Problem 12.18

Given the following functions F(s) find inverse Laplace functions.

$$F(s) = \frac{10}{(s^2 + 2s + 2)}$$

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

Suggested Solution

A

$$F(s) = \frac{10}{(s^2 + 2s + 2)}$$

$$F(s) = \frac{10(s+2)}{s^2 + 4s + 5} = \frac{k_1}{s + 2 - \alpha} + \frac{k_1}{s + 2 + \alpha}$$

for

$s = -1 + \alpha$

$$\frac{10(s+1)}{s + 1 + \alpha} = 5 = k_1$$

so

$$F(s) = \frac{5}{s + 1 - \alpha} + \frac{5}{s + 1 + \alpha}$$

$f(t) = 10e^{-6} \cos t u(t)$

$f(t) = 10e^{-6} \cos t u(t)$

B

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

$$F(s) = \frac{s+1}{s(s^2 + 4s + 5)} = \frac{k_1}{s} + \frac{k_2}{s + 2 - \alpha} + \frac{k_3}{s + 2 + \alpha}$$

$$for$$

$$s = 0$$

$$\frac{s+1}{s^2+4s+5} = 1/5 = k_1$$

$$for$$

$$s = -2+\alpha$$

$$\frac{s+1}{s(s+2+\alpha)} = 0.31\angle -108.43^o = k_2$$

$$F(s) = \frac{1/5}{s} + \frac{0.31\angle -108.43^o}{s+2+\alpha} + \frac{0.31\angle 108.43^o}{s+2+\alpha}$$

$$f(t) = (\frac{1}{5} + 0.62e^{-2t}\cos(t-108.43^o))u(t)$$

$$f(t) = (\frac{1}{5} + 0.62e^{-2t}\cos(t-108.43^o))u(t)$$

Problem 12.39

Find f(t) using convolution if F(s) is

$$F(s) = \frac{1}{(s+1)(s+2)}$$

Suggested Solution

$$F(s) = \frac{1}{(s+1)(s+2)}$$

let

$$F_1(s) = \frac{1}{s+1}$$

$$F_2(s) = \frac{1}{s+2}$$

$$f_1(t) = e^{-t}u(t)$$

$$f_2(t) = e^{-2t}u(t)$$

$$f(t) = \int_0^t e^{-(t-\lambda)}e^{-2\lambda}d\lambda = e^{-t}\int_0^t e^{-\lambda}d\lambda = e^{-t}[e^{-\lambda}]_t^0$$

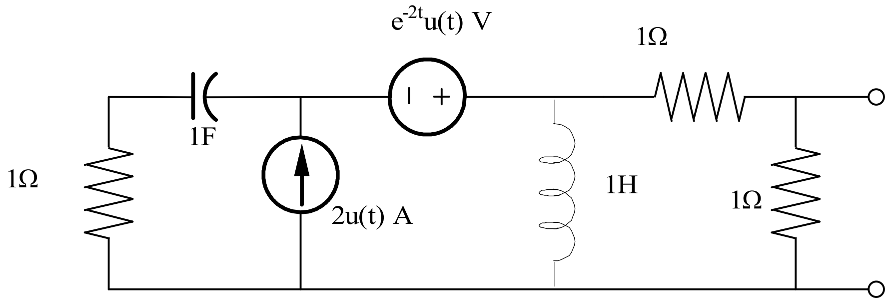
$$f(t) = e^{-t}[1-e^{-t}]$$

$$f(t) = (e^{-t} - e^{-2t})u(t)$$

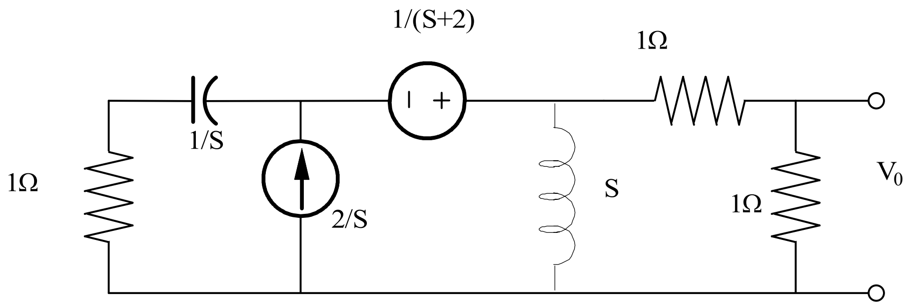
$$f(t) = (e^{-t} - e^{-2t})u(t)$$

Problem 13.6

For the network shown in fig 13.5 find Vo(t),t>0.



Suggested Solution



KCL at the Supernode is:

$$\frac{V_1}{1+1/S}+\frac{V_2}{S}+\frac{V_2}{2}=\frac{2}{3}$$

$$V_1+\frac{1}{S+2}=V_2$$

SOLVING FOR V2 YIELDS:

$$V_2=\frac{2(3S^2+6S+4)}{(S+2)(3S^2+6S+2)}\Rightarrow V_O=\frac{S^2+2S+4/3}{(S+2)(S+0.5-j.646)(S+0.5+j.646)}$$

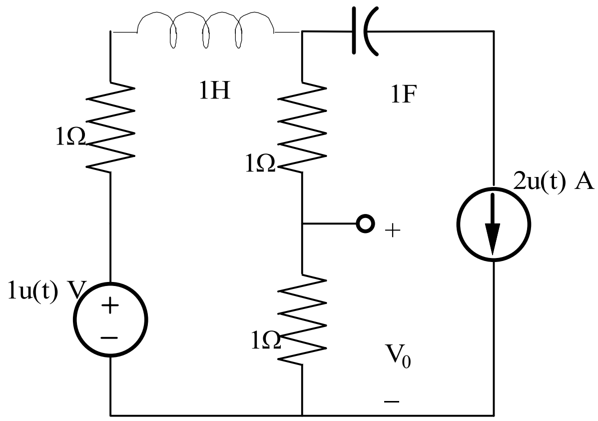
$$=\frac{K_1}{S+2}+\frac{K_2}{(S+0.5-j.646)}+\frac{K_2}{(S+0.5+j.646)}$$

$$K_1=0.5, K_2=0.316\angle{-37.76^0}$$

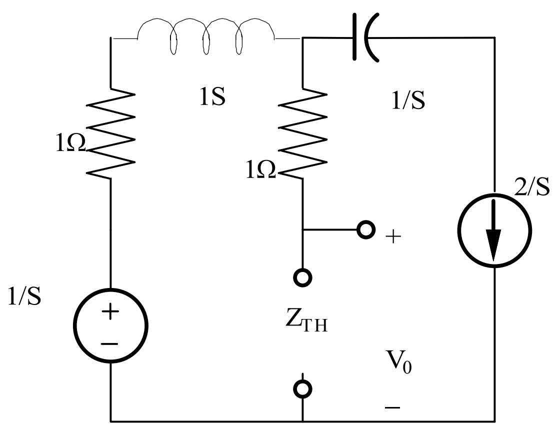
$$\therefore V_o(t)=\left(0.5e^{-2t}+0.632e^{-2t}COS(0.646t-37.76^0)\right)u(t)V$$

Problem 13.23

Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution



Using superposition

$$V_{OC} = \frac{1}{S} - \frac{2}{S}(2s+1) = -\frac{(4s+1)}{s}$$

$$Z_{TH} = 2s + 2$$

$$V_O = V_{OC}\left(\frac{1}{Z_{TH}+1}\right) = \frac{-(4S+1)}{(2S+3)S} = -\frac{(2S+1/2)}{S(S+3/2)}$$

$$V_O = \frac{A}{S} + \frac{B}{S+3/2}$$

$$A = -1/3$$

$$B = -5/3$$

$$V_O(t) = [-\frac{1}{3} - \frac{S}{3}e^{-3/2t}]u(t)V$$