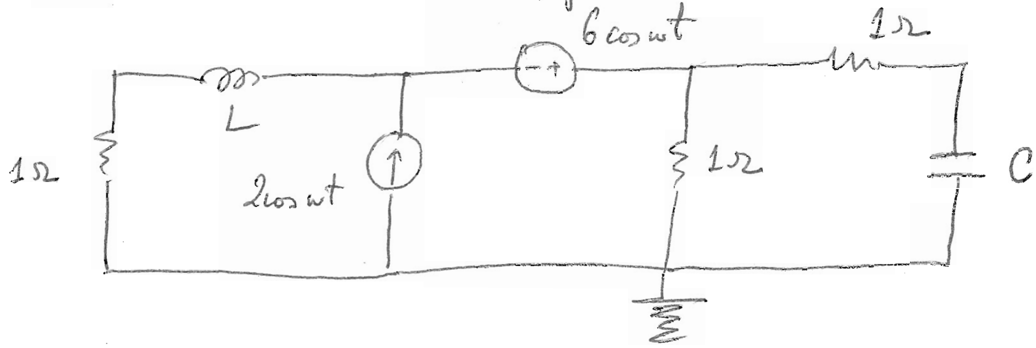
AC Circuit Analysis Techniques:

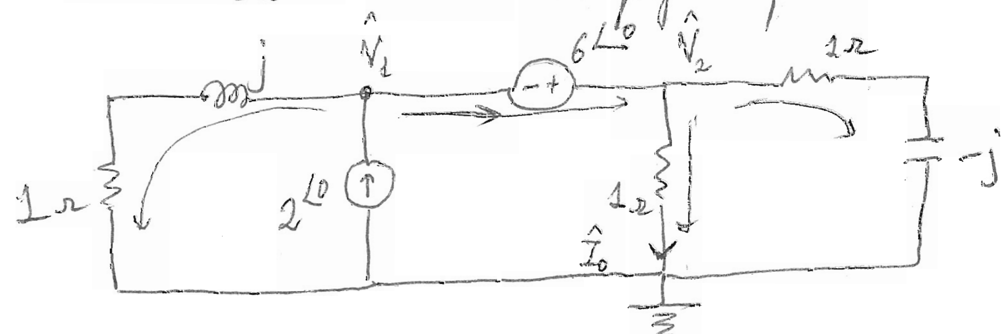


$L = 2.65\,mH$

$C = 2.65\,mF$

$\omega = 2\pi f\,; \quad f = 60\,Hz$

Redraw same circuit in frequency-domain using phasors.



Notes #1

Nodal Analysis:

How many nodes? 2 with phasor voltages $\hat{V}_1$ & $\hat{V}_2$

1) Kirchoff's law for current at node #1: $\sum_i \hat{I}_i = 0$ { current into node: + ; current leaving node: − }

$$+2 - \frac{\hat{V}_1}{1+j} - \left[\frac{\hat{V}_2}{1} + \frac{\hat{V}_2}{1-j} \right] = 0$$

2) $\hat{V}_2 = \hat{V}_1 + 6$

Now this is a system of 2 equations with 2 unknowns. We want to solve for $\hat{I}_o$: eliminate $\hat{V}_1$

$$\rightarrow \quad 2 - \frac{\hat{V}_2 - 6}{1+j} - \hat{V}_2 - \frac{\hat{V}_2}{1-j} = 0$$

$$2 + \frac{6}{1+j} = \hat{V}_2 \left(\frac{1}{1+j} + 1 + \frac{1}{1-j} \right) = \hat{V}_2 \, \frac{1-j + (1+j)(1-j) + (1+j)}{(1+j)(1-j)}$$

$$(a+jb)(a-jb) = a^2+b^2 - abj + abj = a^2+b^2 = |a+jb|^2$$

$$(1+j)(1-j) = (\sqrt{2})^2 = 2$$

$$\rightarrow \quad 2 + \frac{6}{1+j} = \hat{V}_2 \frac{2+2}{2} = 2\hat{V}_2$$

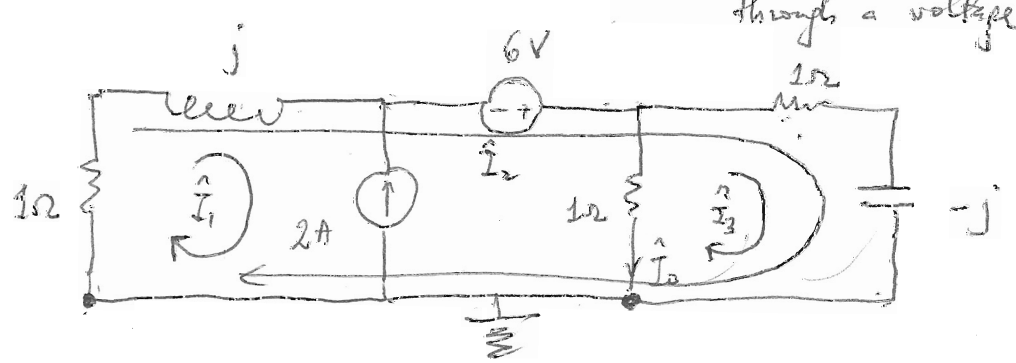
$$\hat{V}_2 = 1 + \frac{3}{1+j} = \frac{1+j+3}{1+j} = \frac{4+j}{1+j}$$

$$\hat{I}_0 = \frac{\hat{V}_2}{1\Omega} = \frac{4+j}{1+j} A = \frac{\sqrt{17}\, e^{j\tan^{-1}(\frac{1}{4})}}{\sqrt{2}\, e^{j45^\circ}} = 2.91^{j(-31^\circ)}$$

$$\rightarrow i_0(t) = 2.91 \cos(377t - 31^\circ) \ A.$$

(assume a standard AC power source at 60 Hz)

Loop Analysis: $\sum_i \hat{V}_i = 0$ $\begin{cases} \hat{V} \text{ negative across an impedance} \\ \hat{V} \text{ positive if going } - \text{ to } + \text{ through a voltage source} \end{cases}$



1) $\hat{I}_1 = -2A$ (since we choose our 3 loops as above)

2) $-(1+j)(\hat{I}_1 + \hat{I}_2) + 6 - (1-j)(\hat{I}_2 + \hat{I}_3) = 0$

3) $-1 \cdot \hat{I}_3 - (1-j)(\hat{I}_2 + \hat{I}_3) = 0$

Since $\hat{I}_0 = -\hat{I}_3$, I need to eliminate $\hat{I}_2$

3) $(-2+j)\hat{I}_3 = +(1-j)\hat{I}_2 \rightarrow \boxed{\hat{I}_2 = -\hat{I}_3 \frac{(2-j)}{1-j}}$

Plug this back to 2)

$$-(1+j)(-2+\hat{I}_2) + 6 - (1-j)\hat{I}_2 - (1-j)\hat{I}_3 = 0$$

$$\hat{I}_2\underbrace{(-1-j-1+j)} + 2(1+j) + 6 - (1-j)\hat{I}_3 = 0$$

$$-2\hat{I}_2 - (1-j)\hat{I}_3 + 8 + 2j = 0$$

$$\underbrace{\left[\frac{2(2-j)}{1-j} - (1-j)\right]}\hat{I}_3 + 8 + 2j = 0$$

$$\frac{4-\cancel{2j}-\cancel{1}+\cancel{2j}+\cancel{1}}{1-j}$$

$$\rightarrow \hat{I}_3 = -\frac{(8+2j)(1-j)}{4} = -\frac{10-6j}{4}$$

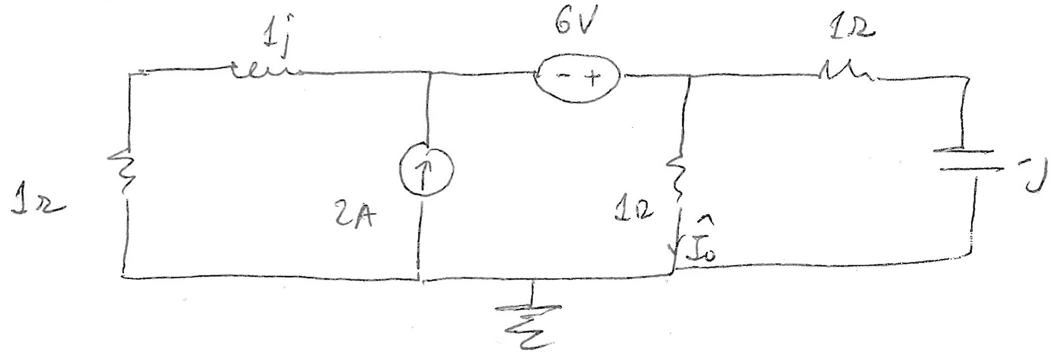
$$\hat{I}_0 = -\hat{I}_3 = \frac{10-6j}{4} = \frac{5-3j}{2} = \frac{\sqrt{34}}{2}e^{-j\tan^{-1}\left(\frac{3}{5}\right)}$$

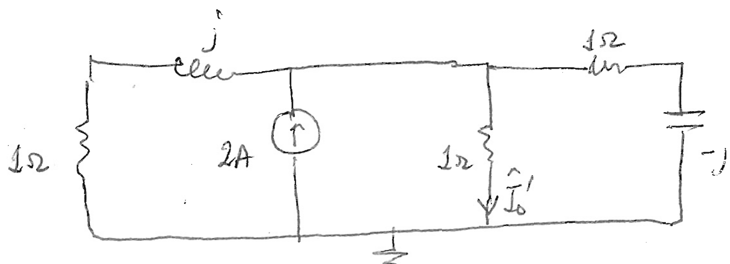
$$\boxed{= 2.91\, e^{j(-31^\circ)} \text{ A}}$$

4th quadrant!

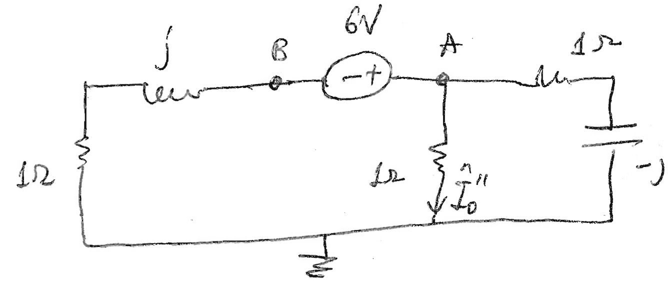
$$i_0(t) = 2.91\cos(377t - 31^\circ) \text{ A}$$

Superposition: (combining results from single sources)





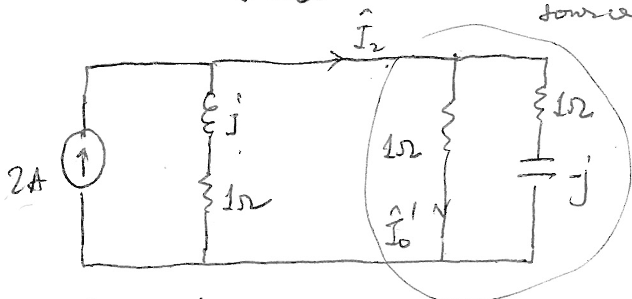
+



Superposition $\hat{I}_o' + \hat{I}_o'' = \hat{I}_o$

$\hat{I}_o'$: short-circuited voltage source

$\hat{I}_o''$: open-circuited the current source



Current division:

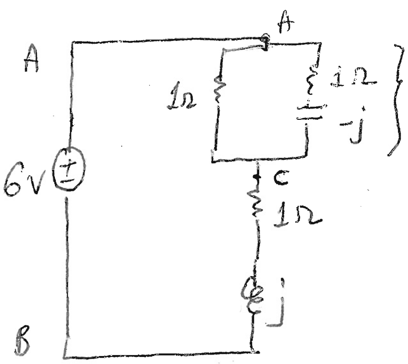
$$\hat{I}_2 = 2A \frac{1+j}{1+j + \frac{(1-j)\cdot 1}{2-j}}$$

2nd current division:

$$\hat{I}_o' = \frac{1-j}{2-j} \cdot 2A \frac{1+j}{1+j + \frac{1-j}{2-j}}$$

$$= \frac{4A}{(2-j)(1+j) + 1-j} = \frac{4A}{3+\cancel{j}+1-\cancel{j}}$$

$$= 1A$$



Voltage div.

$$\hat{V}_{AC} = 6V \frac{(1-j)\cdot 1}{2-j} \Big/ \left(\frac{1-j}{2-j} + 1 + j\right)$$

$$= \frac{(1-j)\,6V}{(2-j)(1+j) + (1-j)}$$

$$= \frac{6}{4}(1-j)$$

$$\hat{I}_o'' = \frac{\hat{V}_{AC}}{1\Omega} = \frac{3}{2}(1-j)\,A$$

$$\rightarrow \hat{I}_o = \hat{I}_o' + \hat{I}_o'' = 1A + \frac{3}{2}(1-j)A$$

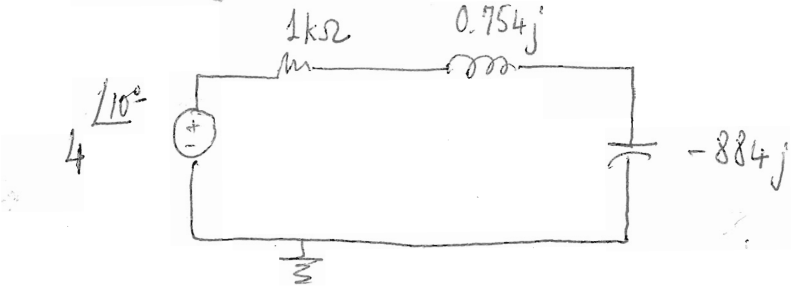
$$= \frac{5-3j}{2}A = 2.91 e^{j(-31°)} A$$

$\rightarrow i(t) = 2.91 \cos(200t - 31°)$ A

Frantzley Paul

Quiz #1 (5 min)

(20)



$f = 60\,Hz$.

Write this circuit back into time domain (for later implementation into PSPICE)

~~7.38~~ ~~$\hat{V}_s = 5.9\angle -23°$~~

~~7.21~~ ~~$v(t) = 5.2\cos(377t + 44.97°)$ V~~

$$V\cos\omega t = Ri + \frac{L\,di}{dt} + \frac{C\,dv}{dt}$$

$$4\angle 10° = 4\cos(377t + 10°)$$

$$R = 1\,K\Omega$$

$$Z = j\omega L = 0.754j = j\,377 \times L$$

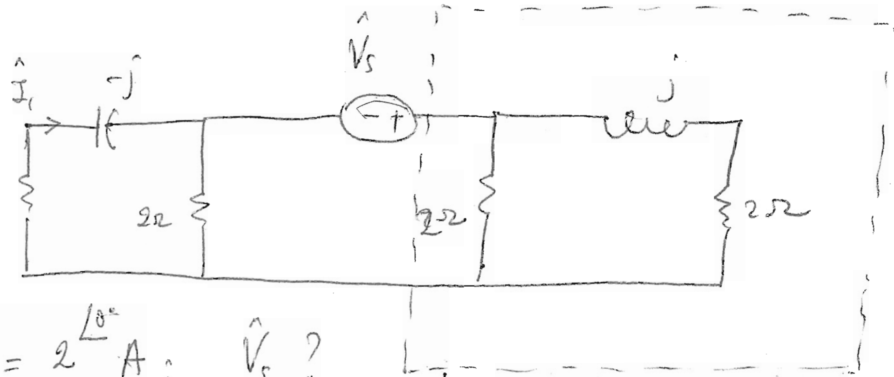
$$L = 0.002\,H = 2\,mH$$

$$C = \frac{1}{j\omega C} = -884j$$

$$\frac{1}{377C} = 884j$$

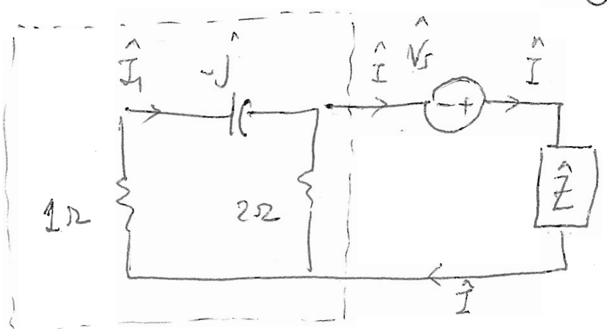
$$C = 3\,\mu F$$

7.38

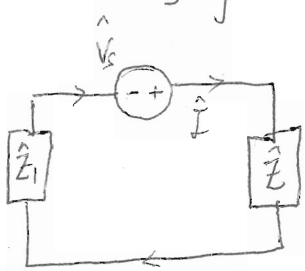


$\hat{I}_1 = 2\angle 0^\circ$ A; $\hat{V}_s$?

$\hat{Z} = 2 \,\|\, (2+j) = \frac{2(2+j)}{4+j}$



$\hat{Z}_1 = 2 \,\|\, (1-j) = \frac{2(1-j)}{3-j}$



$\rightarrow \hat{V}_s = \hat{I}(\hat{Z}+\hat{Z}_1)$

Current div. $\hat{I}_1 = \hat{I}\,\frac{2}{3-j} \rightarrow \hat{I} = \frac{\hat{I}_1(3-j)}{2}$

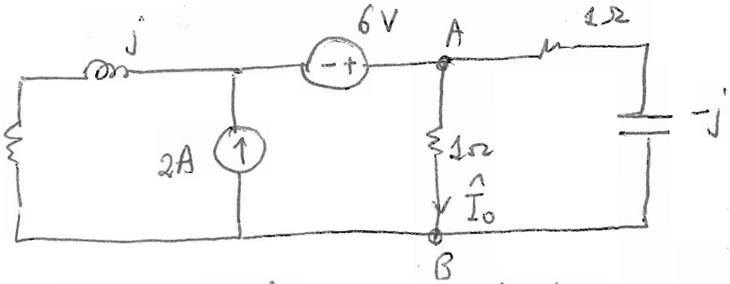
$$\rightarrow \hat{V}_s = \hat{I}_1\,\frac{3-j}{2}\left(\frac{2(2+j)}{4+j} + \frac{2(1-j)}{3-j}\right)$$

$$= \cancel{(3-j)}\;\frac{2(2+j)(3-j) + 2(4+j)(1-j)}{(4+j)\cancel{(3-j)}}$$

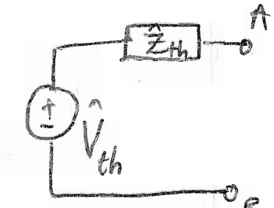
$$= 2\,\frac{(7+j) + (5-3j)}{4+j} = 2\,\frac{12-2j}{4+j} = 4\,\frac{6-j}{4+j}$$

$$= 4\,\frac{\sqrt{37}}{\sqrt{17}}\,e^{j(-\tan^{-1}\frac{1}{6} - \tan^{-1}\frac{1}{4})} = 5.9\angle -23.5^\circ \text{ V}$$

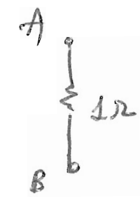
AC Analysis using phasors: Thevenin Equivalent:



Thevenin equivalent: replacing a circuit b/w A & B by



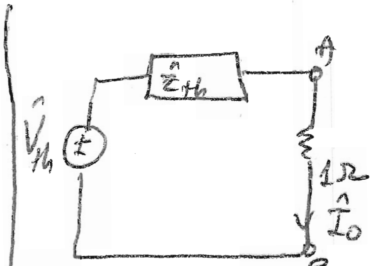
→ We will replace the circuit (without



) by a Thevenin equivalent

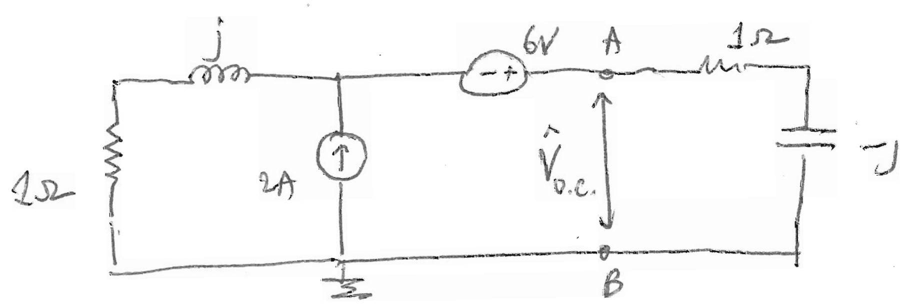
$$\begin{cases} \hat{V}_{th} \equiv \underbrace{\hat{V}_{o.c.}}_{\text{open circuit}} \text{ b/w A \& B} \\ \hat{Z}_{th} \equiv \hat{Z}_{\underbrace{\text{No sources}}} \text{ \& across A\&B} \end{cases}$$

↳ voltage source : short-circuit
current source : open-circuit

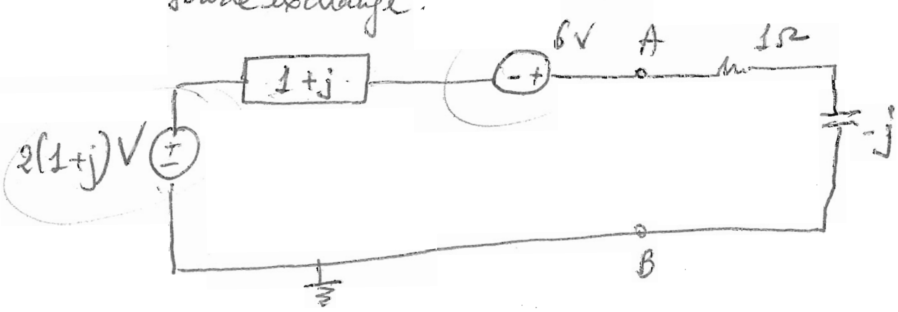


$$\hat{I}_o = \frac{\hat{V}_{th}}{\hat{Z}_{th} + 1}$$

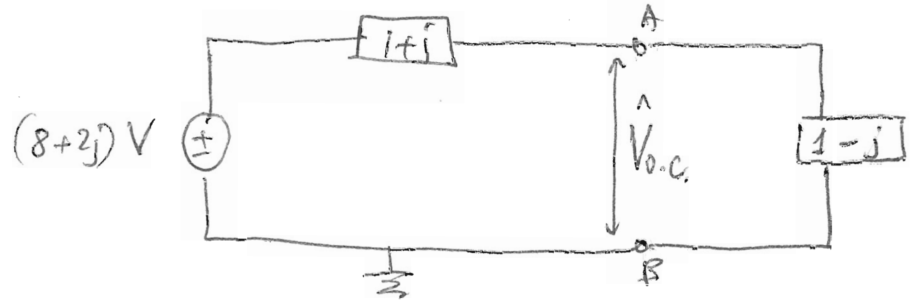
$\hat{V}_{th}$:



source exchange:

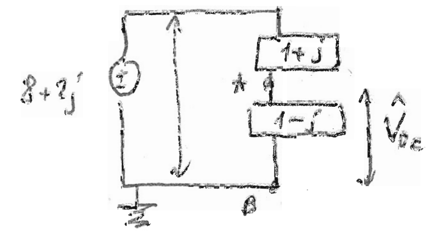


Combining the two voltage sources:



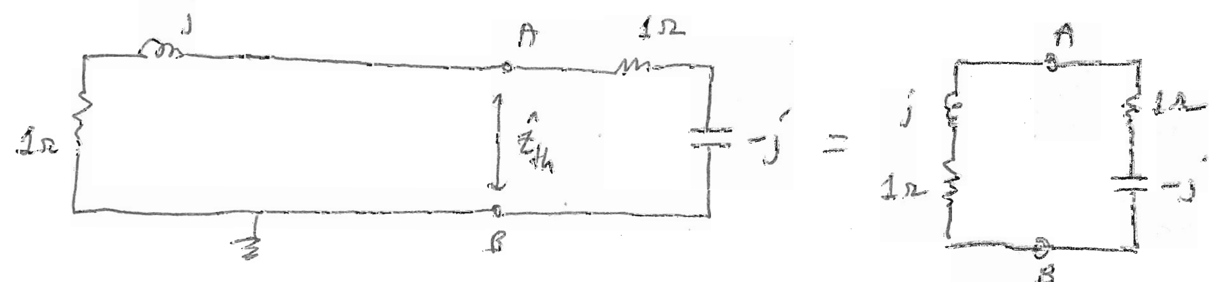
$$\hat{V}_{oc} = (8+2j)\frac{1-j}{1+\cancel{j}+1-\cancel{j}}$$

Voltage div.

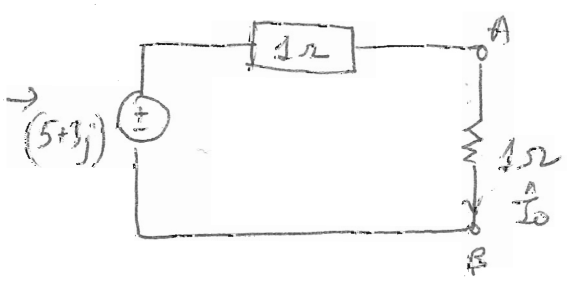


$$\hat{V}_{th} = \hat{V}_{oc} = (8+2j)\frac{(1-j)}{2} = \frac{10-6j}{2} = (5-3j)\ V$$

$\hat{Z}_{th}$:

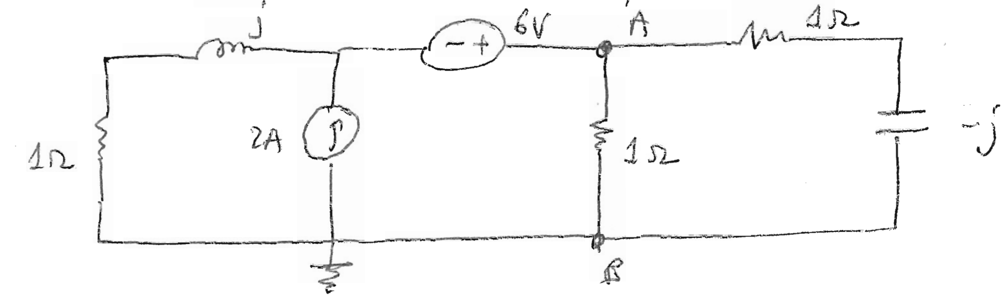


$$\hat{Z}_{th} = \frac{(1+j)\cdot(1-j)}{1+j+1-j} = \frac{2}{2} = 1\Omega$$

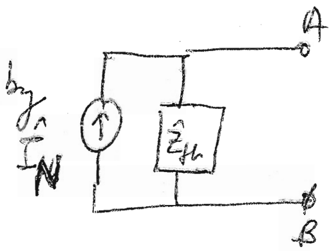


$$\hat{I}_o = \frac{5+3j}{2}\ A = 2.91\, e^{j(-31°)}\ A$$ ✓

AC Analysis using phasors: Norton Equivalent:



Norton equivalent: replace a circuit b/w A & B by



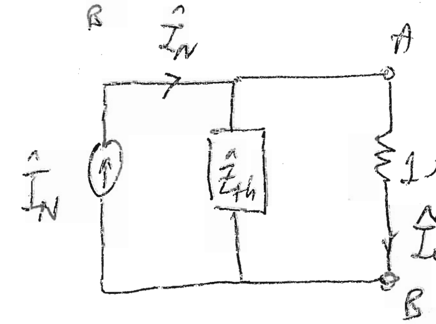
→ We will replace the circuit (without

) by a Norton equivalent

$$\hat{I}_N \equiv \hat{I}_{S.C.} \text{ b/w A \& B}$$

↳ short-circuit

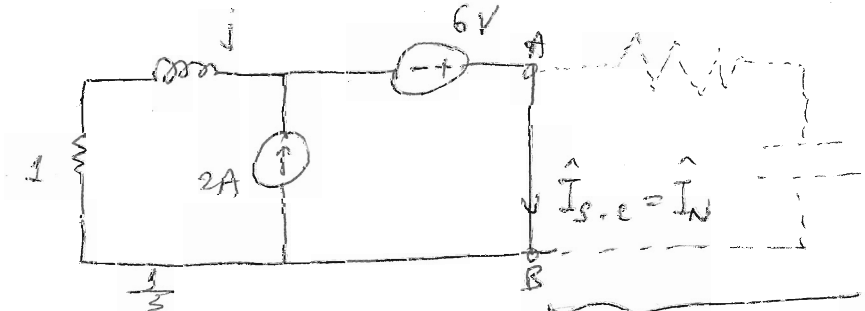
$$\hat{Z}_{th} = 1\,\Omega$$



Current division: $\hat{I}_o = \hat{I}_N \dfrac{\hat{Z}_{th}}{1\Omega + \hat{Z}_{th}}$

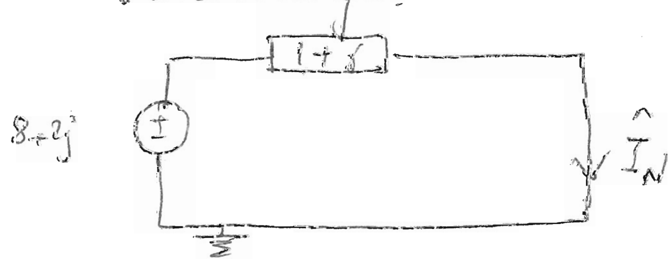
$= \hat{I}_N \dfrac{1}{2}$

$\hat{I}_N$:

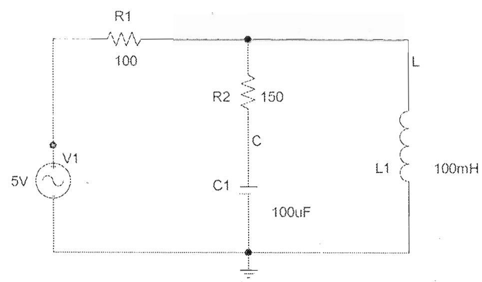


not part of circuit as A & B are short-circuited.

Source exchange:



$$\hat{I}_N = \frac{8+2j}{1+j}\ \text{A}$$



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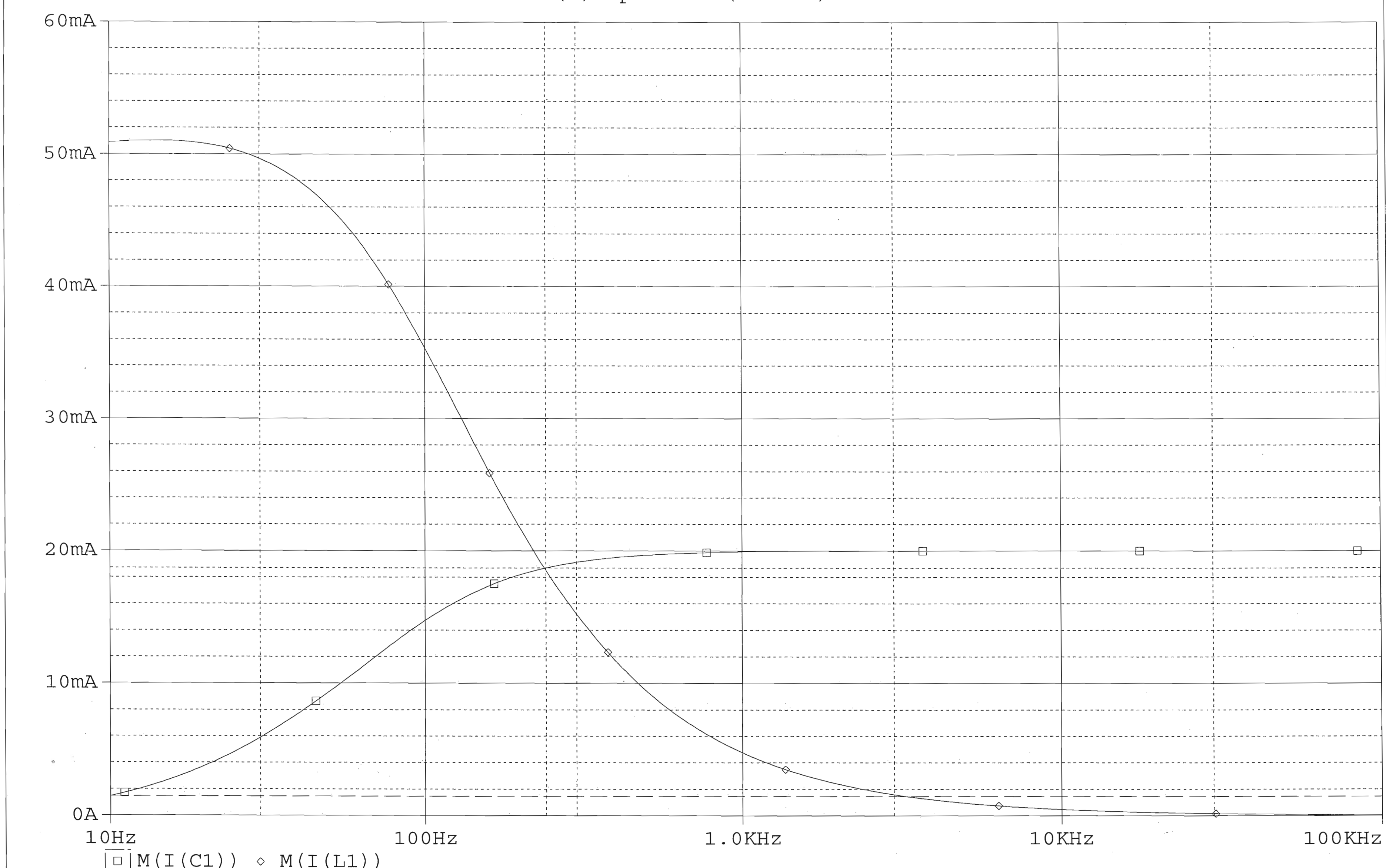
* C:\Temp\232s07\7p76.sch

Date/Time run: 02/20/07 13:57:23 Temperature: 27.0

(A) 7p76.dat (active)



A1:(241.481,18.703m) A2:(10.000,1.4620m) DIFF(A):(231.481,17.241m)

Hz

Date: February 20, 2007 Page 1 Time: 14:03:00

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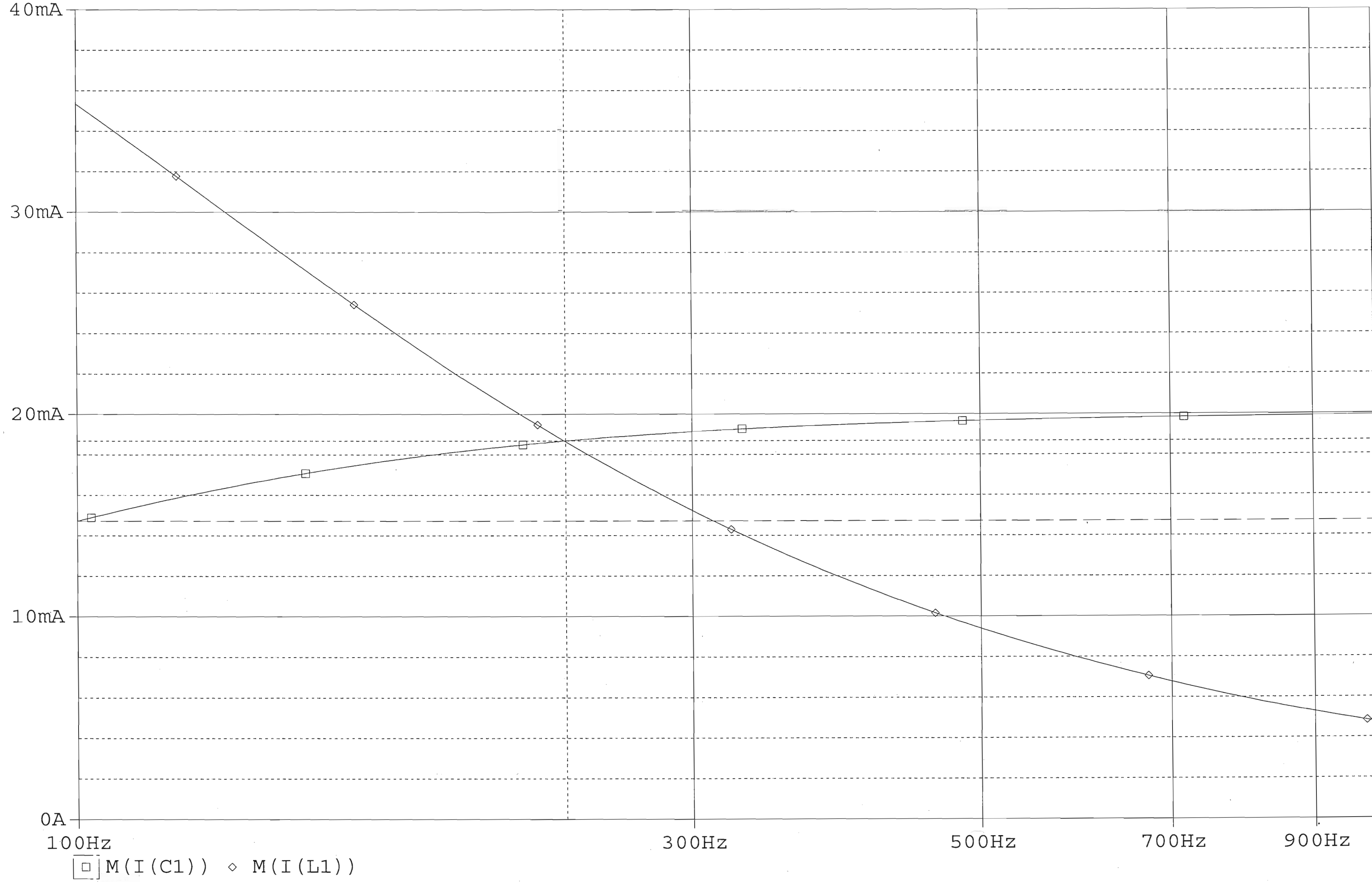
* C:\Temp\232s07\7p76.sch

Date/Time run: 02/20/07 14:07:00 Temperature: 27.0

(A) 7p76.dat (active)

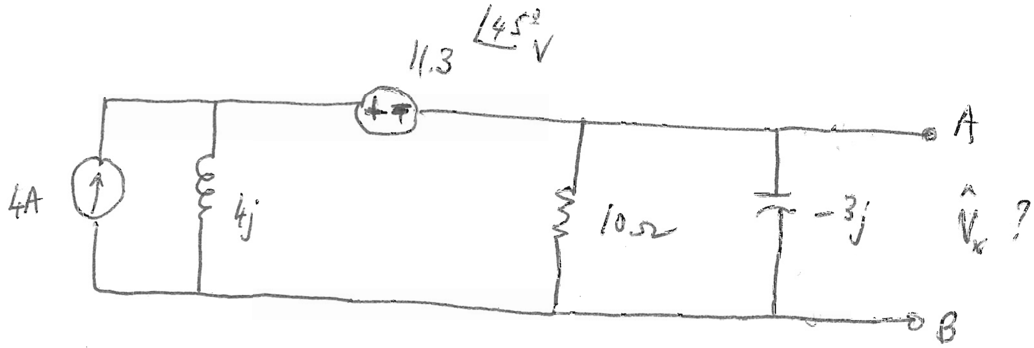


A1:(239.524,18.684m) A2:(100.000,14.724m) DIFF(A):(139.524,3.9603m)

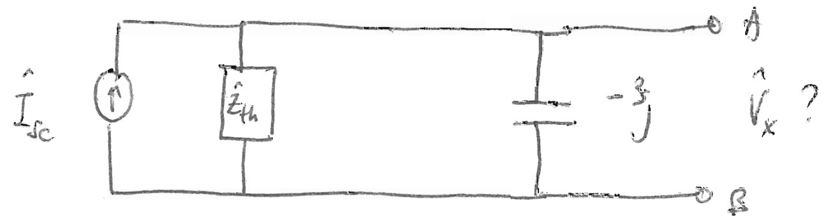
Hz

Date: February 20, 2007 Page 1 Time: 14:07:50

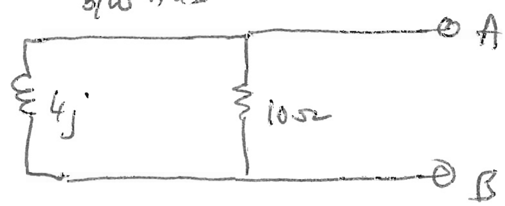
7.70



Norton's equivalent:

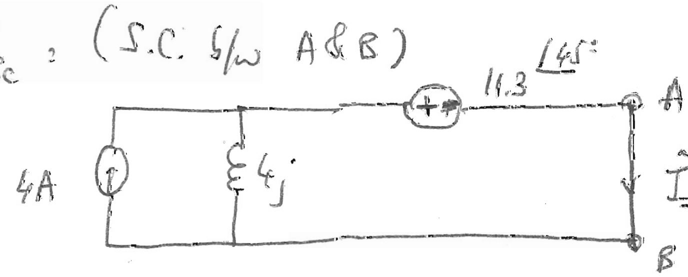


$\hat{Z}_{th}$: (OC element b/w A&B, OC I_{source}, SC V_{source}):

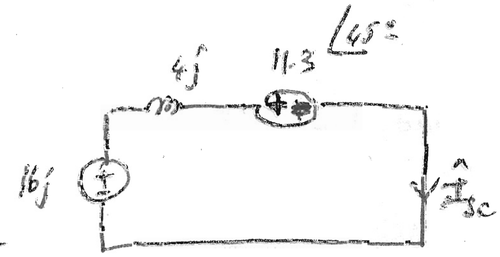


$$\rightarrow \hat{Z}_{th} = \frac{10 \times 4j}{10 + 4j}$$

$\hat{I}_{sc}$: (S.C. b/w A&B)



→ source exchange



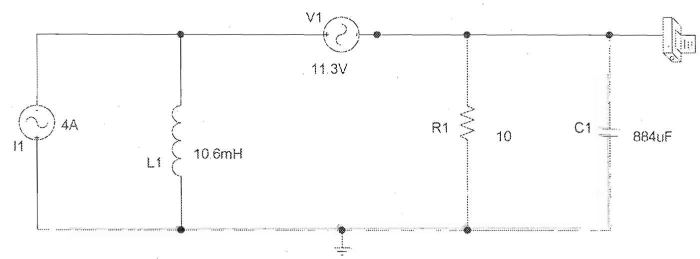
$$\hat{I}_{sc} = \frac{16j - 11.3(\cos 45 + j\sin 45)}{4j} = \frac{16j - 7.99 - j7.99}{4j}$$

$$= \frac{-7.99 + 8.01j}{4j} = \frac{11.3 \angle 135^\circ}{4 \angle 90^\circ} = 2.825 \angle 45^\circ \text{ A}$$

32

7.70 pspice

This is a current source pointing up



→ $\hat{V}_x = 21.7\angle 5.2^\circ$ V

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