Extra credit: 50 Hn's points

Draw 20 log|H(jw)| for 

\[
\begin{align*}
H(jw) &= \frac{25jw}{(jw + 0.5) \left( (jw)^2 + 4jw + 150 \right)} \\
&= \frac{0.5jw}{(1 + jw/0.5)(1 + 4jw/100 + (jw/10)^2)} \\
&= 0.5jw \\
&= \frac{0.5jw}{w_0} \\
&= 0.5 \\
&= 23.6 = \frac{1}{10} \\
&= \frac{1}{10} = 0.2 \\
&= \text{by bump}
\end{align*}
\]

\(w_0 = \frac{1}{10}\)

\(w = \frac{1}{10}\)

Before 0.5: a line up +20 slope by \((jw)\) \(\rightarrow 20\log H 0.5\)

\(\frac{1}{10} \approx 0.5\) \(\rightarrow \) hand in \(\rightarrow 20\log 0.5\) \(\rightarrow\) total flat

\(\frac{1}{10} \approx 0.5\) \(\rightarrow \) quadratic break \(\rightarrow 20\log 0.5\) \(\rightarrow\) flat
Phase Bode Plot:

\[ H(j\omega) = \frac{0.5j\omega}{(1 + j\frac{\omega}{0.5})(1 + \frac{0.2}{10}\frac{j\omega}{10} + (j\frac{\omega}{10})^2)} \]

\[ \angle H(j\omega) = \text{Phase} (0.5j\omega) - \text{Phase} \left( 1 + j\frac{\omega}{0.5} \right) - \text{Phase} \left( 1 + \frac{0.2}{10}\frac{j\omega}{10} + (j\frac{\omega}{10})^2 \right) \]

- Large \( \omega \rightarrow 90^\circ \)
- Small \( \omega \rightarrow 0^\circ \)
- Critical \( \omega_c = \pm 5 \rightarrow 45^\circ \)

\[ H(\omega) \rightarrow 180^\circ \text{ for } \omega = 10 \]
Combining them together:

\[ \text{by piece of } w \]

- Before \( \omega_c = 0.5 \): just from numerator: 90°
- \( \frac{1}{\omega_c} \), \( \omega_c = 0.5 \) & \( \omega_c = 10 \): now also from 1st order pole - total phase going down from 90° to 0°
- After \( \omega_c = 10 \): 2nd pole contributes another 180° down

\[ H(j\omega) \]

\[ w \text{ (log scale)} \]

Resonant Frequency: \( w \) such that \( Z \) is pure resistive

\[ \Theta = 0 \Rightarrow \frac{H}{W} = \omega \Theta = \omega \]

\[ E_\text{in} = R + j\omega L + \frac{1}{j\omega C} \]

\[ V_\text{out} = R + j(\omega L - \frac{1}{\omega C}) \]

\[ w_0 L = \frac{1}{w_0 C} \Rightarrow w_0 = \frac{1}{\sqrt{LC}} \]

"Quality factor": \( Q = \frac{w_0 L}{R} = \frac{1}{\sqrt{LC}} \)
Filter Networks:

- Passive filters: using R, L, C (no gain for a reasonable bandwidth)
  - just select a range of frequency
- Active filters: Op. Amps (gain higher than 1)
  - works and enhances a range of frequency

Passive - Low pass:

\[ H = \frac{V_o}{V_i} = \frac{1}{j\omega C} = \frac{1}{1 + j\omega RC} \]

\[ \omega_c = \frac{1}{RC} \]

This RC arrangement passes components with \( \omega < \omega_c \)
and blocks components with \( \omega > \omega_c \), i.e., it's a low-pass filter.
\[ H = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega RC} \]

Before \( \omega_c = \frac{1}{RC} \): The gain is up with slope +20 (from numerator).

At \( \omega_c = \frac{1}{RC} \): A pole contributes a line of slope -20 to total of flat behavior as freq.

\[ |H| \]

\[ L(\omega) \]

This arrangement makes \( \omega < \omega_c \) block, \( \omega > \omega_c \) high-pass filter.

\[ H = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}} \]

Can reduce to standard form and make Bode plot to determine filter behavior or use \( H = \frac{RC\omega}{RC\omega + j\left(4\pi^2\omega^2 - 1\right)} \)
\[ h = \frac{R C \omega}{R C \omega + j (L C \omega^2 - 1)} \]

\[ \text{Magnitude: } \frac{R C \omega}{\sqrt{(R C \omega)^2 + (L C \omega^2 - 1)^2}} \]

\[ \text{Phase: } -\tan^{-1}\left(\frac{(L C \omega^2 - 1)}{R C \omega}\right) \]

Small \( \omega \) or \( \omega < \frac{1}{V L C} \rightarrow \omega < \frac{1}{L C} \rightarrow L C \omega \ll 1 \rightarrow \text{ Magnitude: } \frac{R C \omega}{\sqrt{(R C \omega)^2 + 1}} \]

Large \( \omega \) or \( \omega > \frac{1}{V L C} \rightarrow \omega^2 \gg \frac{1}{L C} \rightarrow L C \omega^2 \gg 1 \rightarrow \text{ Magnitude: } \frac{R C \omega}{\sqrt{(L C \omega)^2}} \]

\[ \text{Resonant freq: } \frac{1}{V L} \]

Band-pass filter

\[ Band-pass \ filter \ (L, C, R) \]

Question (5 exam points): can we make a band-pass filter with only RC elements? Give concrete values for \( R \)'s and \( C \)'s for this to happen.

Inverting

Non-inverting

\[ V_i \]

\[ V_o \]

\[ (1 + \frac{R_2}{R_1})V_i \]