$\hat{V}_2$? $\hat{I}_0$

$$2 + \frac{6}{1+j} = \hat{V}_2\left(\frac{1}{1+j} + 1 + \frac{1}{1-j}\right) = \hat{V}_2\,\frac{1-\cancel{j}+2+1+\cancel{j}}{2} = 2\hat{V}_2$$

$$\to \hat{V}_2 = 1 + \frac{3}{1+j} = \frac{4+j}{1+j} = \frac{(4+j)(1-j)}{2} = \frac{5-3j}{2} = \frac{\sqrt{34}}{2}e^{-j\tan^{-1}\frac{3}{5}}$$

$$= 2.92\, e^{-j30.96} \text{ V}$$

$$\to \hat{I}_0 = \frac{\hat{V}_2}{1} = 2.92\, e^{-j30.96} \text{ A}$$

Complex conjugate of $1+j$ is $1-j$. Product of any complex number with its conjugate gives its magnitude squared.

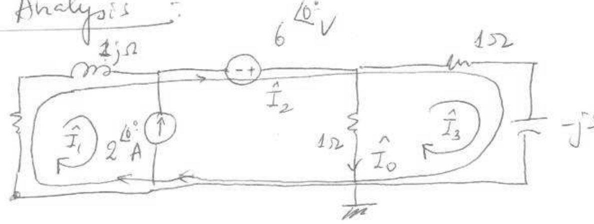
If: $f = 60$ Hz : give $i_0(t) = 2.92\cos(377t - 30.96°)$ A

↳ $\omega = 2\pi f = 377\ s^{-1}$

Polar forms allow quick conversion between frequency & time-domain expressions.

---

2) Loop Analysis :



KVL : voltages (with signs) around a closed loop add up to zero

3 loops → 3 KVL's → choose a direction for current in each loop

↳ loop #1 (left) : a) use current : $\hat{I}_1 = -2$ A

Loop #2 (outer, global) : b) $1(\hat{I}_1+\hat{I}_2) + 1j(\hat{I}_1+\hat{I}_2) - 6 + (1-j)(\hat{I}_2+\hat{I}_3)$

Loop #3 (right) : c) $(1-j)(\hat{I}_1+\hat{I}_3) + \hat{I}_3 = 0$

We need $\hat{I}_0 = -\hat{I}_3$ → eliminate $\hat{I}_2$ from eqns b) & c)

b) $(\hat{I}_2 - 2)(1+j) - 6 + \underbrace{(\hat{I}_2 + \hat{I}_3)(1-j)}_{-\hat{I}_3 \text{ from c)}} = 0$

c) $\hat{I}_2(1-j) + \hat{I}_3(2-j) = 0 \rightarrow \hat{I}_2 = \hat{I}_3 \dfrac{j-2}{1-j}$

$\hookrightarrow \hat{I}_2(1+j) - 8 - 2j - \hat{I}_3 = 0$

$$\underbrace{\hat{I}_3 \frac{(1+j)(j-2)}{1-j}}_{\frac{-3-j}{1-j}} - 8 - 2j - \hat{I}_3 = 0$$

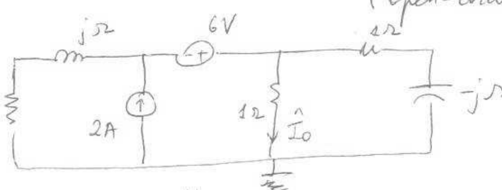
$$\Rightarrow \hat{I}_3 = \frac{8+2j}{\frac{-3-j}{1-j} - 1} = \frac{(8+2j)(1-j)}{(-3-j) - 1 + j} = \frac{10-6j}{-4}$$

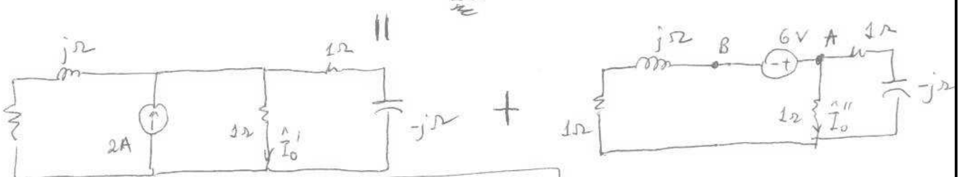
$$\hat{I}_3 = \frac{-5+3j}{2}\ A \quad \text{or} \quad \boxed{\hat{I}_0 = \frac{5-3j}{2}\ A}$$

---

3) Superposition Method using Phasors:

$\hookrightarrow$ adding circuits with single sources {short-circuiting voltage source or open-circuiting current source

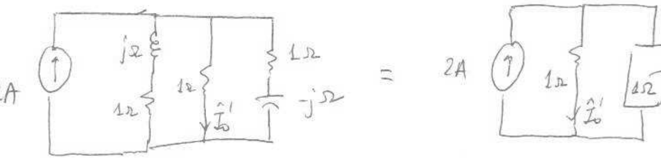




$$\boxed{\hat{I}_0 = \hat{I}_0' + \hat{I}_0''}$$

Instead of one original circuit we solve two single-source circuits

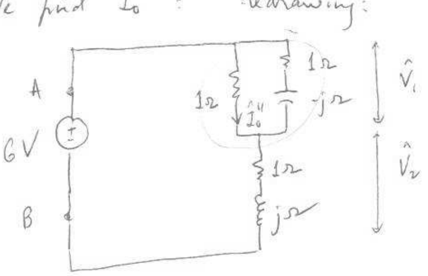
a) We find $\hat{I}_o'$:



$(1+j)\,\|\,(1-j) = \frac{(1+j)(1-j)}{1+j+1-j} = 1\,\Omega$

$\hat{I}_o' = 1A$ ("current division")

b) We find $\hat{I}_o''$: redrawing:



Find $\hat{V}_1$ using "Voltage Division"

$\rightarrow \hat{I}_o'' = \frac{\hat{V}_1}{1\Omega}$

$$\hat{V}_1 = 6V \frac{\frac{1(1-j)}{1+1-j}}{\frac{1-j}{2-j} + 1 + j}$$

$$= 6V \frac{(1-j)}{(1-j)+(1+j)(2-j)}$$

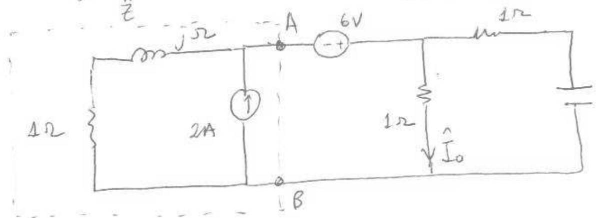
$$= 6V \frac{1-j}{1-\cancel{j}+3+\cancel{j}} = \frac{6}{4}(1-j)V$$

$\hat{I}_o'' = \frac{3}{2}(1-j)\,A$

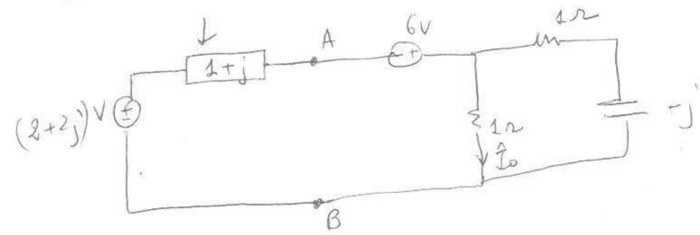
$\Rightarrow \hat{I}_o = \hat{I}_o' + \hat{I}_o'' = 1 + \frac{3}{2}(1-j) = \boxed{\frac{5-3j}{2}\,A}$

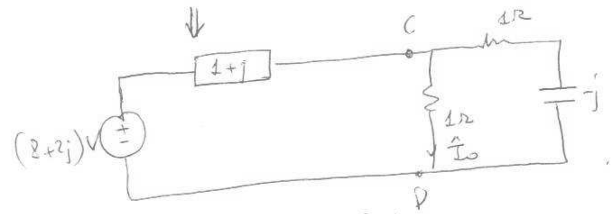
4) Source Exchange using Phasors

$$\hat{I} = \frac{\hat{V}}{\hat{Z}} \qquad \boxed{\hat{V} = \hat{I}\hat{Z}}$$



For example:
change a current source into a voltage source.



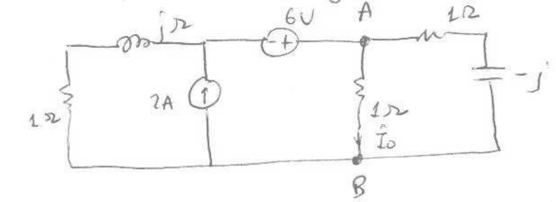


$$Z_L = j\omega L$$
$$Z_C = \frac{1}{j\omega C}$$

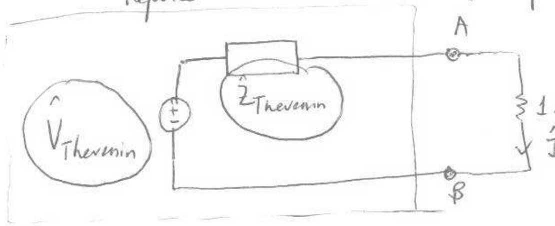
$$\hat{V}_{CD} = (8+2j)\,\frac{\dfrac{(1-j)}{2-j}}{\dfrac{1-j}{2-j} + 1+j} = (8+2j)\,\frac{1-j}{1-j+(1+j)(2-j)} = \frac{10-6j}{3+j+1-j}$$

$$= \frac{10-6j}{4} \rightarrow \hat{I}_o = \frac{\hat{V}_{CD}}{1\,\Omega} = \frac{5-3j}{2}\ A.$$

5) Thevenin Equivalence using Phasors :



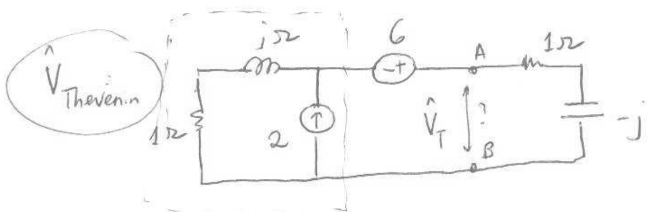
Replace the whole circuit (except for branch AB) by :



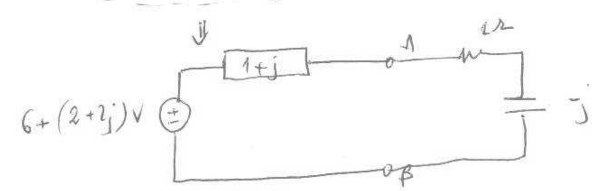
$$\begin{cases} \hat{V}_T = \hat{V}_{OC} \text{ (across AB) open-circuit} \\ \hat{Z}_T = \hat{Z}_{NS} \text{ (across AB)} \end{cases}$$

NS: no sources

(Current source : open
Voltage source : short circuit)

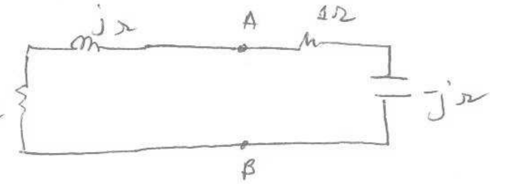


⇓

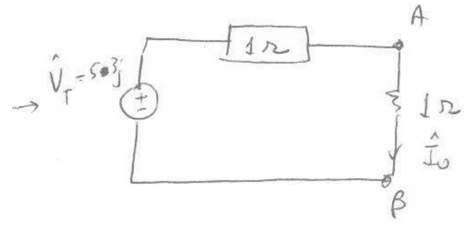


$$\hat{V}_T = \hat{V}_{AB} = (8+2j)\,\frac{1-j}{1-\cancel{j}+1+\cancel{j}} = \frac{(8+2j)(1-j)}{2} = \boxed{5-3j\ V}$$

$\hat{Z}_{Thevenin} =$

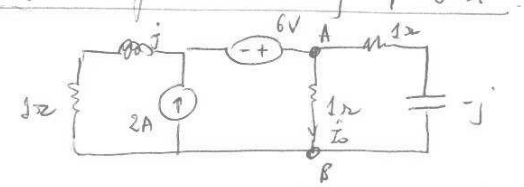


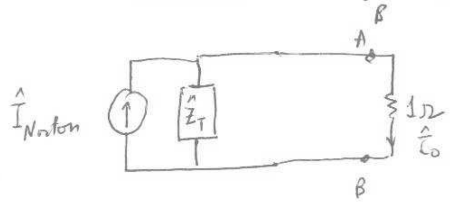
$$\hat{Z}_{AB} = (1+j) \parallel (1-j) = \frac{(1+j)(1-j)}{(1+j)+1-j} = 1\,\Omega$$



$$\rightarrow \hat{I}_0 = \frac{5-3j}{2}\ A.$$

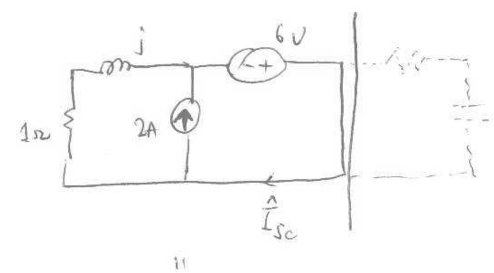
6) Norton Equivalence using Phasors:

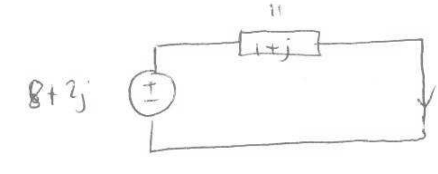




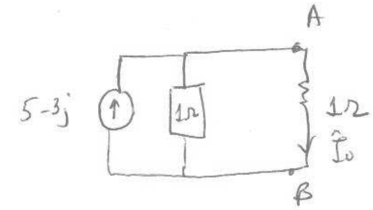
$\hat{I}_{Norton} = \hat{I}_{SC}$ (across AB)
Short-circuit.

$\hat{Z}_T = 1\,\Omega$ as before.





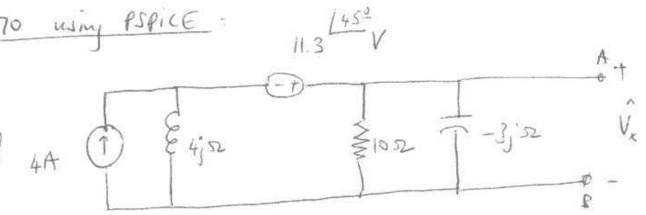
$$\hat{I}_{S.C} = \frac{8+2j}{1+j} = \frac{(8+2j)(1-j)}{2} = \frac{10-6j}{2} = 5-3j\ A$$



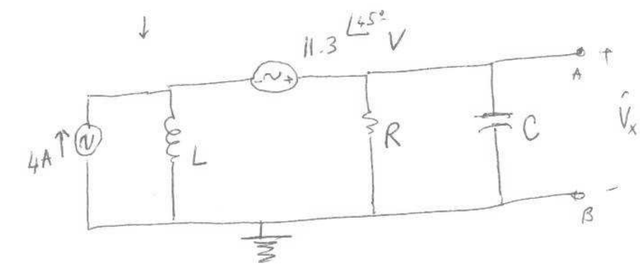
$$\hat{I}_o = \frac{5-3j}{2}\ A = 2.91\, e^{-j31^\circ}\ A$$

7.70 using PSPICE :

By hand



PSPICE



$$\hat{Z}_L = j\omega L = j4 \rightarrow L = \frac{4}{\omega} = \frac{4}{377} = 10.6\ mH$$

$$\hat{Z}_c = \frac{1}{j\omega C} = -j3 \rightarrow C = \frac{1}{3\omega} = \frac{1}{3\times 377} = 884\ \mu F$$

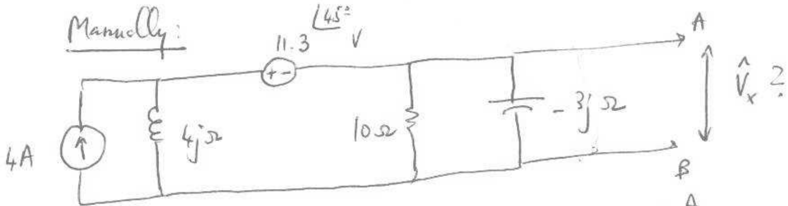
$f = 60\,Hz \rightarrow \omega = 377\ s^{-1}$

EXC = 1HW set

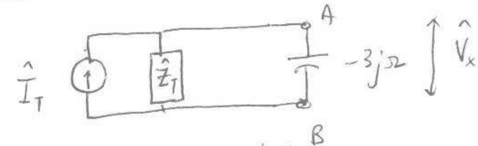
What is the result using Norton's equivalent? By comparing with PSPICE, what is the right order of signs for current source.

$\boxed{7.70}$ Pspice $\begin{cases} \oplus \text{ IAC} & \hat{V}_x = 48.58 \angle 121.8° \ (\downarrow) \\ \ominus \text{ IAC} & \hat{V}_x = 21.7 \angle 5° \ (\uparrow) \end{cases}$

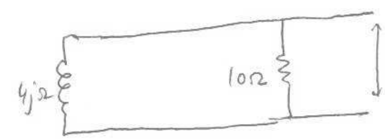
Manually:



Norton's equivalent:

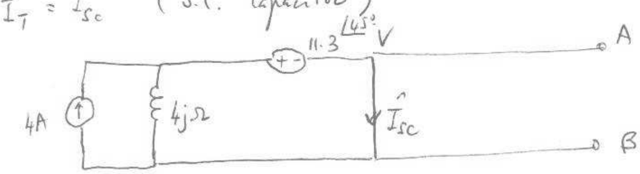


$\hat{Z}_{TH}$ (O.C. capacitor; O.C. 4A (↑); S.C. 11.3 ∠45° V (+ −))

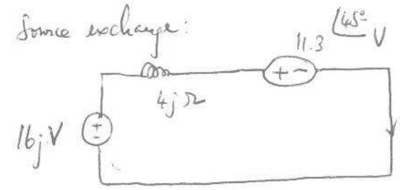


$\hat{Z}_T = \frac{40j}{10+4j}$

$\hat{I}_T = \hat{I}_{sc}$ (S.C. capacitor)



Source exchange:

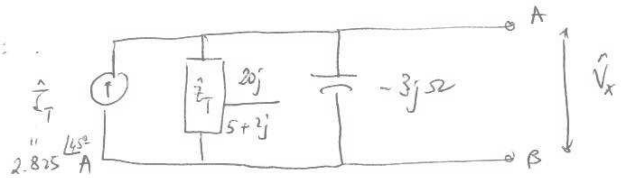


$$\hat{I}_{sc} = \frac{16j - 11.3(\cos 45° + j \sin 45°)}{4j}$$

$$= \frac{-7.99 + 8.01j}{4j} = \frac{11.3 \angle 135°}{4 \angle 90°}$$

$$\hat{I}_{sc} = 2.825 \angle 45° \text{ A}$$

So:



$$\hat{V}_x = \hat{I}_T \cdot \left(\hat{Z}_T \parallel -3j\right) = 2.825\angle 45^\circ \left(\frac{\frac{20j\,(-3j)}{\cancel{5+2j}}}{\frac{20j - 15j + 6}{\cancel{5+2j}}} \right)$$

$$= 2.825\angle 45^\circ \; \frac{60}{6+5j} = \frac{2.825 \times 60\angle 45^\circ}{\sqrt{61}\;\angle \tan^{-1}\left(\frac{5}{6}\right)} = \boxed{21.7\angle 5.2^\circ}$$

PSPICE: IAC ; VAC ; GND ; C ; L ; R ; VPRINT1

IAC: (↑) = (∓)

VPRINT1 → AC = y, MAG = y, Phase = y

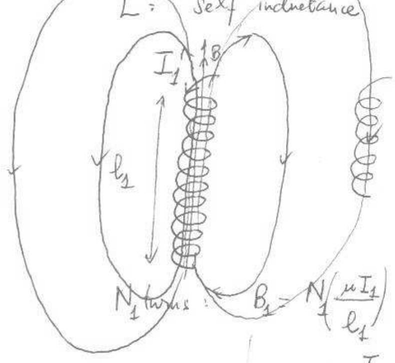
Analysis Setup : AC Sweep
- one freq : 1pt : 60 to 60
- sweep : decade ; 100 pts ; $f_{initial}$ to f_{final}

Simulate.

Results
- one frequency : "examine output" file
- sweep : "add trace" ; M( ) ← V(V_o)

Mutual Inductance & Energy Analysis

L: self inductance (H)   M: mutual inductance (H)



$B_1 = N_1\left(\frac{\mu I_1}{l_1}\right)$ (μ: permeability constant).

$= \underbrace{n_1 \mu I_1}$ ($n_1 \equiv \frac{N_1}{l_1}$: number of turns per unit length)

→ magnetic field created by this solenoid.

This B may go thru some other loop in the same region → creating a flux: $\Phi = B.S$ (surface area). If S is constante (as in our case), and B is constant → nothing interesting. (DC)
If B changes with time → $\frac{d\Phi}{dt} = -V_{\text{induced in second solenoid}}$ = (Faraday's Law). Same with second solenoid: it would induce a voltage in the first solenoid as well → solenoids interact mutually in AC circuits! To account for this we need another term:

DC circuits: $V_1 = L_1 \frac{dI_1}{dt}$ $\frac{d\Phi}{dt} = \frac{dB}{dt}S = \underbrace{n_1\mu S}_{L_1}\frac{dI}{dt}$

AC circuits: $V_1 = L_1\frac{dI_1}{dt} + \underbrace{M\frac{dI_2}{dt}}$ $M = \frac{N_1 N_2 \mu S_1}{l_2}$