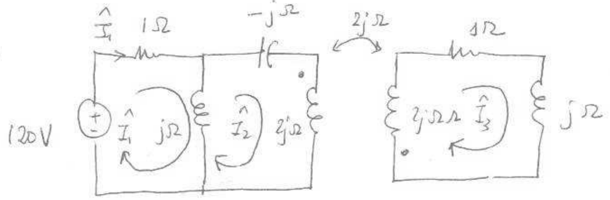
(31)

8.36



Impedance seen by voltage source: $\hat{Z} = \frac{120}{\hat{I}_1}$

Find $\hat{I}_1$: loop analysis.

1) $120 - \hat{I}_1 - (\hat{I}_1 - \hat{I}_2)j = 0 \quad \rightarrow 120 - \hat{I}_1(1+j) + \hat{I}_2 j = 0$

2) $(\hat{I}_2 - \hat{I}_1)j - j\hat{I}_2 + 2j\hat{I}_2 + 2j\hat{I}_3 = 0 \rightarrow -j\hat{I}_1 + 2j\hat{I}_2 + 2j\hat{I}_3 = 0$

3) $\hat{I}_3 2j + 2j\hat{I}_2 + \hat{I}_3(1+j) = 0 \rightarrow 2j\hat{I}_2 + \hat{I}_3(1+3j) = 0$

Need $\hat{I}_1 \rightarrow$ eliminate $\hat{I}_2$ & $\hat{I}_3$: 3) − 2) $\rightarrow \quad j\hat{I}_1 + \hat{I}_3(1+j) = 0$

$$\rightarrow \hat{I}_3 = -\frac{j}{1+j}\hat{I}_1$$

Back in 3) $\quad 2j\hat{I}_2 - \frac{j(1+3j)}{1+j}\hat{I}_1 = 0$

$$\hat{I}_2 = \frac{\cancel{j}(1+3j)}{(1+j)2\cancel{j}}\hat{I}_1$$

Back in 1) $\quad 120 - \hat{I}_1(1+j) + \frac{j(1+3j)}{2(1+j)}\hat{I}_1 = 0$

$$\hat{I}_1 = \frac{120}{1+j - \frac{-3+j}{2(1+j)}}$$

$$\Rightarrow \hat{Z}_{source} = \frac{120}{\hat{I}_1} = 1+j + \frac{3-j}{2(1+j)} = \frac{4j + 3 - j}{2(1+j)} = \boxed{\frac{3}{2}\Omega}$$

Complex Power: $\hat{S} \equiv \hat{V}_{RMS} \cdot \hat{I}^*_{RMS}$

complex # — complex # — complex #

$$= V_{RMS} e^{j\theta_v} \cdot I_{RMS} e^{-j\theta_I} = V_{RMS} I_{RMS} e^{j(\theta_v - \theta_I)}$$

this sign comes from $I^{(*)}_{RMS}$

can be related to average power P

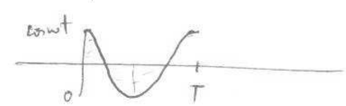
Av. Power: $P = \frac{1}{2} I_M V_M \cos(\theta_v - \theta_i) = Re(\hat{S})$ ✓

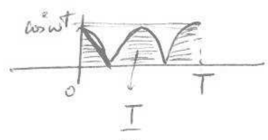
RMS: Root Mean Square

$$I_{RMS} = \sqrt{\frac{1}{T}\int_T i^2(t)\,dt} = I_M \sqrt{\underbrace{\frac{1}{T}\int_T \cos^2(\omega t + \theta_i)\,dt}_{\text{can choose start point so } \theta_i = 0}} = I_M \sqrt{\frac{1}{T}\frac{T}{2}} = \frac{I_M}{\sqrt{2}}$$

$i(t) = I_M \cos(\omega t + \theta_i)$

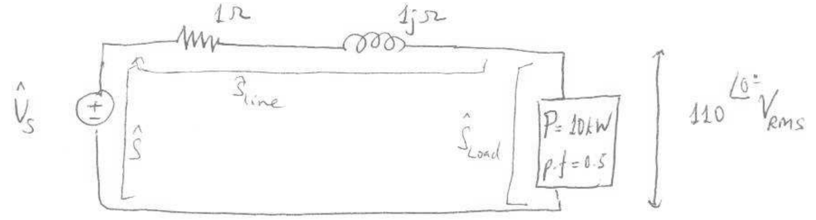
$$\frac{1}{T}\int_T \cos^2 \omega t \, dt$$





$$I_{RMS} V_{RMS} = \frac{I_M}{\sqrt{2}} \frac{V_M}{\sqrt{2}} = \frac{I_M V_M}{2}$$

Application of Complex Power:



→ What is the p.f. at the source?

p.f. = power factor $\equiv \cos(\theta_v - \theta_i)$

$P = \frac{1}{2} I_M V_M \cos(\theta_v - \theta_i) = I_{RMS} V_{RMS} \cos(\theta_v - \theta_i) = I_{RMS} V_{RMS} \cdot p.f.$

(When p.f = 1 → similar expression as DC Power)

Pure resistive element: full power consumption: p.f = 1
Pure reactive element: no power consumption: p.f = 0

1) Find complex power at the load $\hat{S}_L = I_{RMS} V_{RMS} e^{j(\theta_v - \theta_i)}$

$\theta_v - \theta_i = \cos^{-1}(0.5) = 60^\circ$; $P = Re(\hat{S}_L) = I_{RMS} V_{RMS} \cdot p.f$

$I_{RMS} V_{RMS} = \frac{P}{pf} = 20\,000$ V-A

$\rightarrow \hat{S}_L = 20000\, e^{j60^\circ}$ V-A

2) Find complex power at the line (upper branch)

$\hat{S}_{Line} = \hat{V}_{RMS} \hat{I}^*_{RMS} \underset{\text{Ohm's Law}}{=} (\hat{I}_{RMS} \hat{Z}_{Line}) \hat{I}^*_{RMS} = I^2_{RMS} \hat{Z}_{Line} = I^2_{RMS}(1+j)$

$I_{RMS} = \frac{20000}{110} = 181.8$ $\left(\hat{I}_{RMS} = \left(\frac{\hat{S}_L}{\hat{V}_L}\right)^* = \frac{20000\angle -60^\circ}{110} = 181.82\angle -60^\circ A \right)$

$\hat{S}_{Line} = 181.82^2(1+j)$

3) Find complex power at the source → pf at source.

$\hat{S} = \hat{S}_{Line} + \hat{S}_L = (20000\cos 60 + 181.82^2) + j(20000 \sin 60 + 181.82^2)$

(34)

$$\hat{S} = 43058.5 + j\,50379 \quad V\text{-}A$$

$$= 66277.8 \angle 49.5^\circ \quad V\text{-}A$$

$$\rightarrow p.f_{source} = \cos(49.5^\circ) = 0.65$$

$\rightarrow$ What is $\hat{V}_s$?

$$\hat{V}_s = \frac{\hat{S}}{\hat{I}_s^*} = \frac{66227.8 \angle 49.5^\circ}{181.82 \angle 60^\circ} = 364.5 \angle -10.5^\circ \quad V_{RMS}$$

$\rightarrow$ What is $\hat{V}_{Line}$?

$$\hat{V}_{Line} = \frac{\hat{S}_{Line}}{\hat{I}_{Line}^*} = \frac{181.82^2 \sqrt{2} \angle 45^\circ}{181.82 \angle 60^\circ} = 257 \angle -15^\circ \quad V_{RMS}$$

If we are correct: $\hat{V}_{Line} + \hat{V}_L = \hat{V}_s$

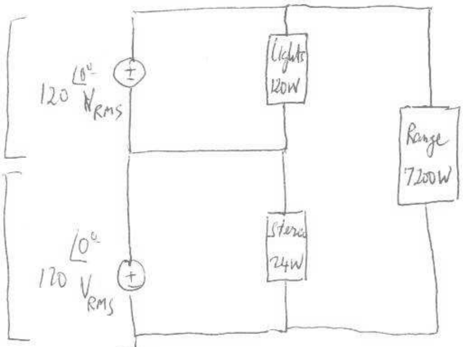
$$(257\cos(15) + 110) + j(257\sin(-15^\circ)) = 358.37 - j66.55$$

$$= 364.5 \angle -10.5^\circ \; V_{RMS}. \rightarrow \text{Yes! Same as } \hat{V}_s$$

(35)

Single-Phase (Three-wire) Circuit:

→ Dryer or Stove (there are gas options for these)



How much is utility bill a month ($0.08 / kWh)

energy.

1) Total energy consumed per day = integrating power over time:

- Lights: on for 15 hours
- Range: on for 4 hours
- Stereo: on for 8 hours

$$E_{①} = \int P_{①}\,dt = \int V_{①} I_{①}\,dt = 120V\left[I_{lights} \times 15hrs + I_{Range} \times 4hrs\right]$$

p.f. at these appliances are 1 (pure resistive)

$$P = I_{RMS} V_{RMS} \cdot 1 \rightarrow I_{RMS} = \frac{P}{V_{RMS}} = \begin{cases} \frac{120}{120} = 1A & \text{at lights.} \\ \frac{7200}{240} = 30A & \text{at range.} \end{cases}$$

$$E_{①} = 120V\underbrace{\left[1A \times 15hrs + 30A \times 4hrs\right]}_{W} = 16.2\,kWh$$

$$E_{②} = \int P_{②}\,dt = \int V_{②} I_{②}\,dt = 120\,V_{RMS}\underbrace{\left[\underset{0.2A}{I_{Stereo}} \times 8hrs + \underset{30A}{I_{Range}} \times 4hrs\right]}_{121.6}$$

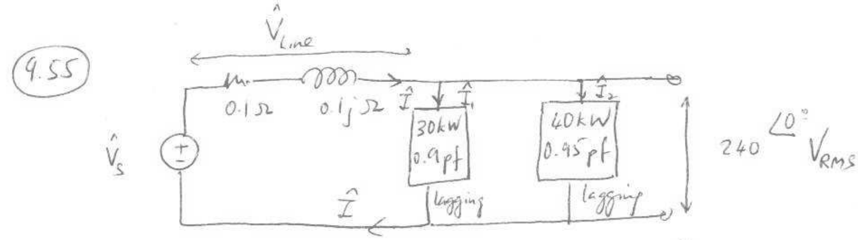
$$= 14.592\,kWh$$

Total energy consumed a day = $(16.2 + 14.592)\,kWh = 30.8\,kWh$ a day

One month $30.8 \times 30\,kWh = 924\,kWh \cdot \frac{\$0.08}{kWh} = \$73.9$ / month.

$\boxed{\text{Exam 2 Apr 6, 01}}$ HW 4 & HW 5 are due / HW 6 due next week / HW 7 following week.

(9.55)



Find $\hat{V}_s$ $\qquad \hat{V}_s = \hat{V}_{Line} + 240 = \hat{I}(0.1 + 0.1j) + 240$

$$\hat{I} = \hat{I}_1 + \hat{I}_2 \quad ; \quad \hat{S} = \hat{V}\hat{I}^*$$

$$= \left(\frac{\hat{S}_1}{240}\right)^* + \left(\frac{\hat{S}_2}{240}\right)^* = \left(\frac{\hat{S}_1 + \hat{S}_2}{240}\right)^*$$

$$\hat{S}_L = I_{RMS} V_{RMS} e^{j(\theta_v - \theta_i)} \rightarrow P_L = I_{RMS} V_{RMS} \underbrace{\cos(\theta_v - \theta_i)}_{pf}$$

$$\rightarrow \hat{S}_1 = \frac{P_{L_1}}{pf_1} \angle \cos^{-1} pf_1 = \frac{30000}{0.9} \angle 25.8^\circ = 33333 \angle 25.8^\circ$$

$$\rightarrow \hat{S}_2 = \frac{40000}{0.95} \angle \cos^{-1} 0.95 = 42105 \angle 18.2^\circ$$

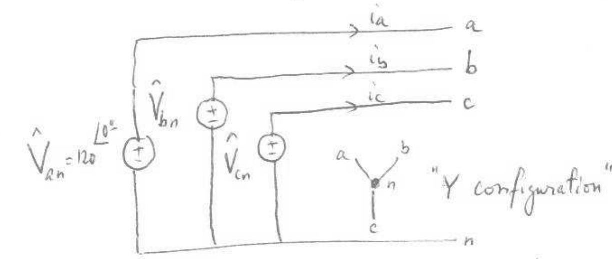
$$\hat{I} = \frac{1}{240}\left[(33333 \cos 25.8) + 42105 \cos 18.2) + j\,(33333 \sin 25.8 + 42105 \sin 18.2)\right]^*$$

$$= \frac{1}{240}\left[7 \times 10^4 + j\, 2.77 \times 10^4\right]^* \text{ A}$$

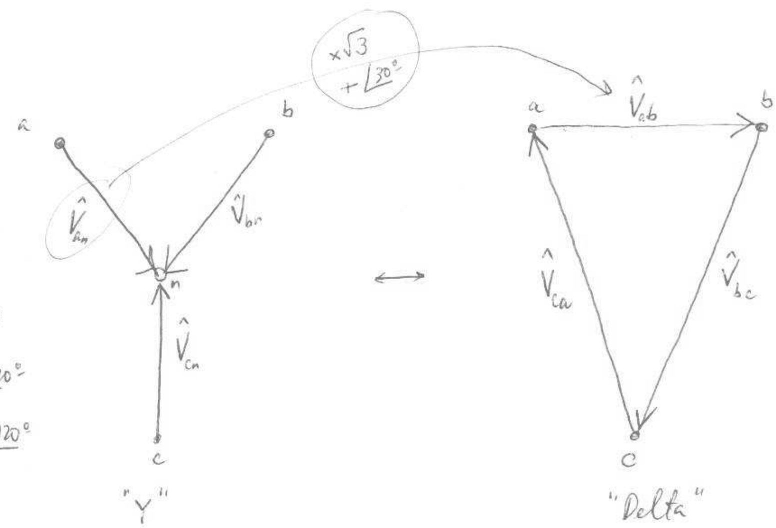
$$\hat{V}_s = \underbrace{\frac{10^3}{240}(7 - j2.77)(1+j)}_{(7 + 2.77 + j4.23)} + 240 = 280.71 + 17.63j$$

$$= \boxed{281.26 \angle 3.59^\circ \text{ V}}$$

Three-Phase Circuits



$\hat{V}_{bn} = 120\angle 120°\ V_{RMS}$; $\hat{V}_{cn} = 120\angle 240°\ V_{RMS}$



$\hat{V}_{an} = V_p\angle 0°$
$\hat{V}_{bn} = V_p\angle 120°$
$\hat{V}_{cn} = V_p\angle -120°$

$$\hat{V}_{ab} \equiv \hat{V}_{an} - \hat{V}_{bn} = V_p - V_p\cos 120 - jV_p\sin 120 = V_p(1-\cos 120) - jV_p\sin 120$$
$$= V_p\sqrt{3}\angle 30°$$

(In vector notation: $\hat{V}_{na} + \hat{V}_{ab} = \hat{V}_{nb}$

$\Rightarrow \hat{V}_{ab} = \hat{V}_{nb} - \hat{V}_{na}$

$= \hat{V}_{an} - \hat{V}_{bn}$)

(38)

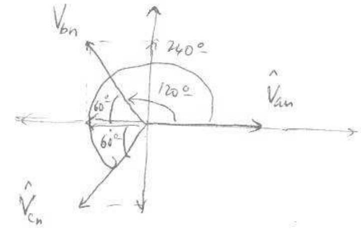
$$\hat{V}_{bc} = \hat{V}_{bn} - \hat{V}_{cn} = V_p(\overbrace{\cos 120^\circ - \cos 120^\circ}^{0}) + V_p(\sin 120^\circ + \sin 120^\circ)j$$

$$= V_p \sqrt{3} \angle 90^\circ$$

$$\hat{V}_{ca} = \hat{V}_{cn} - \hat{V}_{an} = V_p(\cos 120 - 1) + j V_p \sin 120$$

$$= V_p \sqrt{3} \angle 120^\circ$$

Why these phases : $0°, 120°, 240°$



$$\cos 60° = \frac{1}{2} \Rightarrow \hat{V}_{an} + \hat{V}_{bn} + \hat{V}_{cn} = 0$$

Instaneous power supplied by these 3 sources :

$$p(t) = p_{an}(t) + p_{bn}(t) + p_{cn}(t) = v_{an}(t)\, i_a(t) + v_{bn}(t)\, i_b(t) + v_{cn}(t)\, i_c(t)$$

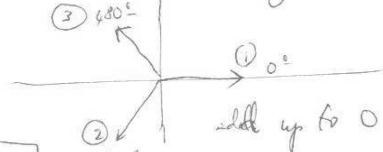
AC circuits

$$\begin{cases} v_{an} = V_M \cos(\omega t) & ; \; i_a = I_M \cos(\omega t + \theta) \\ v_{bn} = V_M \cos(\omega t + 120°) & ; \; i_b = I_M \cos(\omega t + \theta + 120°) \\ v_{cn} = V_M \cos(\omega t + 240°) & ; \; i_c = I_M \cos(\omega t + \theta + 240°) \end{cases}$$

$$p(t) = I_M V_M \left[\cos\omega t \cos(\omega t + \theta) + \cos(\omega t + 120°)\cos(\omega t + \theta + 120°) + \cos(\omega t + 240°)\cos(\omega t + \theta + 240°) \right]$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$= \frac{I_M V_M}{2} \Big[3\cos\theta + \underbrace{\overset{(1)}{\cos}(\underline{2\omega t + \theta}) + \overset{(2)}{\cos}(\underline{2\omega t + \theta} + \overset{240°}{240°}) + \overset{(3)}{\cos}(\underline{2\omega t + \theta} + \overset{480°}{480°})}_{0} \Big]$$



add up to 0.

$\rightarrow \boxed{\text{constant } p(t) = \frac{3}{2} I_M V_M \cos\theta} \rightarrow$ 3 sources of same amplitude and phase changed by 120°

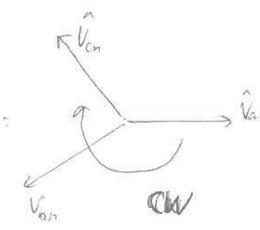
Voltages $\hat{V}_{an}, \hat{V}_{bn}, \hat{V}_{cn}$ have phases in CCW order $(0, 120°, 240°)$

Powers p_a, p_b, p_c " " in CW order $(0, 240, 480=120°)$

↳ if $\hat{V}_{an}, \hat{V}_{bn}, \hat{V}_{cn}$ also have phase in CW order $(0°, 240°, 120°)$

→ still $p(t) = \frac{3}{2} I_m V_m \cos\theta$ constant. (we only need phases separated by $120° \rightarrow \frac{360°}{3}$)

This order is called an abc-sequence:

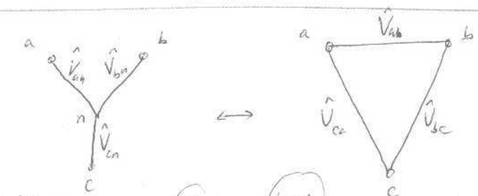


↓

$\boxed{\text{easier ~~than~~ to remember } Y \leftrightarrow \Delta}$

Since sum is 0: balanced abc-sequence

Last time:



$$\left.\begin{aligned} \hat{V}_{an} &: 0° \\ \hat{V}_{bn} &: 120° \\ \hat{V}_{cn} &: 240° \end{aligned}\right\} \text{acb-sequence}$$

$$\overset{0}{\hat{V}_{an}} \xrightarrow{\sqrt{3}\angle{+30°}} \overset{30°}{\hat{V}_{ab}}$$

$$\overset{120}{\hat{V}_{bn}} \xrightarrow{\sqrt{3}\angle{-30°}} \overset{90°}{\hat{V}_{bc}}$$

$$\overset{240}{\hat{V}_{cn}} \xrightarrow{\sqrt{3}\angle{-120°}} \overset{120°}{\hat{V}_{ca}}$$

Different transformations for $\hat{V}_{an}, \hat{V}_{bn}, \hat{V}_{cn}$

Now:

$$\left.\begin{aligned} \hat{V}_{an} &= 0° \\ \hat{V}_{bn} &= -120° \text{ or } 240° \\ \hat{V}_{cn} &= 120° \end{aligned}\right\} \text{abc sequence.}$$

$$\hat{V}_{ab} = \hat{V}_{an} - \hat{V}_{bn} = V_p - V_p \cos 120° + V_p \sin 120° j$$

$$= V_p \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = V_p \sqrt{3} \underbrace{\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)}_{1\angle 30°}$$

(41)

$$\hat{V}_{bc} = \hat{V}_{bn} - \hat{V}_{cn} = V_p\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}j\right) - V_p\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)$$
$$= -V_p\sqrt{3}j = V_p\sqrt{3}\,\underline{/-90^\circ}$$

$$\hat{V}_{ca} = \hat{V}_{cn} - \hat{V}_{an} = V_p\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) - V_p$$
$$= V_p\left(-\frac{3}{2} + \frac{\sqrt{3}}{2}j\right) = V_p\sqrt{3}\underbrace{\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)}_{1\underline{/150^\circ}}$$

abc sequence $Y \to \Delta$

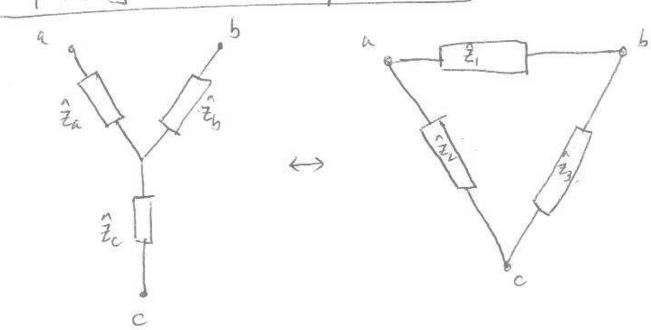
$$\left\{\begin{array}{lcl} \overset{0^\circ}{\hat{V}_{an}} & \xrightarrow{\sqrt{3}\,\underline{/+30^\circ}} & \overset{30^\circ}{\hat{V}_{ab}} \\ \overset{-120^\circ}{\hat{V}_{bn}} & \xrightarrow{\sqrt{3}\,\underline{/+30^\circ}} & \overset{-90^\circ}{\hat{V}_{bc}} \\ \overset{120^\circ}{\hat{V}_{cn}} & \xrightarrow{\sqrt{3}\,\underline{/+30^\circ}} & \overset{150^\circ}{\hat{V}_{ca}} \end{array}\right.$$

$$\underbrace{\phantom{V_{an}}}_{Y} \longrightarrow \underbrace{\phantom{V_{ab}}}_{\Delta}$$

→ Conclusion: same transformation $Y \to \Delta$ for $\hat{V}_{an}, \hat{V}_{bn}, \hat{V}_{cn}$ with phases in abc sequence!

→ For $\Delta \to Y$ (balanced abc sequence) $\frac{1}{\sqrt{3}}\underline{/-30^\circ}$

$Y \to \Delta$ load transformation:



$$\left\{\begin{array}{l} (1)\ \hat{Z}_a + \hat{Z}_b = \hat{Z}_1 \,\|\, (\hat{Z}_2 + \hat{Z}_3) \\ (2)\ \hat{Z}_a + \hat{Z}_c = \hat{Z}_2 \,\|\, (\hat{Z}_1 + \hat{Z}_3) \\ (3)\ \hat{Z}_b + \hat{Z}_c = \hat{Z}_3 \,\|\, (\hat{Z}_1 + \hat{Z}_2) \end{array}\right.$$

const!

$\boxed{Y \rightarrow \Delta}$ (42)

$$\hat{z}_1 = \frac{\hat{z}_a\hat{z}_b + \hat{z}_b\hat{z}_c + \hat{z}_a\hat{z}_c}{\hat{z}_c} = \frac{\hat{z}_a\hat{z}_b}{\hat{z}_c} + \hat{z}_b + \hat{z}_a = \hat{z}_a\hat{z}_b\left(\frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_a} + \frac{1}{\hat{z}_b}\right)$$

$$\hat{z}_2 = \frac{\text{same}}{\hat{z}_b} = \frac{\hat{z}_a\hat{z}_c}{\hat{z}_b} + \hat{z}_a + \hat{z}_c = \hat{z}_a\hat{z}_c\left(\frac{1}{\hat{z}_b} + \frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_a}\right)$$

$$\hat{z}_3 = \frac{\text{same}}{\hat{z}_a} = \frac{\hat{z}_b\hat{z}_c}{\hat{z}_a} + \hat{z}_b + \hat{z}_c = \hat{z}_b\hat{z}_c\left(\frac{1}{\hat{z}_a} + \frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_b}\right)$$

$$\hat{z}_1 + \hat{z}_2 = (\hat{z}_a\hat{z}_b + \hat{z}_b\hat{z}_c + \hat{z}_a\hat{z}_c)\left(\frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_b}\right)$$

$$\hat{z}_3 \| (\hat{z}_1 + \hat{z}_2) = \frac{(\hat{z}_a\hat{z}_b + \hat{z}_b\hat{z}_c + \hat{z}_a\hat{z}_c)^2 \frac{1}{\hat{z}_a}\left(\frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_b}\right)}{\cancel{(\hat{z}_a\hat{z}_b + \hat{z}_b\hat{z}_c + \hat{z}_a\hat{z}_c)}\left(\frac{1}{\hat{z}_a} + \frac{1}{\hat{z}_c} + \frac{1}{\hat{z}_b}\right)}$$

$$= \cancel{(\hat{z}_a\hat{z}_b + \hat{z}_b\hat{z}_c + \hat{z}_a\hat{z}_c)} \frac{\frac{\hat{z}_b + \hat{z}_c}{\cancel{z_a z_b z_c}}}{\frac{\cancel{(z_c z_b + z_a z_b + z_a z_c)}}{\cancel{z_a z_b z_c}}} = \hat{z}_b + \hat{z}_c$$

$\boxed{\Delta \rightarrow Y}$

$$1 + 2 : \quad 2\hat{z}_a + \underbrace{\hat{z}_b + \hat{z}_c}_{3} = \hat{z}_1 \| (\hat{z}_2 + \hat{z}_3) + \hat{z}_2 \| (\hat{z}_1 + \hat{z}_3)$$

$$\hat{z}_a = \frac{1}{2}\left\{ \hat{z}_1 \| (\hat{z}_2 + \hat{z}_3) + \hat{z}_2 \| (\hat{z}_1 + \hat{z}_3) - \hat{z}_3 \| (\hat{z}_1 + \hat{z}_2) \right\}$$

$$\hat{z}_b =$$

$$\hat{z}_c =$$