Variable Frequency Network Performance:

\[ i_R = R \]
\[ i_L = j\omega L \]

\[ \frac{i_c}{j\omega C} \]

Find the transfer function for:

\[ \frac{\hat{v}_0}{\hat{v}} \]

\[ \text{j}\omega \rightarrow s \]

\[
\begin{align*}
\[
\end{align*}
\]

Say it has a pole of what order? (pole = zero of denominator)

By voltage division:

\[ \frac{\hat{v}_0}{\hat{v}} = \frac{R}{\frac{1}{j\omega C} + \frac{1}{j\omega L} + \frac{1}{R}} \]

\[ \frac{R}{sLs + \frac{1}{sC} + \frac{1}{sR}} \]

Denote the denominator:

\[ s^2 + \frac{R}{L} + \frac{1}{LC} = 0 \]

\[ s = \frac{-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}{2} \]
\[ S_t = -\frac{R}{2L} \pm \frac{1}{2L} \sqrt{\frac{R^2C - 4L}{LC}} = -\frac{R}{2L} \pm \frac{1}{2L} \sqrt{\frac{R^2C - 4L}{C}} \]

- If \( R^2C - 4L = 0 \) or \( \frac{R^2C}{L} = 4 \) \( \Rightarrow S_t = -\frac{R}{2L} \)

Then \( S^2 + s \frac{R}{L} + \frac{1}{LC} = \left( s + \frac{R}{2L} \right)^2 \) \( \rightarrow \) Transfer function has a pole of 2\(^{nd}\) order

- If \( R^2C - 4L > 0 \) \( \Rightarrow S_+ \)

\[ S_+^2 + s \frac{R}{L} + \frac{1}{LC} = \left( s + \frac{R}{2L} - \frac{1}{2L} \sqrt{\frac{R^2C - 4L}{C}} \right) \left( s + \frac{R}{2L} + \frac{1}{2L} \sqrt{\frac{R^2C - 4L}{C}} \right) \]

\( \rightarrow \) Transfer function has two poles of 1\(^{st}\) order

- If \( R^2C - 4L < 0 \) \( \Rightarrow S_\pm = -\frac{R}{2L} \pm \frac{1}{2L} \sqrt{4L - R^2C} \)

\[ S_\pm^2 + s \frac{R}{L} + \frac{1}{LC} = \left( s + \frac{R}{2L} - \frac{1}{2L} \sqrt{\frac{4L - R^2C}{C}} \right) \left( s + \frac{R}{2L} + \frac{1}{2L} \sqrt{\frac{4L - R^2C}{C}} \right) \]

\( \rightarrow \) Transfer function has a pair of complex conjugate poles

We will learn about frequency variation in connection with the type of poles of the transfer function.
Transfer function: \( \hat{H}(j\omega) \) or \( \hat{H}(s) \) (e.g. \( \hat{H}(j\omega) = \frac{\hat{V}_o(j\omega)}{\hat{V}(j\omega)} \))

Bode plots:
- Magnitude: \( 20 \log |\hat{H}(j\omega)| \) versus \( \log \omega \)
- Phase: \( \angle \hat{H}(j\omega) \) versus \( \log \omega \)

We now learn how to make Bode plots from a transfer function. (two-way connection). Let's start with a standard form of \( \hat{H} \):

\[
\hat{H}(j\omega) = \frac{K_0 (j\omega)^N}{(1+j\omega \tau_1)(1+j\omega \tau_2)(1+j\omega \tau_3)(1+j\omega \tau_4)}
\]

(\( \tau_1 \) “tau sub one”)
(\( \tau_3 \) “tau sub three”)

From our example:

\[
\hat{H} = \frac{\hat{V}_o}{\hat{V}} = \frac{RCj\omega}{1 + (j\omega)^2LC + j\omega RC}
\]

\[
\begin{cases}
R \cdot C \cdot N = 0; \\
\tau_1, \tau_2, \tau_3, \tau_4: \text{none} \\
\tau_1 < \text{none} \\
\tau_2 = LC \Rightarrow \tau_3 = \sqrt{LC} \\
2\tau_3 \tau_4 = RC \\
\frac{1}{\tau_3} = \frac{RC}{2\tau_3} \Rightarrow \frac{1}{\tau_3} = \frac{RC}{2\tau_3}
\end{cases}
\]

Review:

\[
\left| \frac{\hat{V}_o}{\hat{V}} \right| = \frac{\sqrt{2 \cdot 2 \cdot \hat{V}_o^2 \hat{V}^2}}{\hat{V}} = \sqrt{2 \cdot \hat{V}_o^2 \hat{V}^2}
\]

1) \[
\log_{10} \left| \frac{\hat{V}_o}{\hat{V}} \right| = \log_{10} \left| \frac{\hat{V}_o}{\hat{V}} \right| + \log_{10} 1 = \log_{10} |\hat{V}_o| - \log_{10} |\hat{V}|
\]

2) \[
\log_{10} \left| \frac{\hat{V}_o}{\hat{V}} \right| = \log_{10} \frac{1}{|\hat{V}_o|} = \log_{10} \frac{1}{|\hat{V}|}
\]
Magnitude Bode Plot

\[ 20 \log_{10} |H(w)| = \begin{cases} \frac{1}{20} \log_{10} \left| \frac{\omega}{\omega_0} \right|^2 + \frac{1}{20} \log_{10} \left| 1 + j\omega \right|^2 + \frac{1}{20} \log_{10} \left| 1 + 2j\omega \right|^2 + \frac{1}{20} \log_{10} \left| \left( j\omega \right)^2 + 1 \right| - \cdots \end{cases} \]

Let's see how to plot different terms:

1) \[ 20 \log_{10} |\omega_0| \] (DC)

\[ \begin{array}{c|c}
\omega & \log_{10} \omega \\
\hline
0 & 100 \\
\end{array} \]

2) \[ \pm 20 \log_{10} \omega \] (FS)

3) Asymptotic analysis:
   \[ 20 \log_{10} \left| 1 + j\omega \right| \]
   \[ w \to 0 \] (low freq): sufficiently low such that \( j\omega \ll 1 \)
   \[ 20 \log_{10} \left| 1 + j\omega \right| \approx 0 \]
   \[ w \to \infty \] (high freq): sufficiently high such that \( j\omega \gg 1 \)
   \[ 20 \log_{10} \left| 1 + j\omega \right| \approx 20 \log_{10} \omega \approx 20 \log_{10} \frac{\omega}{\omega_0} \approx 2 \log_{10} \omega \]
   \[ j\omega \approx \omega \] or \[ \omega_0 \approx \frac{1}{2} \] \[ 20 \log_{10} \left| 1 + j\omega \right| \approx 3 \text{ dB} \]
\[ \log_{10} \left(1 + jw \epsilon_1 \right) = \begin{cases} w = 0 & 0 \\ w = \frac{1}{\epsilon_2} \epsilon_0 & 3 \text{ dB} \\ w = \infty \end{cases} \]

When \( \epsilon_2 < 1 \)

This term is similar to 2) except for the negative slope:

\[ y = -20x - 20 \log_{10} \frac{1}{w_0} \]
\[ H(jw) = \frac{100 \cdot jw}{(1+jw)(10+jw)(50+jw)} \]

To apply previous results, we put all factors in standard format:

\[ H(jw) = \frac{\frac{100}{500} \cdot jw}{(1+jw)(1+j\frac{w}{10})(1+j\frac{w}{50})} \]

\[ 20 \log_{10} |H| = 20 \log_{10} 0.2 + 10 \log_{10} w - 20 \log_{10} (1 + j\frac{w}{10}) - 20 \log_{10} (1 + j\frac{w}{50}) \]

\[ y = 20 \log_{10} w + 20 \log_{10} 0.2 \]

This line does not stop at crossing with horizontal axis as with the asymptotic analysis. We have 2 terms and valid for any \( f \).

\[ \log_{10} w = x \]
when adding: do by pieces of
repeating intervals
In summary, we are plotting the magnitude bode plot for

\[ |H(j\omega)| = \frac{\omega}{(1+j\omega)(1+j\frac{\omega}{\omega_c})} \]

The numerator gives a line up of slope +20 at all \( \omega \) (there is no asymptotic analysis on this term). The critical \( \omega \) are 1, 10, 50 before \( \omega = 1 \) only contribution from the +20 slope line that ends at \( \log_{10} 0 = -2 \text{ dB} \).

- At \( \omega = 1 \) term \( 1+j\omega \) provides a -20 slope line (from asymptotic analysis of large \( \omega \) ), this combine with the +20 slope line to give a horizontal piece. No contribution from \( (1+j\frac{\omega}{\omega_c}) \), because they are 0 to the left of their critical points (from asymptotic analysis of small \( \omega \) ). This continue until \( \omega_c = 10 \).
- At \( \omega_c = 10 \) \( 1+j\omega \) turns down as a -20 slope line while \( (1+j\frac{\omega}{\omega_c}) \) continues the same behavior as before. This gives total behavior of a -20 slope line until \( \omega_c = 50 \), since we are \( \frac{\omega}{\omega_c} \) 50 & 50, i.e. to the left of the \( (1+j\frac{\omega}{\omega_c}) \)'s critical point, it has zero contribution.
- At \( \omega_c = 50 \) \( 1+j\omega \) kicks in with a -20 slope line that gives a total behavior: -40 slope line.
Bode Plot with graphed poles and zeros.

\[ \frac{\text{expr}}{1 + j\omega_3 \tau_3} \times \text{asymptotic analysis} \]

1. Small \( \omega \), \( \omega \tau < 1 \rightarrow \)
\[ -20 \log_{10} |1 - j\omega_0 \tau_3| = 0 \]

2. Large \( \omega \), \( \omega \gg 1 \rightarrow \)
\[ -20 \log_{10} |1 - j\omega \tau_3^2| = -20 \log_{10} \left( \frac{\omega}{\tau_3} \right)^2 \]
\[ = -20 \log_{10} \omega - 40 \log_{10} \tau_3^2 \]
\[ = -40 \log_{10} \omega - 40 \log_{10} \tau_3^2 \]

3. \( \omega \tau_3 \ll 1 \) or \( \omega \tau_3 \gg 1 \)
\[ -20 \log_{10} |1 - j\omega \tau_3| = -20 \log_{10} \left( \frac{\omega}{2\tau_3} \right) \]

\[
\begin{array}{c|c}
\tau_3 & -20 \log_{10} \left( \frac{\omega}{2\tau_3} \right) \\
0.2 & 9.96 \\
0.4 & 9.94 \\
0.5 & 0 \text{ (no real subject to curve)} \\
0.6 & -1.53 \\
0.8 & -4.1 \\
\end{array}
\]

Lower is now controlled by \( \tau_3 \), not a constant of
\[ -3.16 \text{ any more.} \]