j} = \frac{6}{\sqrt{29}\angle -\tan^{-1}\frac{2}{5}}$$

$$= 1.11 \angle 21.8°$$



Solution answer is correct but signs for mutual inductance term in loop #2 should be +.

$$\angle \theta_v - \theta_i = \cos^{-1}(p.f) = \cos^{-1}(0.5) = 60°$$

a) $\hat{S}_L = \underbrace{V_{RMS} I_{RMS}}$

We know $P = 10\,kW$; $p.f = 0.5 \rightarrow V_{RMS} I_{RMS}$?

Since $Re[\hat{S}_L] = P = V_{RMS} I_{RMS}\, p.f \rightarrow V_{RMS} I_{RMS} = \dfrac{P}{pf} = \dfrac{10\,kW}{0.5}$

$= 20\,kW$

$$\hat{S}_L = 20000 \angle 60° \;\underbrace{V.A}_{\text{same as } W}$$

b) $\underbrace{\hat{S}_{Line}}_{\text{def. of Complex power}} = \hat{V}_{Line} \hat{I}^*_{Line} = \underbrace{(\hat{I}_{Line} \hat{Z}_{Line})}_{\text{Ohm's law}} \hat{I}^*_{Line} = I^2_{line} \hat{Z}_{line}$

$= I^2_{line} (1+j)$

$\hat{I}_{line} = \hat{I}_L$ (series) $\rightarrow \hat{I}_L = \left(\dfrac{\hat{S}_L}{\hat{V}_L}\right)^* = \left(\dfrac{20000 \angle 60°}{110 \angle 0°}\right)^* = 181.82 \angle -60°\, A$

$$\hat{S}_{Line} = 181.82^2 (1+j) \;\; V.A$$

c) $\hat{S}_s = \hat{S}_{Line} + \hat{S}_L = 181.82^2 (1+j) + 20000 (\cos 60 + j \sin 60)$

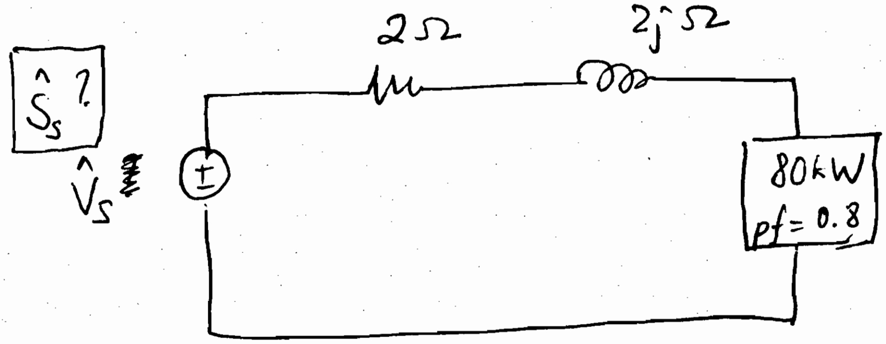
$= 43058.5 + j\,50379 \;\; VA = 66272.8 \angle 49.5° \;\; VA$

p.f at source : = cos of the angle of $\hat{S}_s$ = $\cos(49.5°) = 0.65$

d) $\hat{V}_s = \dfrac{\hat{S}_s}{\hat{I}^*_L} = \dfrac{66272.8 \angle 49.5°}{181.82 \angle 60°} = 364.49 \angle -10.5° \; V$

$\hat{S}_s = \hat{V}_s \underbrace{\hat{I}^*_s}_{} \underset{\text{series}}{=} \hat{V}_s \hat{I}^*_L$

Quiz #2 March 27, 2007



\+

$\hat{V}_L = 300\ V_{RMS}$

lagging $(\theta_v - \theta_i > 0)$

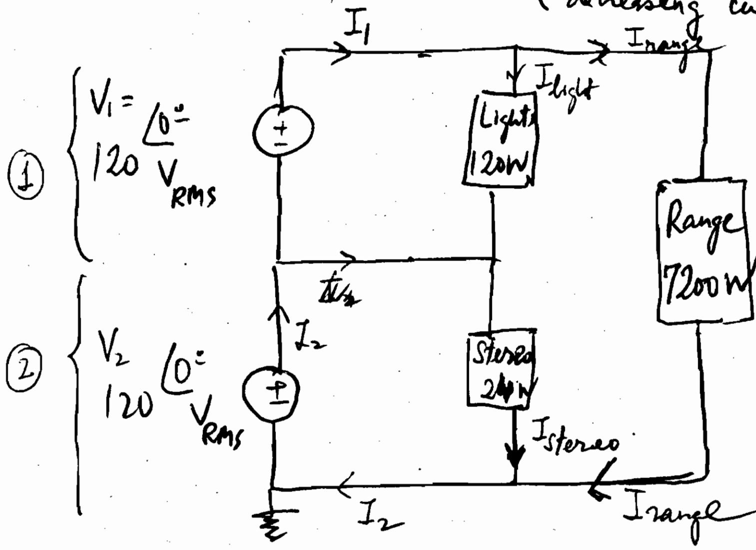
\-

Find the complex power at the source $\hat{S}_s$.

HW 6 due 4/3/07
HW 7 due 4/10/07
Exam 2 4/12/07 (HW's 4–7)

Single-phase three-wire circuit:

increase voltage difference for stove and dryer
(decreasing current required)



- Single-phase of 0°
- Three-wire

Utility bill per month ?
\$0.08 / kWh

Total energy consumed a day (lights on 15 hours, Range on 4 hours, stereo on 8 hours)

Given Power and time → energy consumed is $P \times$ time.

Energy supplied by source #1 is

$$E_1 = \int P_1\,dt = \int V_1 I_1\,dt = V_1 \times (I_1 \times \text{time})$$

$$= 120V\Big[\underbrace{I_{lights}}_{\frac{120\,W}{120\,V}} \times 15\text{ hrs} + \underbrace{I_{Range}}_{\frac{7200\,W}{240\,V}} \times 4\text{ hrs}\Big] = \begin{matrix}120\times \\ 15\text{ Whrs} \\ +14400\text{ Wh}\end{matrix}$$

$$= 14.4\text{ kWh} + 1.8\text{ kWh} = 16.2\text{ kWh}.$$

Energy supplied by source #2 is

$$E_2 = \int P_2\,dt = \int V_2 I_2\,dt = V_2 \times [I_2 \times \text{time}]$$

$$= 120V \times \Big[\underbrace{I_{Stereo}}_{\frac{24W}{120V}} \times 8h + \underbrace{I_{Range}}_{\frac{7200W}{240V}} \times 4h\Big] = 192\text{Wh} + 14{,}400\text{Wh}$$

$$= 14.592\text{ kWh}$$

Total energy consumed = total energy delivered = $E_1 + E_2$

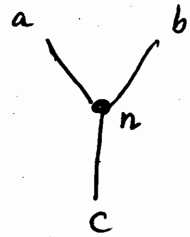
$$= (16.2 + 14.592)\text{ kWh} = 30.8\text{ kWh}$$

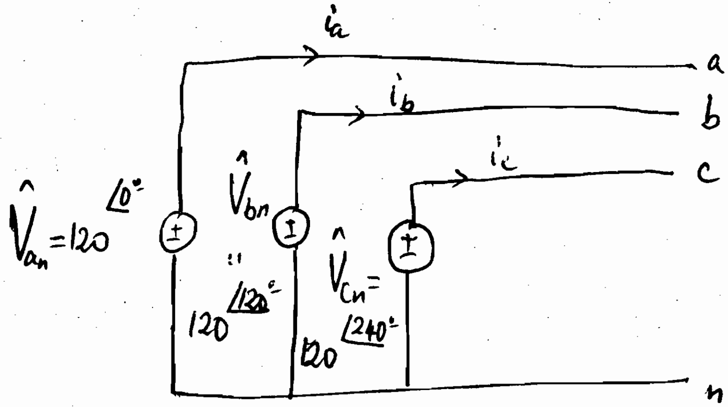
(a day).

→ Electric cost a month = $30.8\,\cancel{\text{kWh}} \times 30 \times \dfrac{\$0.08}{\cancel{\text{kWh}}} = \$73.9$ a month

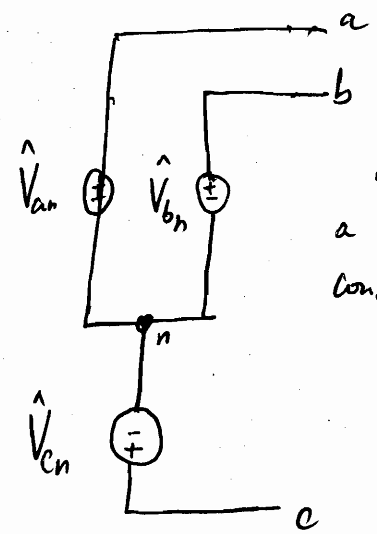
Three - Phase Circuits :

"Y" configuration

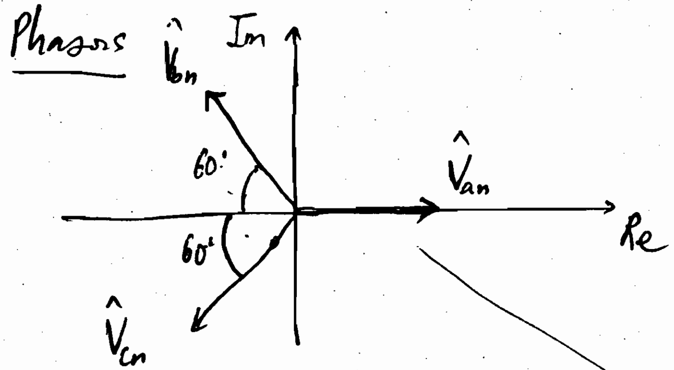




or



a "Y" configuration

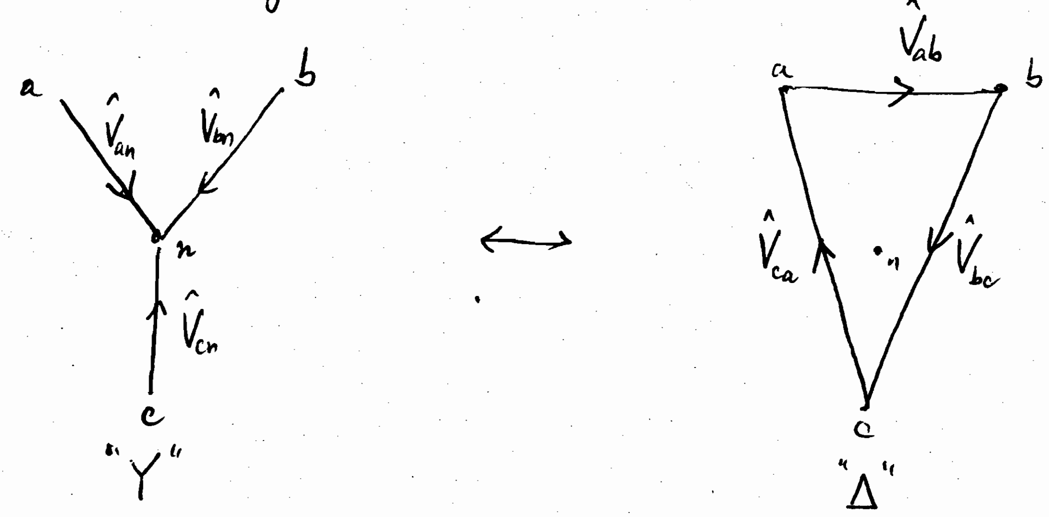


$$\underbrace{\quad}_{120 \times \underbrace{\cos 60}_{0.5} \times 2} = 120$$

$\hat{V}_{an} + \hat{V}_{bn} + \hat{V}_{cn} = 0 \rightarrow$ "Balanced three phase circuit"

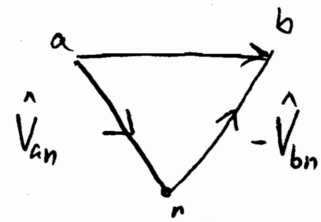
Ordered in CCW direction $\rightarrow$ "~~abc~~ acb balanced three-phase circuit".

"Δ" or delta configuration:



Through geometrical consideration, we can go from "Y" voltages to "Δ" voltages and viceversa

$$\hat{V}_{ab} = \hat{V}_{an} - \hat{V}_{bn} \quad ;$$



rectangular form

$$= V_p - (V_p\cos 120 + V_p\sin 120\, j)$$

$$(V_p = 120) \quad = V_p(1-\cos 120) - jV_p \underbrace{\sin 120}_{\frac{\sqrt{3}}{2}} = V_p\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$=$$

polar form

$$= V_p\sqrt{\frac{9}{4}+\frac{3}{4}} \angle \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = V_p\sqrt{3} \angle -30^\circ$$

$$\hat{V}_{bc} = \hat{V}_{bn} - \hat{V}_{cn} = V_p(\cancel{\cos 120^\circ} + j\sin 120^\circ) - V_p(\cancel{\cos(-120^\circ)} + j\sin(-120^\circ))$$

$$= V_p j \underbrace{\sin(120)}_{\frac{\sqrt{3}}{2}} \times 2 = V_p \sqrt{3}\, j$$

$$= V_p \sqrt{3} \angle 90^\circ$$

$$\hat{V}_{ca} = \hat{V}_{cn} - \hat{V}_{an} = V_p(\cos(-120^\circ) + j\sin(-120^\circ)) - V_p$$

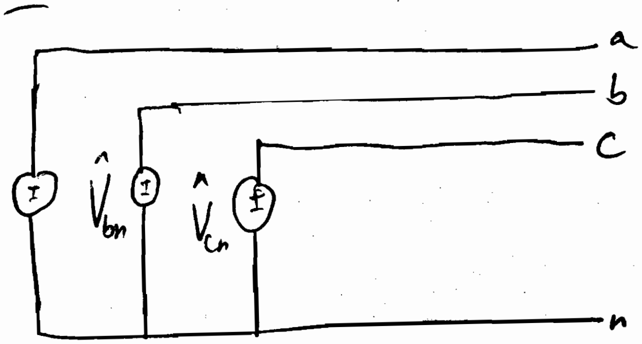
$$= V_p(\cos 120 - 1) - j V_p \sin 120 = V_p \underbrace{\left(-\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}_{\text{3rd quad.}}$$

$$= V_p \sqrt{3} \angle \tan^{-1}\frac{1}{\sqrt{3}} + 180^\circ$$

$$= V_p \sqrt{3} \angle 210^\circ$$

"<u>ABC</u> balanced three-phase circuit":

$\hat{V}_{an} =$



$$\hat{V}_{an} = 120 \angle 0^\circ = V_p \angle 0^\circ$$

$$\hat{V}_{bn} = 120 \angle 240^\circ = V_p \angle -120^\circ$$

$$\hat{V}_{cn} = 120 \angle 120^\circ$$

Phasors: