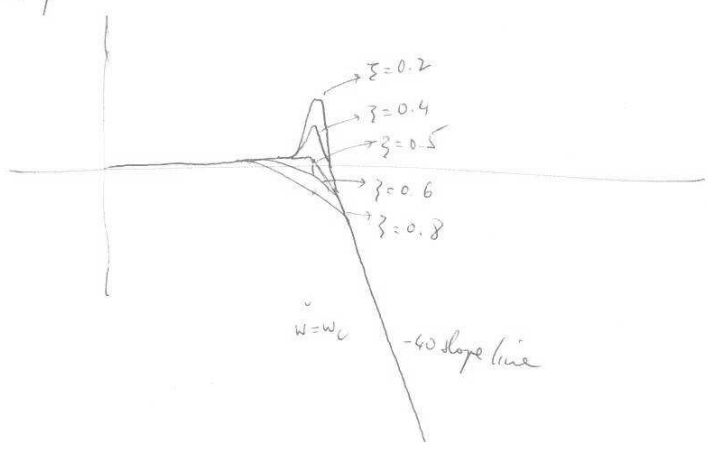
quadratic pole



<u>Extra credit:</u> 50 HW's points

Draw $20\log_{10}|\hat{H}|$ for $\hat{H}(jw) = \dfrac{25jw}{(jw+0.5)\left[(jw)^2 + 4jw + 100\right]}$

$$= \frac{0.5\,jw}{\left(1 + j\frac{w}{0.5}\right)\left(1 + 4j\frac{w}{100} + \left(j\frac{w}{10}\right)^2\right)}$$

$w_c = 0.5$ $\qquad$ $w_c = 10$

$2\zeta_s = \frac{4}{10} \rightarrow \zeta_s = 0.2$

(big bump)

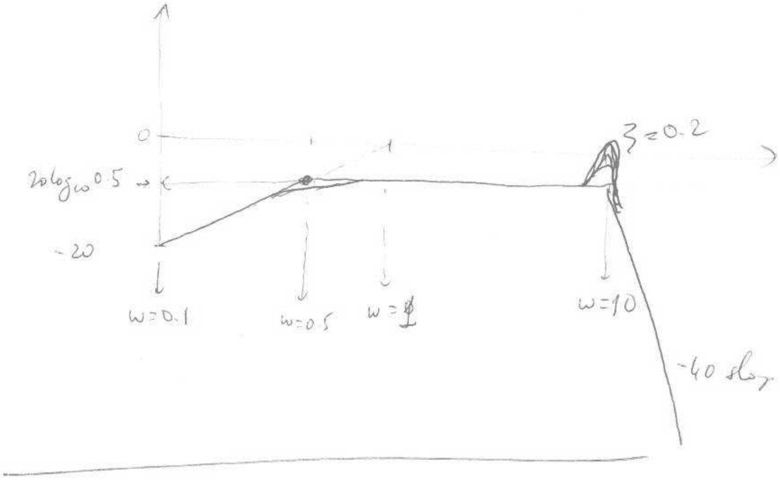
$w_c = 0.5\ ;\ 10$

→ Before 0.5 : a line up +20 slope by (jw) → $20\log_{10} 0.5$

→ B/w 0.5 & 10 : $\left(1 + j\frac{w}{0.5}\right)$ kicks in with -20 slope line

→ total flat.

→ After $w = 10$: quadratic kicks in with -40 slope.



Phase Bode Plot:

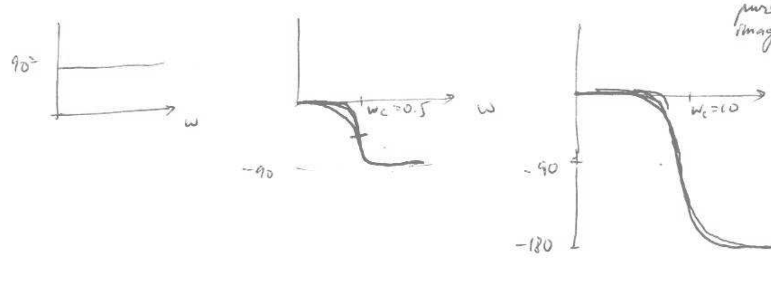
$$\hat{H}(j\omega) = \frac{0.5 j\omega}{\left(1 + j\frac{\omega}{0.5}\right)\left(1 + \frac{4}{10} j \frac{\omega}{10} + \left(j\frac{\omega}{10}\right)^2\right)}$$

$$\angle \hat{H}(j\omega) = \text{Phase}(0.5 j\omega) - \underbrace{\text{Phase}\left(1 + j\frac{\omega}{0.5}\right)}_{\text{asymptotic analysis}} - \underbrace{\text{Phase}\left(1 + \frac{4}{10} j \frac{\omega}{10} + \left(j\frac{\omega}{10}\right)^2\right)}_{\text{asymp. analysis}}$$

$$= 90^\circ$$

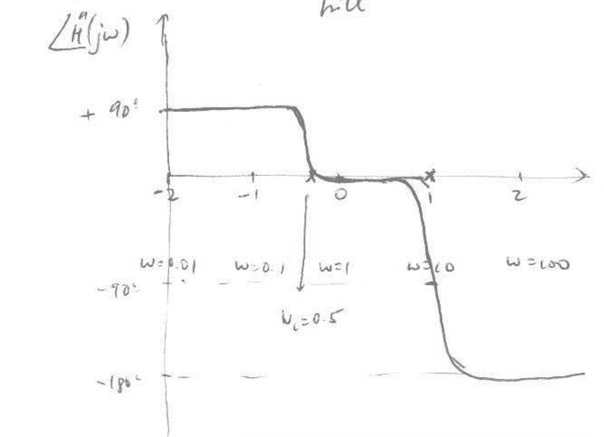
- large $\omega \rightarrow 90^\circ$
- small $\omega \rightarrow 0^\circ$
- critical $\omega_c = 0.5 \rightarrow 45^\circ$

- large ω: 180°
- small ω: 0°
- $\omega_c = 10 \rightarrow \text{Phase}(\underbrace{0.4j}_{\text{pure imaginary}}) =$



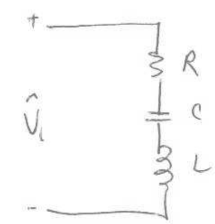
<u>Combining them together:</u> by pieces of ω

- Before $\omega_c = 0.5$: just from numerator : 90°
- B/w $\omega_c = 0.5$ & $\omega_c = 10$: now also from 1st order pole : total phase going down from 90° to 0°
- After $\omega_c = 10$: quad. pole contributes another 180° down hill



<u>Resonant Frequency:</u> ω such that $\hat{Z}$ is pure resistive or $\theta = 0$ or $pf = \cos\theta = 1$

E.g:



$$\hat{Z}(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

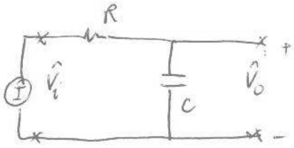
$$\omega_0 L = \frac{1}{\omega_0 C} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

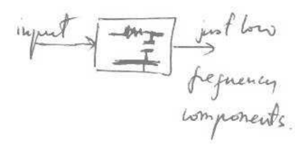
"Quality factor": $Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

Filter Networks:

- Passive filters: using R, L, C (no gain for a reasonable bandwidth)
  ↓ just select a range of frequency
- Active filters: Op-Amps (gain higher than 1)
  ↓ selects and enhance a range of frequency

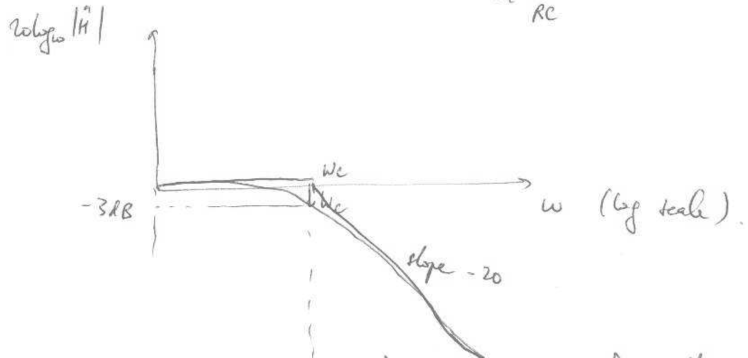
Passive - Low pass:



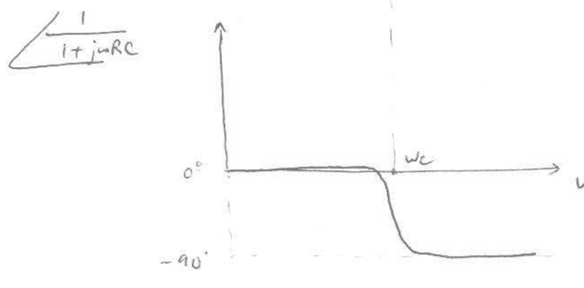


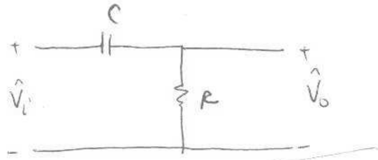
$$\hat{H} = \frac{\hat{V}_o}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$w_c = \frac{1}{RC}$$



This RC arrangement passes components with $\omega < \omega_c$, and blocks components with $\omega > \omega_c$, i.e. it is a Low-pass filter.



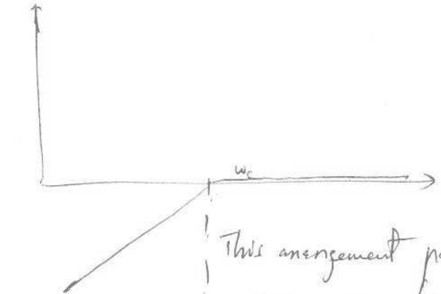


$$\hat{H} = \frac{\hat{V}_o}{\hat{V}_i} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

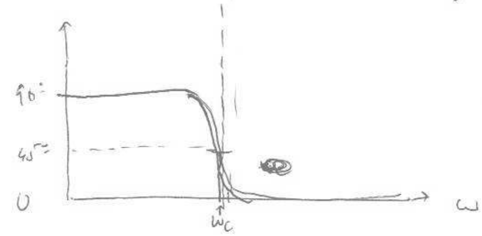
$$\omega_c = \frac{1}{RC}$$

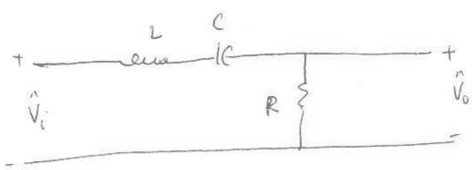
→ Before $\omega_c = \frac{1}{RC}$: line goes up with slope +20 (from numerator)

→ ~~Afte~~ At $\omega_c \frac{1}{RC}$: pole contributes a line of slope −20 → total of flat behavior vs freq.



This arrangement passes $\omega > \omega_c$, blocks $\omega < \omega_c$ → high-pass filter!





$$\rightarrow \hat{H} = \frac{\hat{V}_o}{\hat{V}_i} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Can reduce to standard forms and make Bode Plot to determine filter behavior or use $\hat{H} = \frac{RC\omega}{RC\omega + j(LC\omega^2 - 1)}$

$$\hat{H} = \frac{RC\omega}{RC\omega + j\,(LC\omega^2 - 1)} \quad \begin{cases} \text{Magnitude} : \dfrac{RC\omega}{\sqrt{(RC\omega)^2 + (LC\omega^2-1)^2}} \quad \left\{ \text{Small } \omega \text{ or } \omega \ll \dfrac{1}{\sqrt{LC}} \right. \\[2ex] \text{Phase} : -\tan^{-1}\left(\dfrac{LC\omega^2 - 1}{RC\omega}\right) \end{cases}$$

Small ω or $\omega \ll \frac{1}{\sqrt{LC}} \rightarrow \omega^2 \ll \frac{1}{LC} \rightarrow LC\omega^2 \ll 1 \rightarrow$ Magn: $\frac{RC\omega}{\sqrt{(RC\omega)^2 + 1}} \simeq RC\omega \rightarrow 0$

large ω or $\omega \gg \frac{1}{\sqrt{LC}} \rightarrow \omega^2 \gg \frac{1}{LC} \rightarrow LC\omega^2 \gg 1$

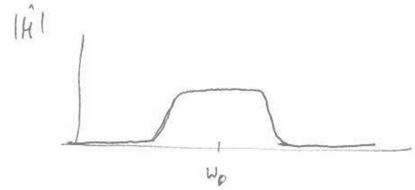
$\hookrightarrow$ Magn $\simeq \frac{RC\omega}{\sqrt{(LC\omega^2)^2}}$

$= \frac{RC\omega}{LC\omega^2} = \frac{R}{L}\frac{1}{\omega}$

$\rightarrow 0$

$\omega_c = \frac{1}{\sqrt{LC}} \rightarrow$ magn $\frac{RC\omega}{\sqrt{(RC\omega)^2}} = 1$

(Resonant freq.)

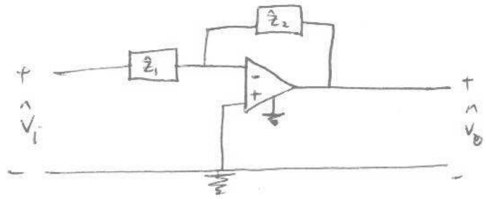


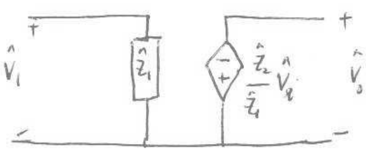
Band-pass filter
(L, C, R)

$\rightarrow$ Question (5 exam points) : can we make a band-pass filter with only RC elements? Give concrete values for R's and C's for this to happen.

Active filters. Op. Amp.

Inverting





Non-inverting