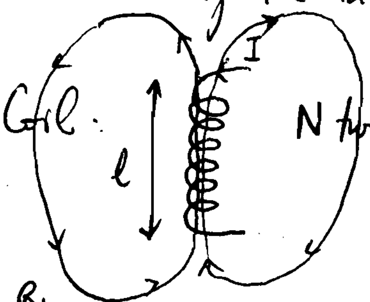
Magnetically Coupled Circuits

(7ed: Ch. 8 or 8th ed: Ch 10)

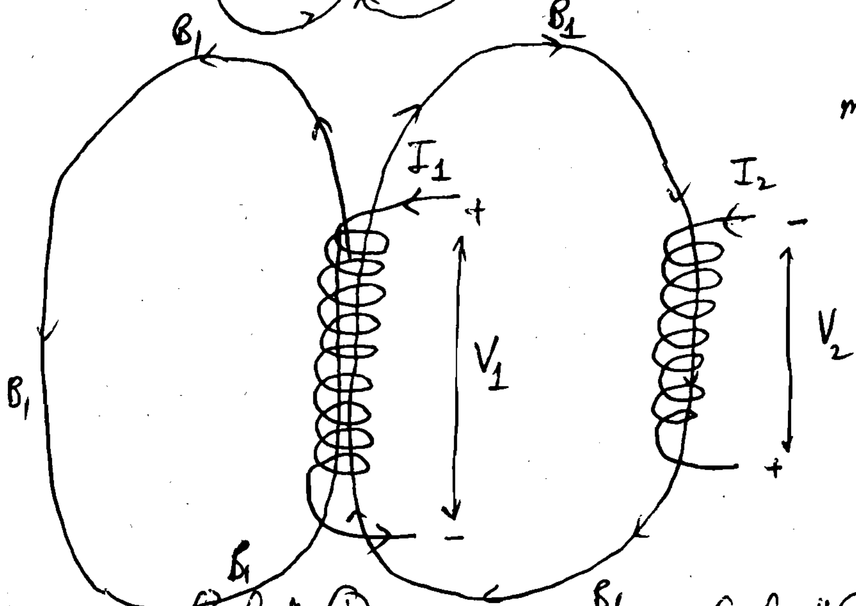
So far inductance L is actually self-inductance: we haven't considered magnetic interaction leading to mutual inductance.

Coil: 

N turns : Magnetic field created by this coil is $B = \mu \frac{N}{l} I$

μ → magnetic permeability (1 in vacuum)

$\underbrace{\frac{N}{l}}_{n}$ → # turns per unit length.



- B_1: magnetic field due to coil #1 interacts with coil #2 and vice versa!

Coil # 1	Coil # 2
Current $= I_1$	Current $= I_2$
# turns $= N_1$	# turns $= N_2$
Cross section $= S_1$	Cross section $: S_2$
Length $= l_1$	Length $: l_2$

Magnetic flux: $\phi = \vec{B}.\vec{S}$ (larger field and larger cross section → larger flux)

Faraday's law: $\underbrace{\text{induced e.m.f.}}_{\text{potential}} = \frac{d\phi}{dt}$

$$V = \frac{d\phi}{dt} = \frac{d}{dt}(B.S) \overset{\text{coil not expanding}}{=} S\frac{dB}{dt} = S\frac{d}{dt}(\mu n I)$$

$$V = \underbrace{S\mu n}_{L} \frac{dI}{dt}$$ This is what we have been using so far.

$$V_1 = L_1 \frac{dI_1}{dt}$$ one coil, one current.

→ Two coils, two currents I_1 & I_2:

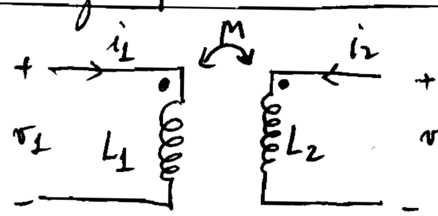
$$V_1 = \underbrace{L_1}_{\text{self-inductance}} \frac{dI_1}{dt} + \underbrace{M}_{\text{mutual-inductance}} \frac{dI_2}{dt}$$

due to magnetic interaction b/w the two coils.

Similarly: $$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} \qquad (M_{12} = M_{21})$$

Conclusion: for our purposes of doing circuit analysis, the implication of including magnetic interaction is the consideration of one <u>extra term with the correct sign</u>: using the "dot convention"

<u>Correct sign for the mutual inductance term</u> (dot convention)

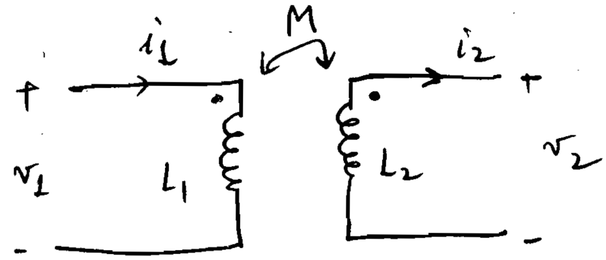


- currents go into dots, then coils
- potential polarities as shown

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

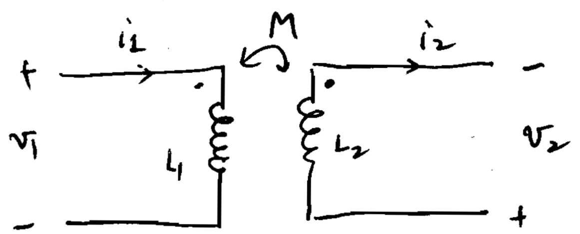
$\xrightarrow{\text{for phasors}}$ $v \to \hat{V} e^{j\omega t}$, $i \to \hat{I} e^{j\omega t}$

$$\hat{V}_1 = j\omega L_1 \hat{I}_1 + j\omega M \hat{I}_2$$
$$\hat{V}_2 = j\omega L_2 \hat{I}_2 + j\omega M \hat{I}_1$$

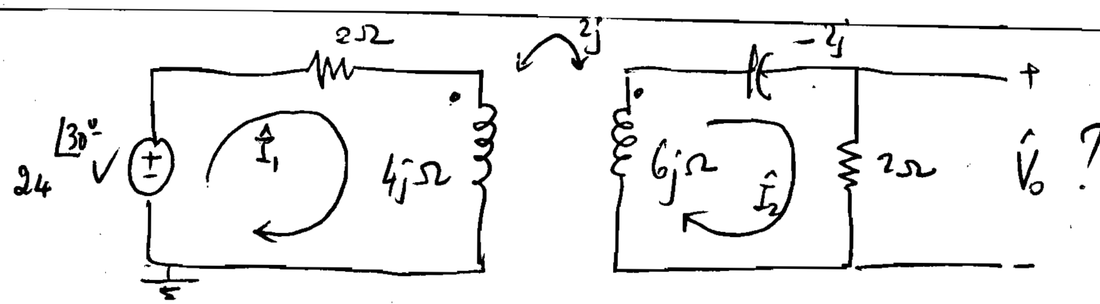


- one current goes into dot, then coil
- one current leaving dot
- polarities as shown

$$\left.\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ v_2 &= -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned}\right\} \xrightarrow{\text{phasors}} \left\{\begin{aligned} \hat{V}_1 &= j\omega L_1 \hat{I}_1 - j\omega M \hat{I}_2 \\ \hat{V}_2 &= -j\omega L_2 \hat{I}_2 + j\omega M \hat{I}_1 \end{aligned}\right.$$



$$\left.\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ v_2 &= L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \end{aligned}\right\} \xrightarrow{\text{phasors}} \left\{\begin{aligned} \hat{V}_1 &= j\omega L_1 \hat{I}_1 - Mj\omega \hat{I}_2 \\ \hat{V}_2 &= j\omega L_2 \hat{I}_2 - Mj\omega \hat{I}_1 \end{aligned}\right.$$



$$\left\{\begin{aligned} \omega L_1 &= 4 \\ \omega L_2 &= 6 \\ \omega M &= 2 \\ \frac{1}{\omega C} &= 2 \end{aligned}\right.$$

Loop analysis:

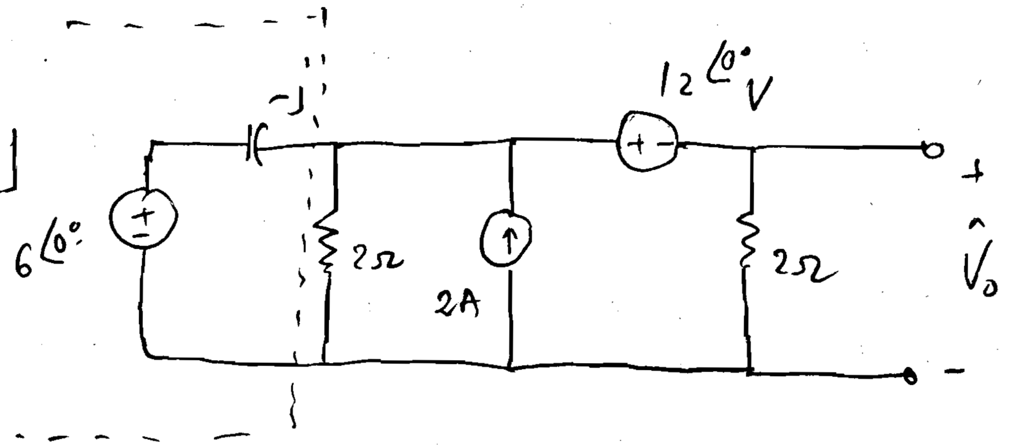
1) $$24\angle 30° = 2\hat{I}_1 + 4j\hat{I}_1 \underbrace{- 2j\hat{I}_2}_{\text{New !!}}$$

2) $$0 = 6j\hat{I}_2 \underbrace{- 2j\hat{I}_1}_{\text{New !!}} + (2-2j)\hat{I}_2$$

25% HW3 $\hat{V}_0 = 2\hat{I}_2 = 5.36\angle 3.43° \; V$

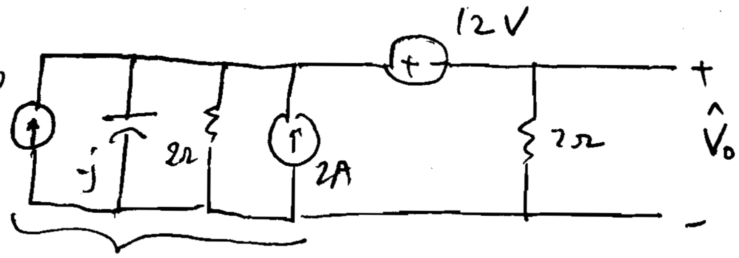
polar form

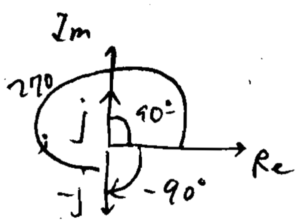
7.60

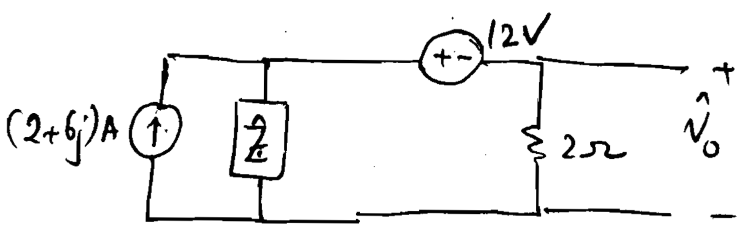


Source exchange

$6j = 6\angle 90$

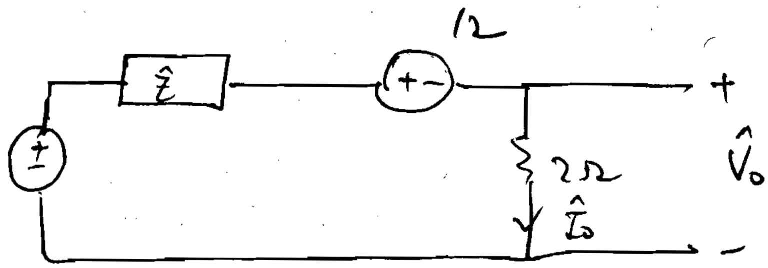




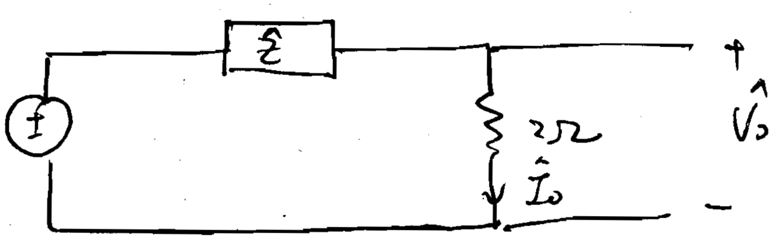


$$\hat{Z} = 2 \parallel -j = \frac{-2j}{2-j} = \frac{2}{2j+1}$$

$(2+6j)\dfrac{2}{2j+1}$



$-12 + \dfrac{2(2+6j)}{1+2j}$



Voltage div.:

$$\hat{V}_o = \left[-12 + \frac{2(2+6j)}{1+2j}\right] \frac{2}{2 + \frac{2}{2j+1}}$$

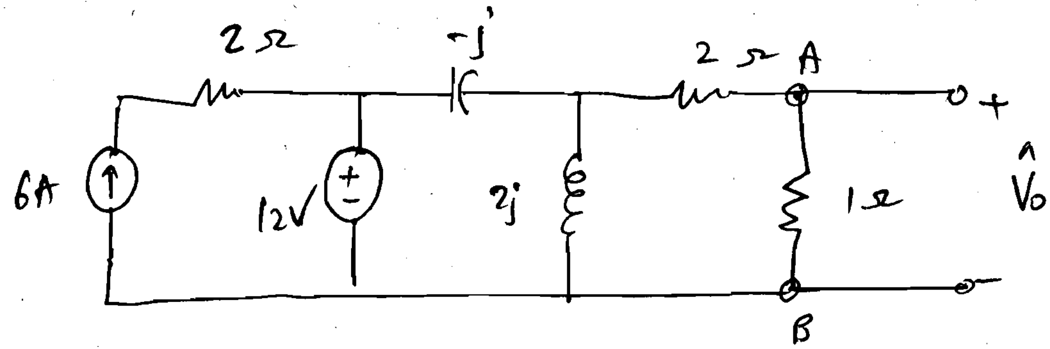
$$= \left[-12(1+2j) + 2(2+6j)\right] \frac{2}{2+4j+2}$$

$$\hat{V}_0 = [-8-12j]\,\frac{2}{4(1+j)}$$

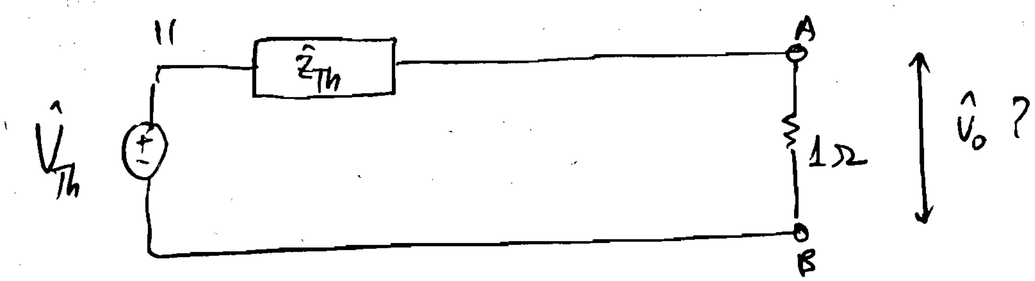
$$= \frac{(-2-3j)\,2}{1+j} = 2\angle 180^\circ\,\frac{(2+3j)}{1+j}$$

$$= 2\angle 180^\circ\,\frac{\sqrt{13}\,\angle \overbrace{\tan^{-1}\tfrac{3}{2}}^{56^\circ}}{\sqrt{2}\,\angle 45^\circ} = 5.1\,\angle 191.3^\circ \quad \checkmark$$

7.63



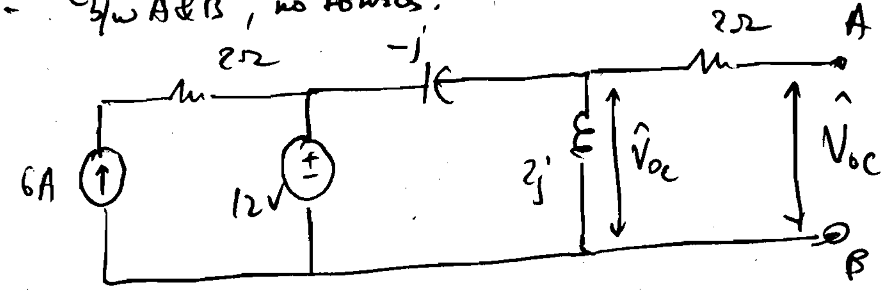
Use Thevenin's equivalent.



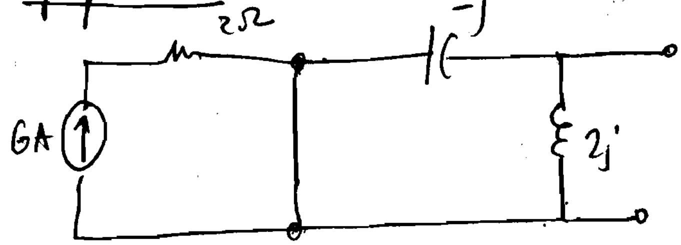
$\hat{V}_{Th} = \hat{V}_{o.c.}$ b/w A & B

$\hat{Z}_{Th} = \hat{Z}$ b/w A & B, no sources.

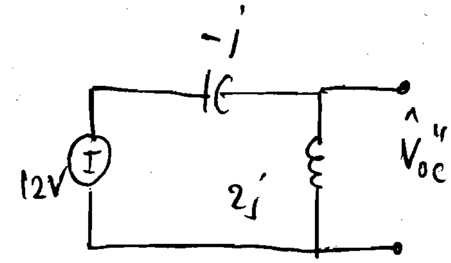
$\hat{V}_{oc}$:



Superposition:



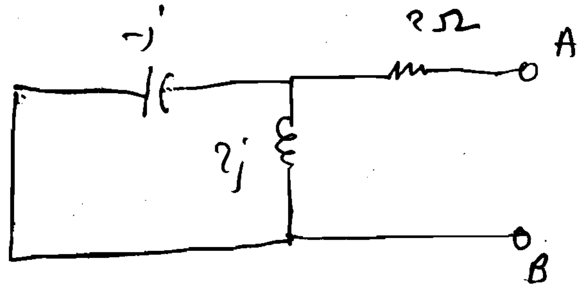
+



$$\hat{V}_{oc}' = 0 \qquad \hat{V}_{oc}'' = 12\,\frac{2j}{1j} = 24\,V$$

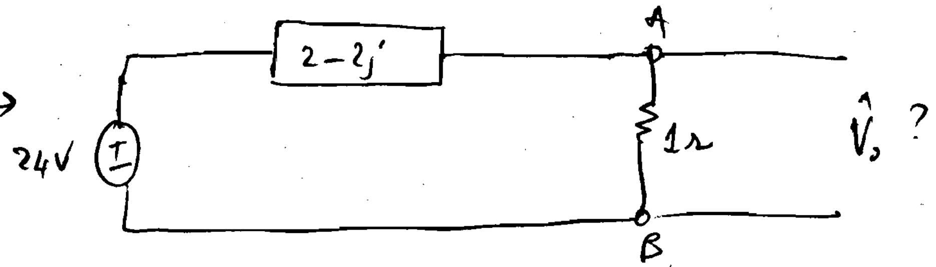
$\hat{V}_{th} = 24\ V.$

$\hat{Z}_{th} =$



$$\hat{Z}_{th} = 2 + \frac{2j(-j)}{2j-j} = 2 + \frac{2}{j} = 2 - 2j$$

→



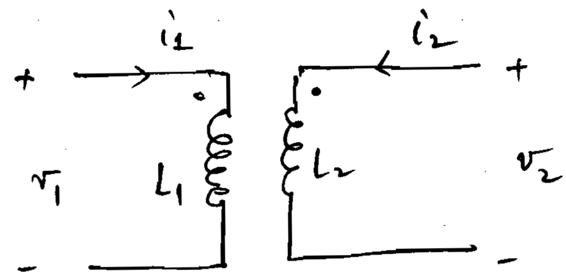
$$\hat{V}_o = 24\,\frac{1}{3-2j} = \frac{24}{\sqrt{13}\ \angle -\tan^{-1}\frac{2}{3}} = \frac{24}{\sqrt{13}}\angle \tan^{-1}\frac{2}{3}$$

$$= 6.66\angle 33.69^\circ\ V$$

(solution is good).

Mutual inductance (Cont.)

Energy analysis:



Power P = energy per unit time $\rightarrow \mathcal{E} = \int P(t)\, dt$

We look at two time intervals:

$$0 \longrightarrow t_1 \longrightarrow t_2$$

$$i_1 : 0 \longrightarrow \underset{(max)}{I_1} \xrightarrow{\text{const}} I_1$$

$$i_2 : 0 \xrightarrow{\text{const.}} 0 \longrightarrow \underset{(max)}{I_2}$$

At any time: $P(t) = i_1 v_1 + i_2 v_2$

$$= i_1(t)\left[L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right] + i_2(t)\left[L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}\right]$$

$$\mathcal{E} = \underbrace{\int_0^{t_1} P(t)\, dt}_{i_2 \text{ const} = 0} + \underbrace{\int_{t_1}^{t_2} P(t)\, dt}_{i_1 = I_1 \text{ const}} = \int_0^{t_1} L_1 i_1 \frac{di_1}{\cancel{dt}} \cancel{dt} + \int_{t_1}^{t_2} \left(I_1 M \frac{di_2}{\cancel{dt}} \cancel{dt} + L_2 i_2 \frac{di_2}{\cancel{dt}} \cancel{dt} \right)$$

$$\mathcal{E} = L_1 \int_0^{I_1} i_1\, di_1 + M I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2\, di_2$$

mutual magnetic interaction

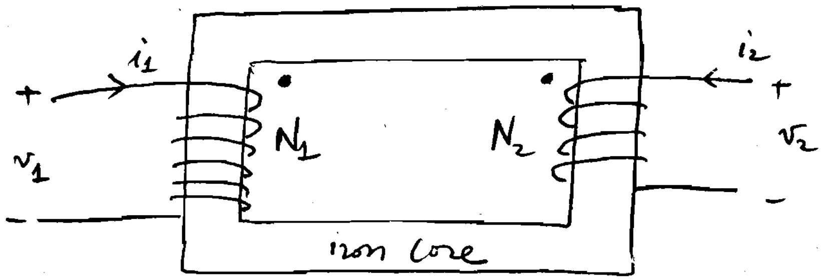
$$= L_1 \frac{I_1^2}{2} + M I_1 I_2 + L_2 \frac{I_2^2}{2} \rightarrow \mathcal{E} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

Ideal Transformer:

Two coils + iron core

To change the voltage using different number of turns

↓



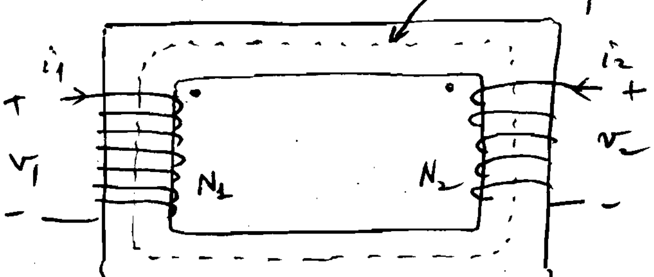
coils $\begin{cases} v_1 = N_1 \dfrac{d\phi_1}{dt} \\ v_2 = N_2 \dfrac{d\phi_2}{dt} \end{cases}$ $\phi_1 = \phi_2$ (iron core) $\boxed{\dfrac{v_1}{v_2} = \dfrac{N_1}{N_2}}$

Faraday's Law: change of magnetic flux w.r.t. time → induced voltage.

Iron core: similar to a short-circuit for electrons (electric fields), the iron core channels all magnetic field lines → can fix the flux: $\Phi_1 = \Phi_2$

Ampere's Law: $\oint_{loop} \vec{H} \cdot \vec{dl} = i_{(\text{enclosed by loop})} = i_1 N_1 + i_2 N_2$

loop ↓ amperian loop.



$\vec{B} = \mu \vec{H}$ → m. field

$\vec{B}$ ↓ m. induction

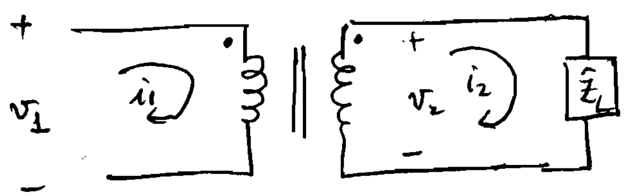
μ ↓ m. permeability ↓ $\begin{cases} 1 \text{ for vacuum} \\ \infty \text{ for iron core} \end{cases}$

$\boxed{\dfrac{i_1}{i_2} = -\dfrac{N_2}{N_1}}$

If $\mu = \infty$ ⇒ for a finite value for B
H has to be 0 ⇒ $0 = i_1 N_1 + i_2 N_2$

Ideal transformer in a circuit:

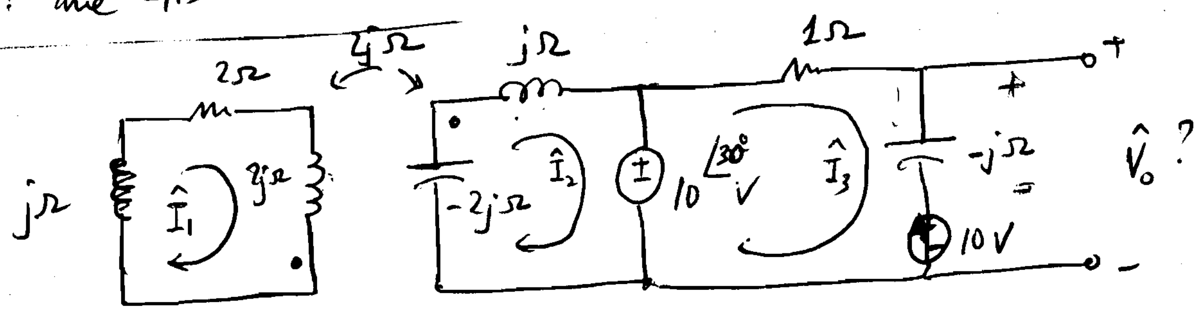
$N_1 : N_2$



$$\begin{cases} \dfrac{v_1}{v_2} = \dfrac{N_1}{N_2} \\[2ex] \dfrac{i_1}{i_2} = \dfrac{N_2}{N_1} \end{cases}$$

HW 4: due 3/13

(8.29)



1) $3j\hat{I}_1 + 2\hat{I}_1 - \hat{I}_2 2j = 0$ → ($\hat{I}_1$ goes into L_1 then dot, $\hat{I}_2$ goes into dot then L_2)

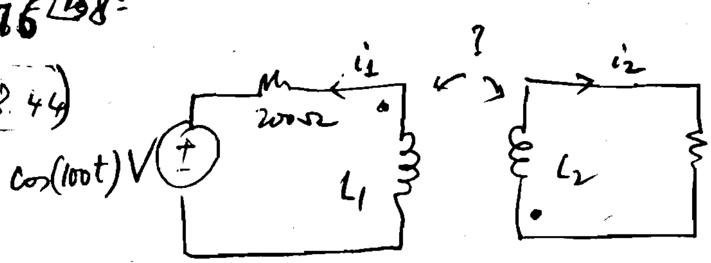
2) $-j\hat{I}_2 + 10\angle 30° - 2j\cdot\hat{I}_1 = 0$

3) $-10\angle 30° + \hat{I}_3(1-j) + 10 = 0 \rightarrow \hat{I}_3 = \dfrac{-10 + 10\angle 30°}{1-j} = 3.66\angle 150°$

→ $\hat{V}_o = \hat{I}_3(-j) + 10$ → (Eliminate $\hat{I}_1$ & $\hat{I}_2$)

$\hat{V}_o = 3.66\angle 150°-90° + 10$
$= 8.76\angle 158°$

(8.44)



$K^2 = \dfrac{M^2}{L_1 L_2}$: coupling coeff.

$K = 0.8$; $L_1 = L_2 = 4H$

Then solve for $\hat{I}_2 =$