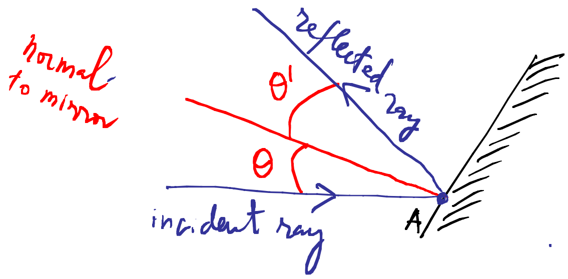
Ch 30 Reflection & Refraction

Geometrical Optics: (Ch 30 & 31) uses geometry to solve for direction of light rays which travel in straight paths

Physical Optics: (Ch 32) when these rays travel through small openings $\sim$ wavelength λ

Law of Reflection: $\theta' = \theta$ (Reflected angle = incident angle)



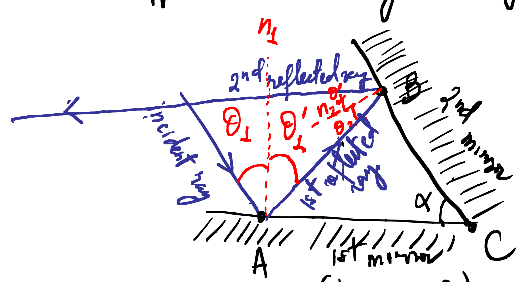
Light doesn't travel through this side of mirror!

→ Normal direction: perpendicular to mirror @ incident point A

↳ Angles are defined wrt. Normal direction $\begin{cases} \text{Incident angle } \theta \\ \text{Reflected angle } \theta' \end{cases}$

→ Law of Reflection also applies to a system of mirrors:

Law of Reflection $\begin{cases} \theta_1' = \theta_1 \\ \theta_2' = \theta_2 \end{cases}$



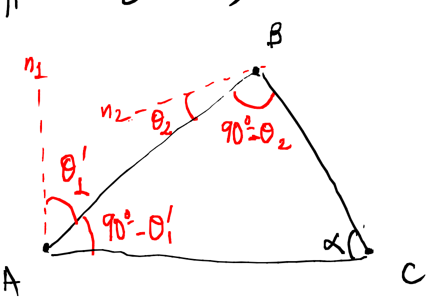
n_1: normal to 1st mirror

n_2: normal to 2nd mirror

Goal: to determine θ_2' (direction of outgoing ray)

↳ (i) Law of reflection
(ii) Geometry

α will affect θ_2' $\Rightarrow$ look at triangle $\hat{ABC}$



Geometry: $90° - \theta_1' + 90 - \theta_2 + \alpha = 180°$

$\alpha - \theta_1' - \theta_2 = 0$

$\theta_2 = \alpha - \theta_1'$

Laws of Reflection: $\theta_2' = \theta_2 = \alpha - \theta_1' = \alpha - \theta_1$

$\Rightarrow \boxed{\theta_2' = \alpha - \theta_1}$

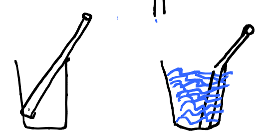
Law of Refraction: when light ray travels from one medium (n_1) to another (n_2)

(Reflection: light rays travel in only one medium – can't travel through the other side of mirror)
↳ incident & reflected

Index of refraction: $n \equiv \frac{c}{v}$ $\frac{\text{(speed of light in vacuum)}}{\text{(speed of light in medium)}} \geq 1$

↳ intuition { material of higher density: water $= n_w > n_0$; $n_w = 1.33$
air $= n_0 = 1$

↳ "Broken straw" effect



straw looks broken

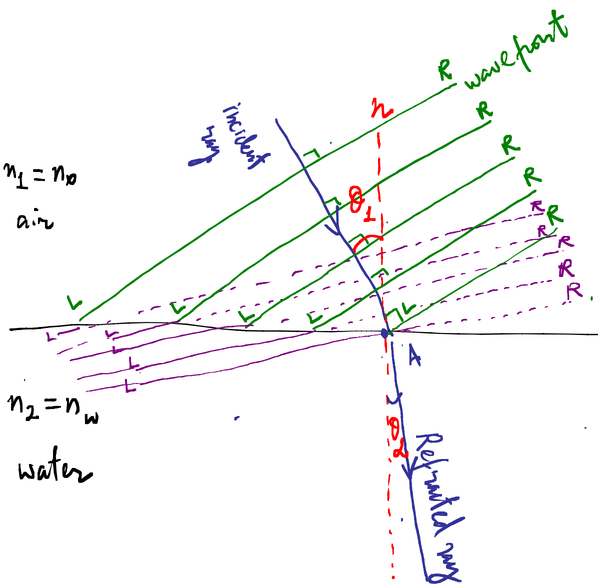
Geometry:

$n_1 = n_0$
air

$n_1 < n_2$

Light travels from lower index medium to higher index medium

$n_2 = n_w$
water



Wave fronts: are equally spaced lines that are perpendicular to light rays.

interface surface

↳ (i) Each wave-front left side L hits medium 2 ($n_2 > n_1$) before right side R

(ii) L slows down while R still travels at original speed:
→ wavefronts rotate toward interface → refracted ray is closer to normal ($\theta_2 < \theta_1$) when $n_2 > n_1$

$\theta_2 < \theta_1$
($n_2 > n_1$)

(iii) Smaller spacing between wavefronts (purple) ⟺ slower propagation in medium 2 ($n_2 > n_1$)

(iv) Refracted ray is ⊥ to new purple wave fronts

(v) Vice versa for light rays traveling from n_1 to n_2 where $n_1 > n_2$

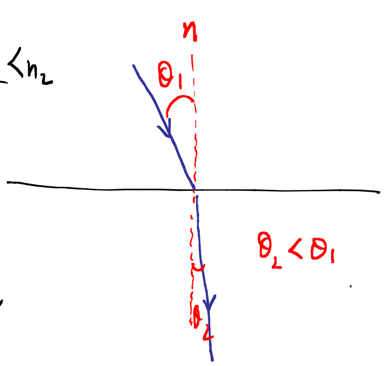
Refraction: { n_1: index of refraction medium 1
n_2: " " " 2
θ_1: incident angle; θ_2: refracted angle

↓

Snell's Law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

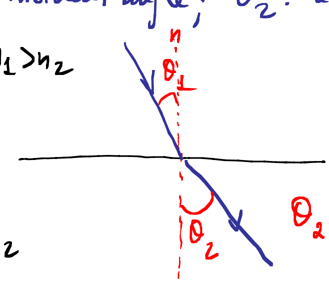
$n_1 < n_2$



n_2

$\theta_2 < \theta_1$

$n_1 > n_2$

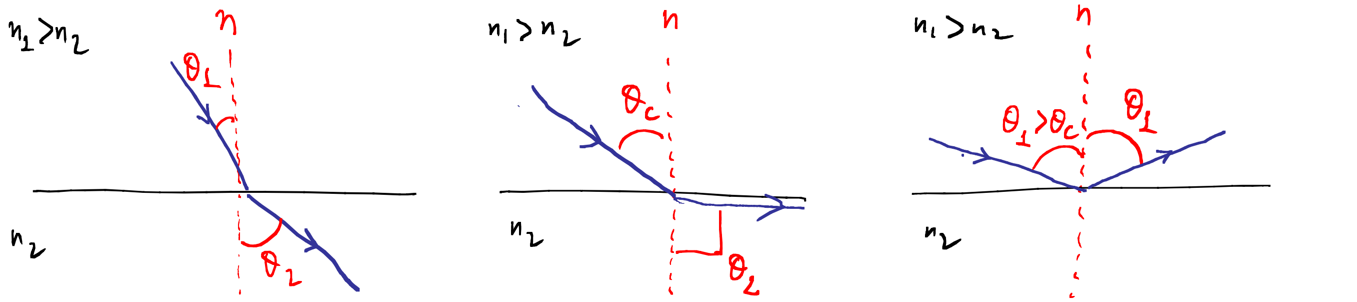


n_2

$\theta_2 > \theta_1$

Law of Refraction or Snell's Law & Critical Angle θ_c when $n_1 > n_2$

$\theta \geq \theta_c \leftrightarrow$ total internal reflection:



$\theta_2 > \theta_1$

at certain $\theta_1 = \theta_c$
$\theta_2 = 90°$ or refracted ray is parallel to boundary b/w two media

Technically there is no "refraction" as the "refracted ray" doesn't travel in medium 2

↓ Light ray stays in medium 1
→ "Total internal reflection"
(Incident ray stays internally within medium 1)

Total internal reflection for $\theta_1 > \theta_c$

⇓

Snell's Law:
$$\left\{\begin{array}{l}\theta_1 = \theta_c \\ \theta_2 = 90°\end{array}\right\} \rightarrow n_1 \sin\theta_c = n_2 \underbrace{\sin 90°}_{1}$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Note: only defined when $n_2 < n_1$ or $n_1 > n_2$
(from higher index to lower index!)

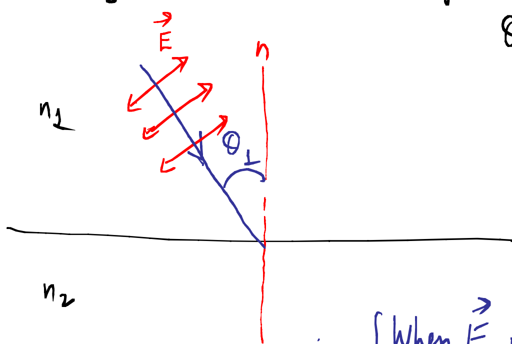
Brewster's angle or polarizing angle:

Critical angle: $\left\{\begin{matrix} \theta_1 \geq \theta_c \\ \boxed{(n_1 > n_2)} \end{matrix}\right\} \rightarrow$ all rays are reflected, none is refracted

$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\boxed{\text{Brewster's angle}}$: $\theta_1 = \theta_B$ or $\theta_p \rightarrow$ all rays are refracted, none is reflected

θ_p or $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$



When $\vec{E}$ oscillates $\perp$ to incident ray and in plane with page as shown and if $\theta_1 = \theta_p$ then there is no reflection at the boundary, all energy travels in second medium n_2

(i) If $\vec{E}$ oscillates not in the plane of page and $\theta_1 = \theta_p$ there would still be some reflection at the boundary!

(ii) If that is the case, the direction of polarization (direction of oscillation of $\vec{E}$) for the reflected rays (at $\theta_1 = \theta_p$) is $\perp$ to plane of page

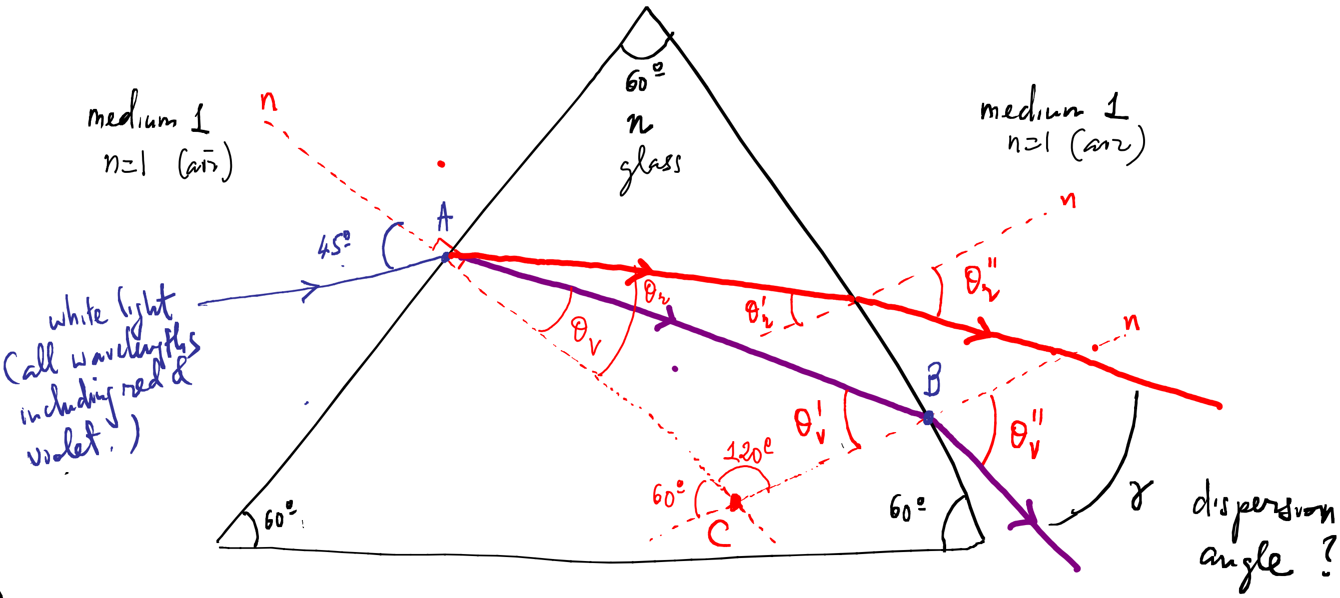
→ Reason for $\theta_1 = \theta_p$ or polarizing angle (this angle "polarizes" the reflected rays)

30.27 Prism spreads white light (incident beam ⟶ outgoing cone)

disperses → behaves differently for different wavelengths

$$\begin{cases} n_{red} = 1.582 = \dfrac{c}{v_{red}} \\ n_{violet} = 1.633 = \dfrac{c}{v_{violet}} \end{cases}$$

Determine dispersion angle γ



$n_{red} = 1.582$

$n_{violet} = 1.633$

$\boxed{\gamma = \theta_V'' - \theta_r''}$?

White light beam incident of left side of prism is spread out after refractions by prism → determine dispersion angle γ.

Geometry: triangle $\triangle ABC$ where C is intersection of normals to left & right sides

↳ Violet ray: $\theta_1 = 45°$
Refracted angle after hitting left boundary is θ_V
Incident angle on right boundary is θ_V'
Refracted angle after right boundary is θ_V''

$\theta_V + \theta_V' + 120° = 180°$ or $\boxed{\theta_V' = 60° - \theta_V}$

Law of Refraction or Snell's Law: (Violet ray)

(i) Left boundary: $1 \sin 45° = n_V \sin \theta_V$

$$\theta_V = \sin^{-1}\left(\frac{\sin 45°}{n_V}\right)$$

$$\theta_V = \sin^{-1}\left(\frac{\frac{\sqrt{2}}{2}}{1.633}\right) = 25.5°$$

(ii) Right boundary: incident angle is $\theta_V' = 60 - \theta_V = 34.5°$

$$n_V \sin 34.5° = 1 \cdot \sin \theta_V''$$

$$\theta_V'' = \sin^{-1}(n_V \sin 34.5°) = \sin^{-1}(1.633 \cdot \sin 34.5°)$$

$$\boxed{\theta_V'' = 67.7°}$$

Similarly work out red rays: ⇒ (i) Left boundary: $1 \sin 45° = n_r \sin \theta_r$

$$\theta_r = \sin^{-1}\left(\frac{\frac{\sqrt{2}}{2}}{1.582}\right) = 26.5°$$

(ii) Right boundary: incident angle $= \theta_r' = 60 - \theta_r = 33.5°$

$$n_2 \sin 33.5^\circ = 1 \cdot \sin \theta_2''$$

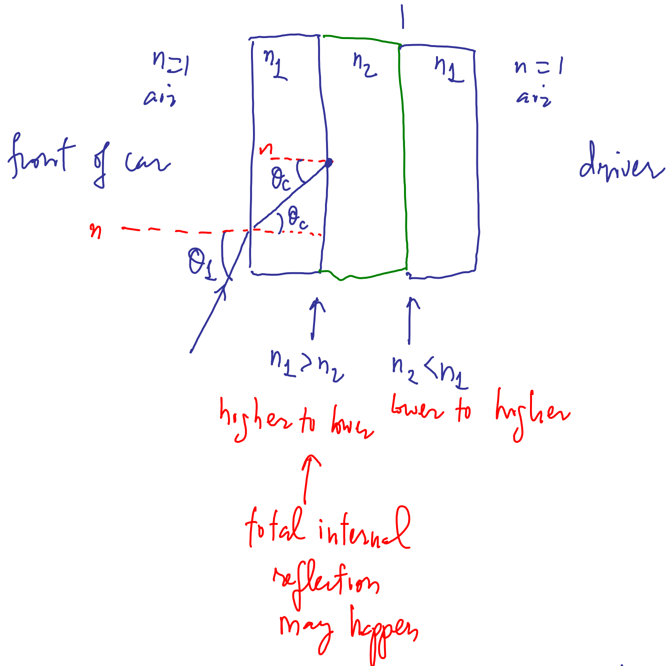
$$\rightarrow \theta_2'' = \sin^{-1}(1.582 \cdot \sin 33.5^\circ)$$

$$\boxed{\theta_2'' = 60.7^\circ}$$

$$\Rightarrow \gamma = \theta_v'' - \theta_2'' = 67.7^\circ - 60.7^\circ = 7^\circ$$

30.60 New materials for car windshields: glass - plastic - glass

$n_1 = 1.55$ $\quad n_2 = 1.48$ $\quad n_1 = 1.55$



We want to calculate what incident angle θ_1 would refract into a ray that hits the 2nd boundary @ critical angle θ_c which leads to total internal reflection (reduces visibility)

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.48}{1.55}\right) = 73^\circ$$

$\rightarrow$ At 1st boundary (air to glass): $\quad 1 \sin\theta_1 = 1.55 \sin\theta_c \rightarrow \theta_1 = \sin^{-1}(1.55 \cdot \sin 73^\circ)$

$$= \sin^{-1}(1.48)$$

Not possible!

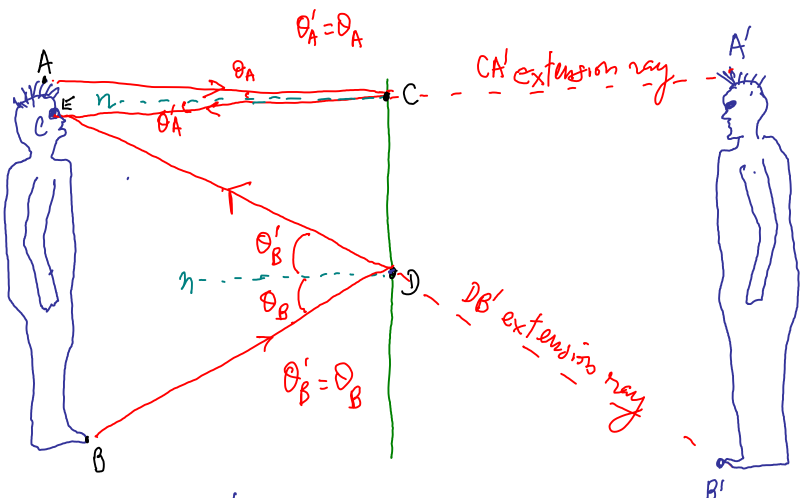
$\rightarrow$ No problem w/ total internal reflection with this new windshield!

Ch 31 Images & Optical Instruments (lenses & mirrors)

Form an image for a object through mirrors & lenses.

Mirrors:

(i) Mirror size so we can see our entire body in the mirror:



$AB = h$

$CD = \frac{h}{2}$

a) Our eyes: we see two rays come in: 1) From C 2) From D

b) We can't tell they originate at C or D

c) Through mirrors our image is formed by extension rays

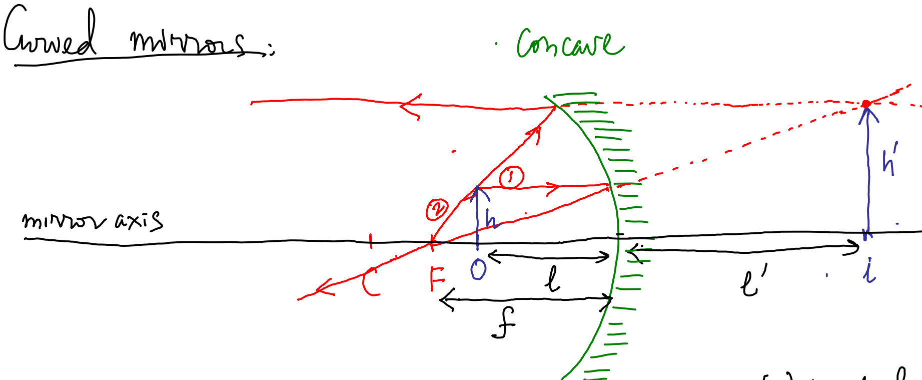
d) Extension rays are virtual (only in our mind). There is no light @ A' & B'
Image by mirrors is virtual

e) Mirror size: h, $\frac{h}{2}$, $\frac{h}{3}$

Virtual image: { 1) Formed by extension rays
2) No actual light rays converge at a virtual image

Real image: { 1) Formed by real light rays
2) Actual light rays converge at real image

(ii) Curved mirrors:

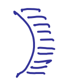


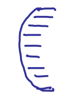
Mirrors & lenses: Standard notation:
{ O: location of object
i: location of image
h: object height
h': image height
l: position of object
l': position of image
f: focal length

Curved mirror { F: focal point → { 1) Incident ray ∥ axis → reflected ray passes thru F
2) Incident ray thru F → reflected ray emerges ∥ axis
C: center of curvature

Mirror equation: $\boxed{\frac{1}{\ell}+\frac{1}{\ell'}=\frac{1}{f}}$

↳ Sign convention:

f { + concave mirror 

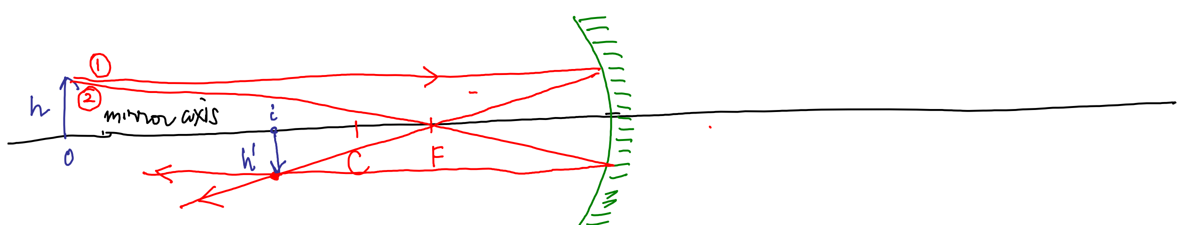
− convex mirror 

ℓ' { + if image is located same side of mirror as the object

− if image is located the other side of mirror compared to object.

Magnification factor: $M=\frac{h'}{h}\underset{\text{geometry}}{=}-\frac{\ell'}{\ell}$

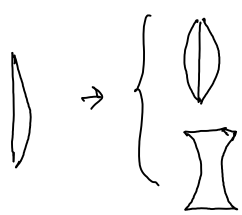
iii) Can a curved mirror form a real image?



Real image, inverted { formed by real light rays which converge @ i

Lenses: { Thin
Thick

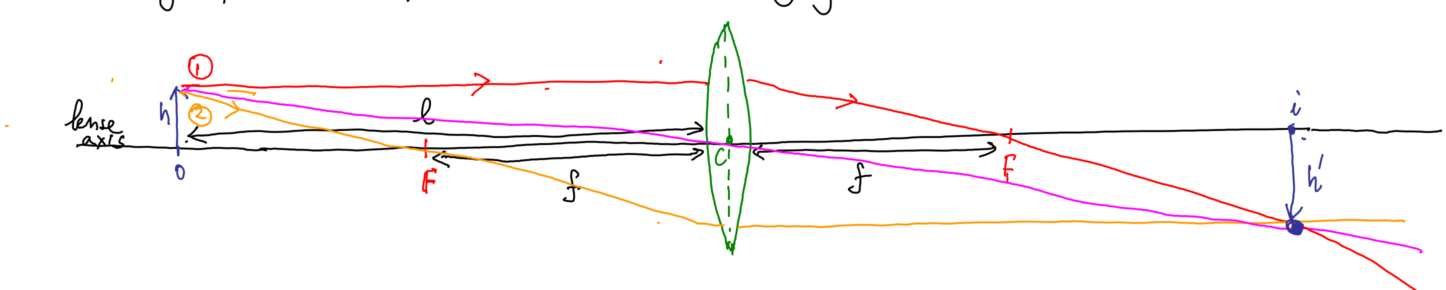
(i) Thin lenses:



converging lense or convex lense

diverging lense or concave lense

Image formation for a thin converging lense:



Ray tracing
1. Incident ray ∥ axis emerges thru F the other side of lense
2. Incident ray thru left F, emerges ∥ axis the other side of lense
3. Incident ray thru center of lense C, keeps its direction.

↳ All three converge at top of image (inverted)

(Thin) Lense Equation: $\boxed{\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}}$

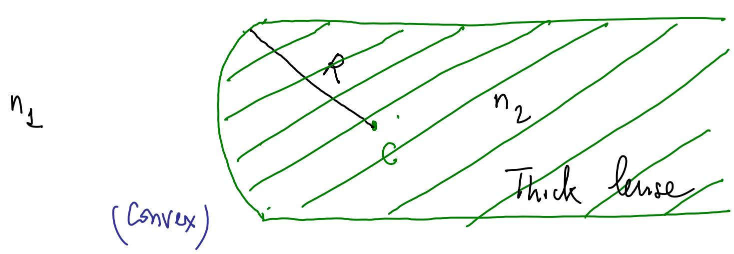
↳ Sign convention for lenses:

$f = \begin{cases} - & \text{concave or diverging lense} \\ + & \text{convex or converging lense} \end{cases}$

$\ell' = \begin{cases} + & \text{if image located other side of lense} \\ - & \text{if image located same side of lense} \end{cases}$

Magnification factor $\boxed{M = \frac{h'}{h} = -\frac{\ell'}{\ell}}$

(ii) Thick lense #1



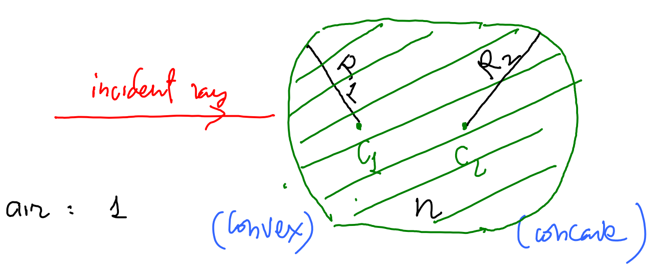
Thick lense #1 equation:

$$\boxed{\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}}$$

Sign convention:

$R = \begin{cases} + & \text{if convex} \\ - & \text{if concave} \end{cases}$ looking from incident side

(iii) Thick lense #2 (two curved boundaries)

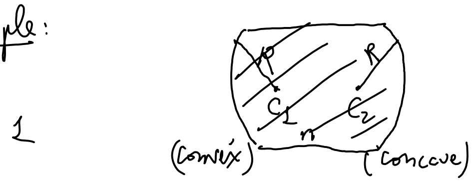


Thick lense #2 equation:

$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

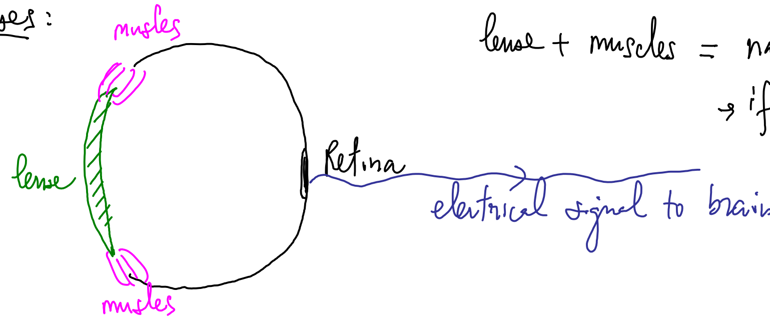
Signs: $\begin{cases} R_1 + & (\text{convex}) \\ R_2 - & (\text{concave}) \end{cases}$

Example:



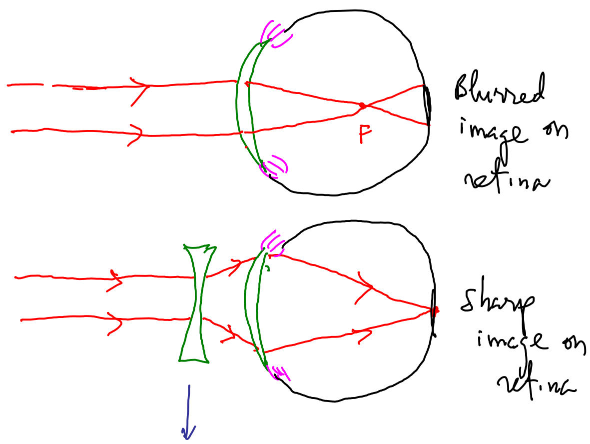
$$\Rightarrow \frac{1}{\ell} + \frac{1}{\ell'} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = (n-1)\frac{2}{R}$$

(iv) Eyes: lenses:



lense + muscles = natural focusing mechanism
→ if this is not enough to form an image on retina
↓
Corrective lenses

Near-sighted eye (myopic)
↓
Focal point short of retina

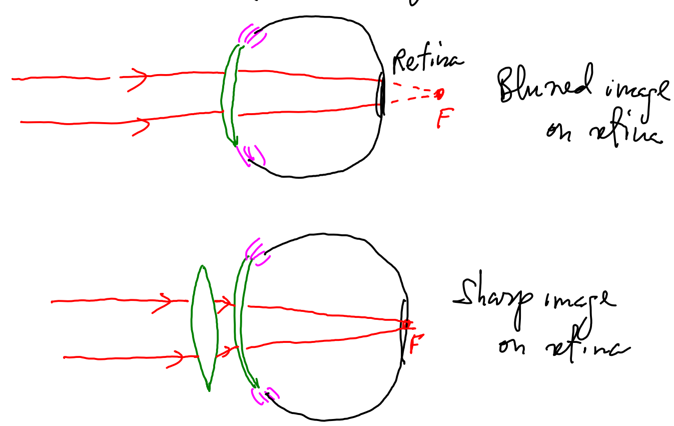


↓
Diverging corrective lense (f negative)

Diopter $= \frac{1}{f}$ (f in m)

Far-sighted eye (hyperopic)

Focal point beyond retina



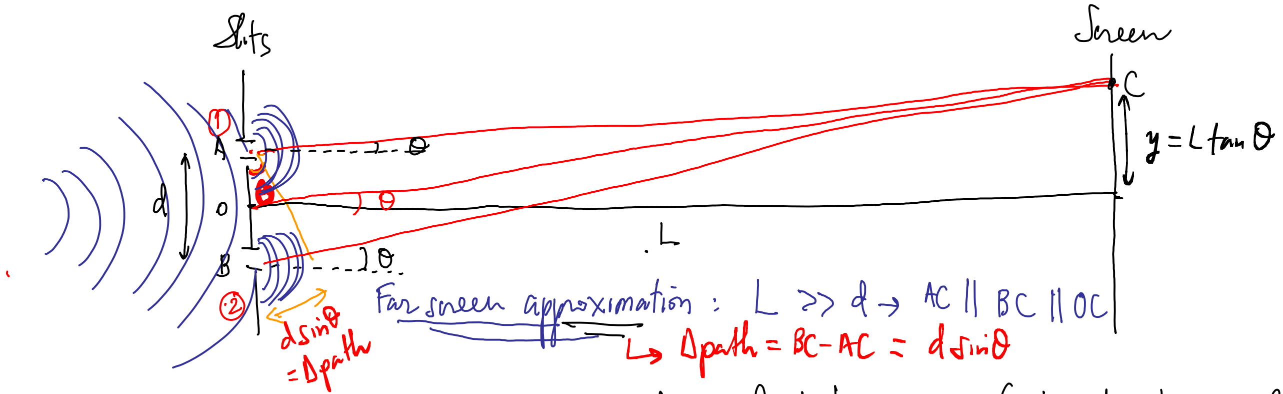
Converging corrective lense (f +)

Diopter $= \frac{1}{f}$ (f in m)

Ch 32 Interference & Diffraction
(Physical Optics)

(i) When light rays travel through small openings : no longer straight rays
(ii) Superposition of light waves similar to that of sound waves or water waves (mechanical waves) in Physics I
↳ 1) Double-slit
2) Tripple-slit
3) Diffraction
4) Thin film interference

Double-slit Interference:



Far screen approximation : $L \gg d \to AC \parallel BC \parallel OC$

$\hookrightarrow \Delta path = BC - AC = d\sin\theta$

1) EM toward two slits creating two identical baby waves (Huyghens' principle)
(same amplitudes A, same wavelengths λ, frequency f, same phase $\to$ when one is max, the other is max as well)

2) These two identical waves travel different paths AC & BC to point C on screen a distance L from slits. Superposition of 2 waves of different phase @ C :

C Bright spot or constructive interference

$$\Delta path = \underbrace{BC - AC}_{d\sin\theta} = m\lambda \quad (m = 0, 1, 2, 3, \ldots)$$

$\hookrightarrow \theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$

$\hookrightarrow y_m = L\tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$

Far screen approximation

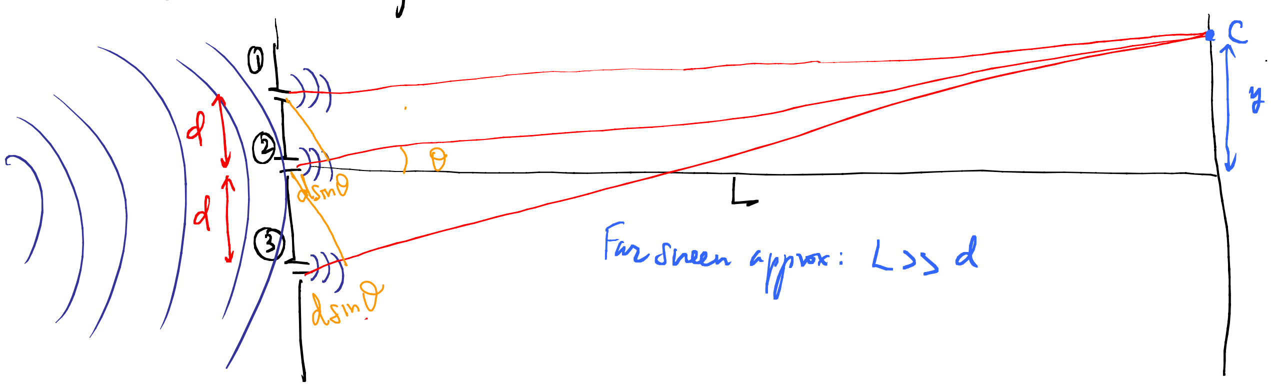
C dark spot or destructive interference

$$\Delta path = \underbrace{BC - AC}_{d\sin\theta} = (2m+1)\frac{\lambda}{2} \quad (m = 0, 1, 2, 3, \ldots)$$

$\hookrightarrow \theta_m = \sin^{-1}\left(\frac{(2m+1)\frac{\lambda}{2}}{d}\right)$

$\hookrightarrow y_m = L\tan\left(\sin^{-1}\left(\frac{(2m+1)\frac{\lambda}{2}}{d}\right)\right)$

Tripple-slit Interference :



(i) EW wave onto 3 slits creating 3 identical baby waves (same A, λ, ω, phase ϕ)

(ii) They arrive @ C after travelling different paths:

$$\Delta path_{12} = d\sin\theta \ ; \quad \Delta path_{23} = d\sin\theta \ ; \quad \Delta path_{13} = 2d\sin\theta$$

(iii) C : Bright spot $\Leftrightarrow$ constructive interference : $\Delta path = m\lambda$

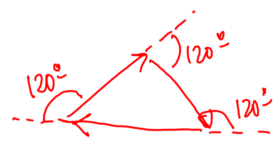
$d\sin\theta = m\lambda$ (1 & 2 or 2 & 3)

$2d\sin\theta = 2m\lambda$ (1 & 3)

$$\boxed{d\sin\theta_m = m\lambda}$$

(iv) C: dark spot ⇔ destructive interference:

↳ 3 waves → out of phase by $\frac{\lambda}{3}$, $\frac{4\lambda}{3}$, $\frac{7\lambda}{3}$, etc...



→ $\frac{360°}{3}$

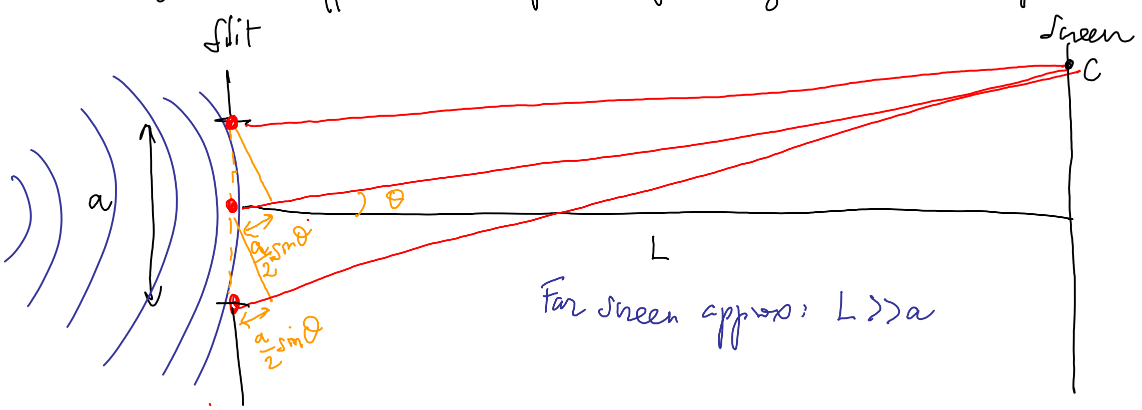
↳ $d\sin\theta_m = \left(m+\frac{1}{3}\right)\lambda$

$$\theta_m = \sin^{-1}\left[\frac{\left(m+\frac{1}{3}\right)\lambda}{d}\right]$$

⇒ Generalize for N slits! : $d\sin\theta_m = \left(m+\frac{1}{N}\right)\lambda = (Nm+1)\frac{\lambda}{N}$

Diffraction: Superposition of waves out of a same slit: each point in a slit is the source of a baby wave.

↳ Single-slit diffraction → pattern of bright & dark spots on a screen



Far screen approx: $L \gg a$

(i) EM wave to slit → Huyghens principle → ∞ number of baby waves in slit.

(ii) Far screen approx. $L \gg a$ → $\Delta path = \begin{cases} \frac{a}{2}\sin\theta & (1 \& 2 \text{ or } 2 \& 3) \\ a\sin\theta & (1 \& 3) \end{cases}$ } looking @ only 3 sources

(iii) Dark spots : $\Delta path = (2n+1)\frac{\lambda}{2}$

$\frac{a}{2}\sin\theta$ → $a\sin\theta = (2n+1)\lambda = \lambda, 3\lambda, 5\lambda$, etc...

$a\sin\theta$ → $a\sin\theta = (2n+1)\frac{\lambda}{2} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$, etc...

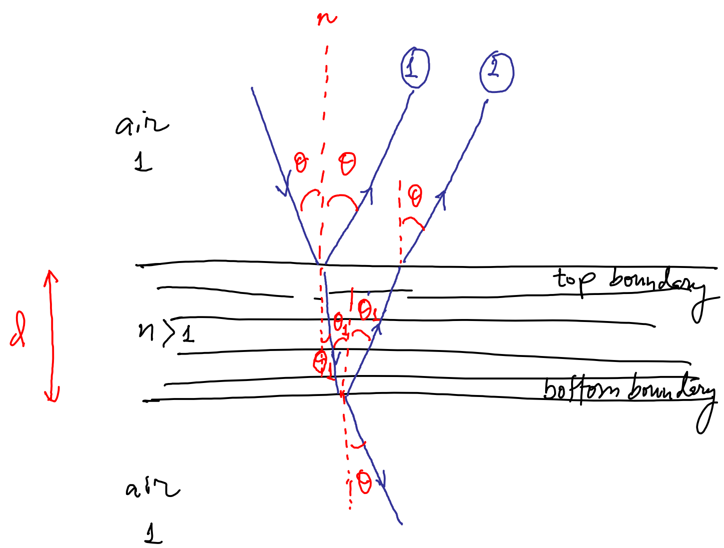
looking @ all sources in slit → $\boxed{a\sin\theta = n\lambda \quad (n=1,2,3,\text{ etc.})}$

dark spot in single-slit diffraction. ($n \neq 0$)

Diffraction limit: (optical instrument resolution): $\theta_{min} = \frac{1.22\lambda}{D}$ ← diameter of lense!

Thin film interference : geometrical + physical optics

Light incident on a thin film :



Rays 1 & 2 are parallel

1 Reflected ray off top boundary of thin film. Since it reflects off a higher index medium → wave is inverted → acquires an extra phase of π or $\Delta path = \frac{\lambda}{2}$

2 Comes from a reflection off bottom boundary or it reflects off a lower index medium → wave is not inverted.
By travelling top – bottom – top this ray acquired $\Delta path = 2d$ (thin film)

Superposition of waves 1 & 2 :

↳ Constructive interference (bright rings) :

$$\Delta path_{12} = 2d - \frac{\lambda}{2} = n\lambda \quad (n = 0, 1, 2, \ldots)$$

$$\boxed{2d = n\lambda + \frac{\lambda}{2}} = \left(n + \frac{1}{2}\right)\lambda = \boxed{(2n+1)\frac{\lambda}{2}} \quad n = 0, 1, 2, \ldots$$

↳ Destructive interference (dark rings) :

$$\Delta path_{12} = 2d - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad (n = 0, 1, 2, \ldots)$$

$$2d = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2} = 2n\frac{\lambda}{2} + \lambda$$

$$\boxed{2d = (n+1)\lambda \quad (n = 0, 1, 2, \ldots)}$$

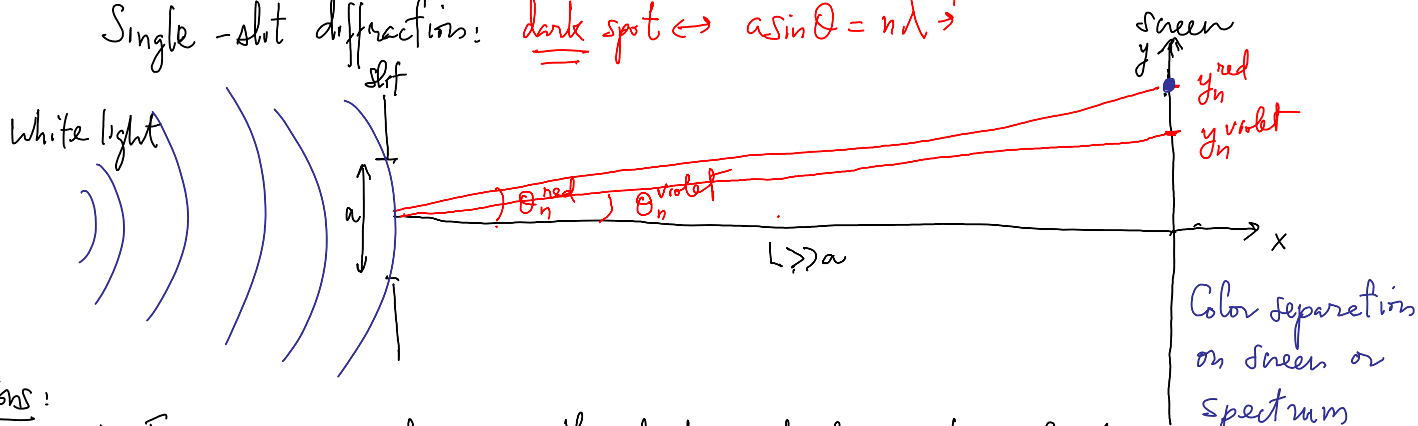
32.41

Visible Light Spectrum $\begin{cases} \lambda_v = 400\,nm \\ \lambda_r = 700\,nm \end{cases}$

$\theta_n^{red} = \sin^{-1}\left(\frac{n\lambda_{red}}{a}\right)$

$\theta_n^{violet} = \sin^{-1}\left(\frac{n\lambda_{violet}}{a}\right) < \theta_n^{red}$

Single-slit diffraction: dark spot $\leftrightarrow$ $a\sin\theta = n\lambda$



Conclusions:

(i) For a same order n, the dark spot for red is further up from center of screen than that for violet $\theta_n^{red} > \theta_n^{violet}$

(ii) A red dark spot of order n may overlap with the next violet dark spot of order $n+1$

$$\sin\theta_n^{red} = \sin\theta_{n+1}^{violet}$$

$$\frac{n\lambda_{red}}{\cancel{a}} = \frac{(n+1)\lambda_{violet}}{\cancel{a}}$$

$$n\lambda_{red} = n\lambda_{violet} + \lambda_{violet}$$

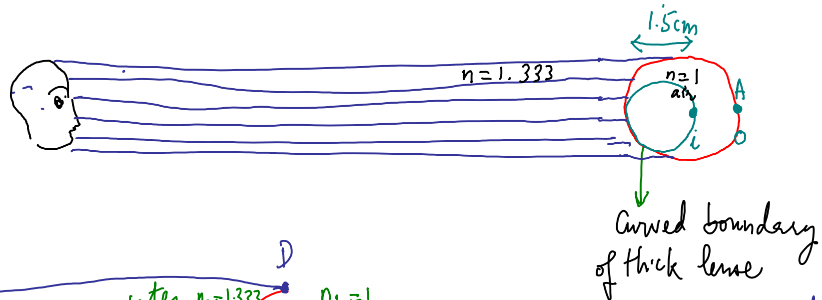
$$n = \frac{\lambda_{violet}}{\lambda_{red} - \lambda_{violet}} = \frac{400\,nm}{700\,nm - 400\,nm} = \frac{4}{3} = 1.33$$

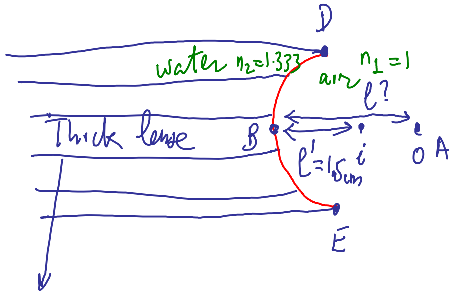
→ Any order above 1.33 or 2 or higher will have some overlap (dark red and next dark violet)

→ Only order 1 (1st in spectrum) has no overlaps!

31.32

Spherical air bubble under water (through a thick lense made of water) appears to have $d = 1.5\,cm$ → Actual diameter of air bubble?





$$\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}$$

(i) i image of A through concave lens DBE position is $\boxed{\ell' = -1.5\text{ cm}}$ → bubble appears to have a diameter of 1.5 cm

Sign convention: a) ℓ' is negative due to sign convention for lenses: image is on same side of lense as object

b) Light incident from the right → lense DBE is concave thick lense

$\boxed{R \text{ is negative}}$

(ii) Light incident from air to water

$\boxed{n_1 = 1 \text{ (air)}; \quad n_2 = 1.333 \text{ (water)}}$

(iii) location of object is actual diameter of bubble: $\boxed{\ell = 2R}$ (distance BA)

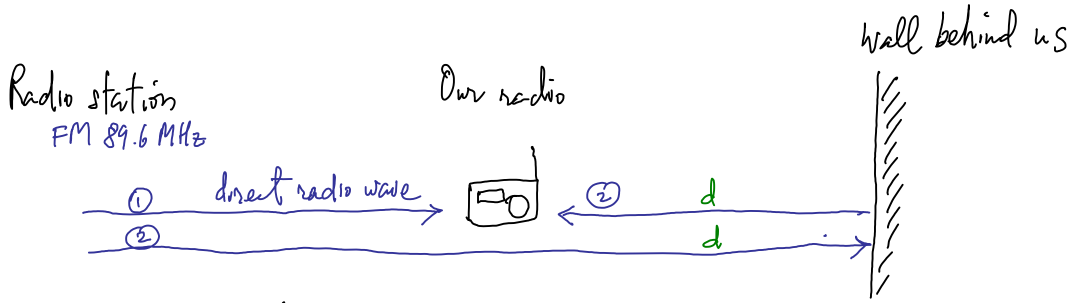
$$\frac{1}{2R} + \frac{1.333}{-1.5\text{ cm}} = \frac{1.333 - 1}{-R}$$

$$= \frac{0.333}{-R} = \frac{0.666}{-2R} \rightarrow \frac{1.666}{2R} = \frac{1.333}{1.5} \rightarrow 2R = \frac{1.666 \cdot 1.5}{1.333}$$

$$\boxed{2R = 1.87\text{ cm}}$$

Actual diameter of bubble

32.56



1. Direct wave from radio station
2. Reflected wave off wall (extra phase of π or extra path $\frac{\lambda}{2}$) with extra path of $2d$

$$\Delta path_{12} = 2d - \frac{\lambda}{2}$$

→ Dead spot (destructive interference) at current location of our radio:

$$\Delta path_{12} = 2d - \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \quad (m = 0, 1, 2, 3, \text{etc...})$$

$$\hookrightarrow d = \frac{1}{2}\left[(2m+1)\frac{\lambda}{2} + \frac{\lambda}{2}\right] = \frac{1}{2}\left[m\lambda + \lambda\right]$$

$$\boxed{d_{dead} = (m+1)\frac{\lambda}{2}}$$

→ Clear spot (constructive interference)

$$\Delta path_{12} = 2d - \frac{\lambda}{2} = m\lambda \quad (m = 0, 1, 2, 3, \text{etc...})$$

$$\boxed{d_{clear} = \frac{1}{2}\left(m\lambda + \frac{\lambda}{2}\right) = \left(m + \frac{1}{2}\right)\frac{\lambda}{2}}$$

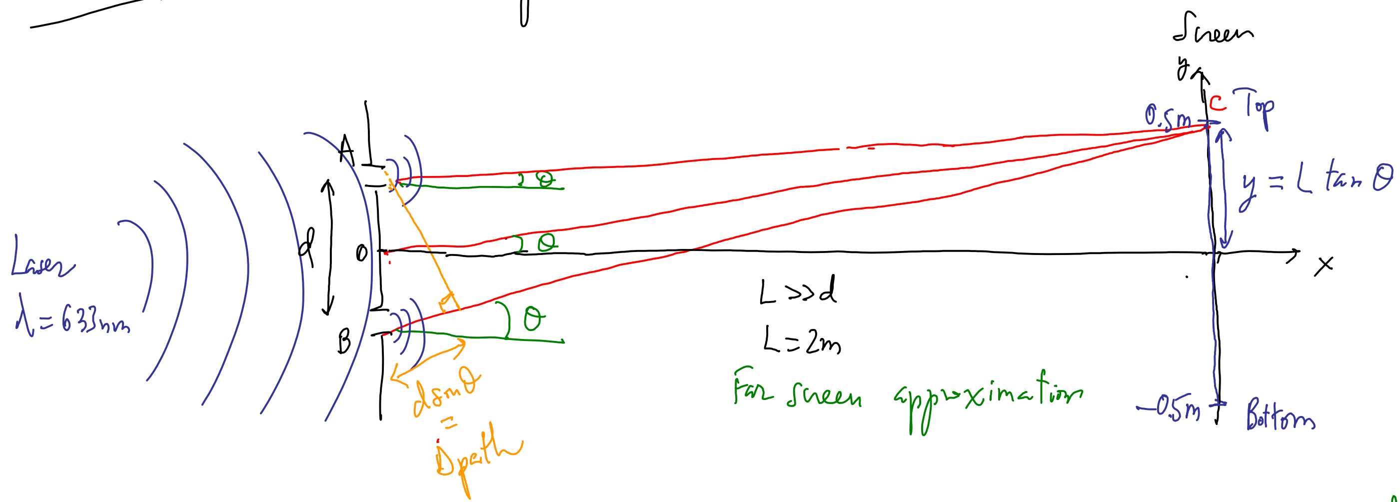
Current dead spot → m → next clear spot → $m+1$: what is the distance to move?

$$\underset{(m+1)}{d_{clear}} - \underset{(m)}{d_{dead}} = \left(m+1+\frac{1}{2}\right)\frac{\lambda}{2} - (m+1)\frac{\lambda}{2} = \frac{\lambda}{4} = \frac{3.352}{4} = 0.838\,m$$

$$f = 89.5 \cdot \underbrace{10^6}_{M}\,Hz \rightarrow c = \lambda \cdot f \rightarrow \lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \cdot 10^{\cancel{8}\,2}}{89.5 \cdot \cancel{10^6}} = 3.352\,m$$

32.37 Two-slit interference:



Highest order bright fringe that fits on screen?

$\downarrow$ m_{max}? $\downarrow$ constructive interference $\downarrow$ $y = 0.5\,m$ (top of screen)

$d = 0.1\,mm = 10^{-4}\,m$

$d = 10\,\mu m = 10^{-5}\,m$

$\downarrow$

$$D_{path} = m\lambda \quad (m = 0, 1, 2, 3 \text{ etc...})$$

$$d\sin\theta = m\lambda \rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

$$y = L\tan\theta = L\tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$$

$$0.5 = 2\tan\left(\sin^{-1}\left(m_{max}\cdot\frac{633\cdot 10^{-9}}{d}\right)\right)$$

(i) $d = 10^{-4}\,m$ $\rightarrow$ $$\tan^{-1} 0.25 = \sin^{-1}\left(m_{max}\frac{633\cdot 10^{\cancel{-9}\,-5}}{10^{\cancel{-4}}}\right)$$

$$m_{max} = \frac{\sin\left(\tan^{-1} 0.25\right)}{633\cdot 10^{-5}} = 38.22$$

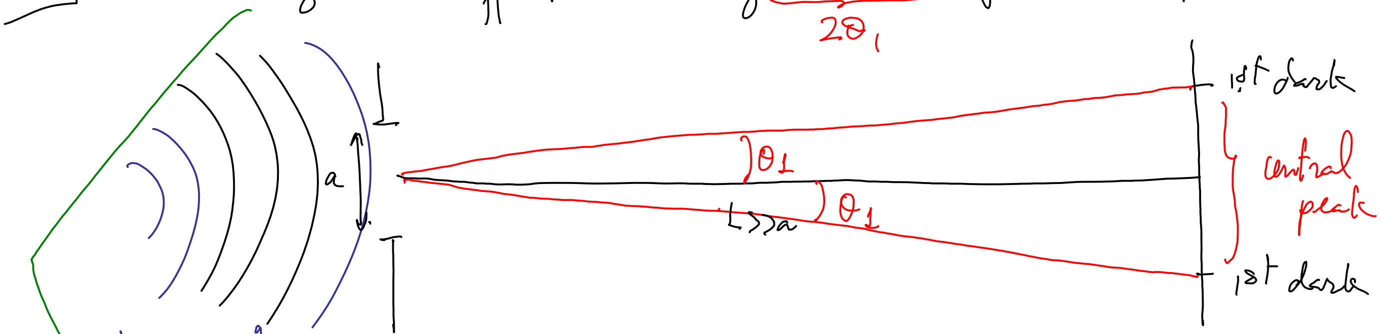
$\downarrow$ orders in whole numbers $\rightarrow$ $\boxed{m_{max} = 38}$

(ii) $d = 10^{-5}\,m$ $\rightarrow$ $$m_{max} = \frac{\sin\left(\tan^{-1} 0.25\right)}{633\cdot 10^{-4}} = 3.822$$

$\boxed{m_{max} = 3}$

$\downarrow$ reason: smaller slit spacing fringes are further apart $\rightarrow$ only 3 will fit.

32.27 Single-slit diffraction: angular width of central peak.
$\underbrace{\text{angular width}}_{2\theta_1}$



$\lambda = 633 \cdot 10^{-9}$ m
$a = 2.5 \cdot 10^{-6}$ m

Dark spots: $a \sin\theta = n\lambda$ $(n = 1, 2, 3, \text{etc.})$

1st dark spot: $\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{633 \cdot 10^{-9}}{2.5 \cdot 10^{-6}}\right) = 14.7°$

$$\boxed{2\theta_1 = 29.4°}$$

31.42 (i) Concave mirror (f negative)
(ii) Virtual image (the other side of mirror) (ℓ' negative)
(iii) $M = \frac{h'}{h} = -\frac{\ell'}{\ell} = 1.8$ (positive → image is upright!) ⇒ $\ell' = -1.8\,\ell$
(iv) $\ell = 22$ cm

↳ Calculate radius of curvature R for concave mirror: from focal length: $|f| = \frac{R}{2}$

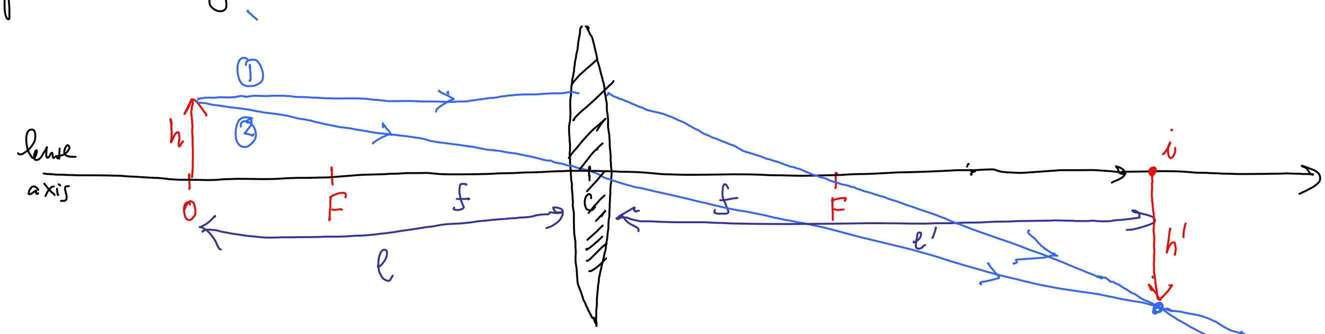
focal length f: from mirror equation:

$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f} \rightarrow f = \left(\frac{1}{22\text{cm}} + \frac{1}{-1.8 \cdot 22}\right)^{-1} = \left[\frac{1}{22}\left(1 - \frac{1}{1.8}\right)\right]^{-1}$$

$f = 49.4$ cm → $R = 98.8$ cm

31.50 Calculate position of image through a converging lense - or convex lense → f is positive: $f = +35$ cm

a) $\ell = 40$cm



ℓ' positive: image the other side of lense → sep b/w o & i is $\ell + \ell'$

lense equation: $\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \rightarrow \frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{l-f}{lf} \rightarrow l' = \frac{lf}{l-f}$

$\downarrow$

$l' = \frac{40 \cdot 35}{40-35} = 280\,cm \rightarrow$ sep. o & i is $l + l' = 40 + 280 = 320\,cm$

b) $l = 30\,cm$

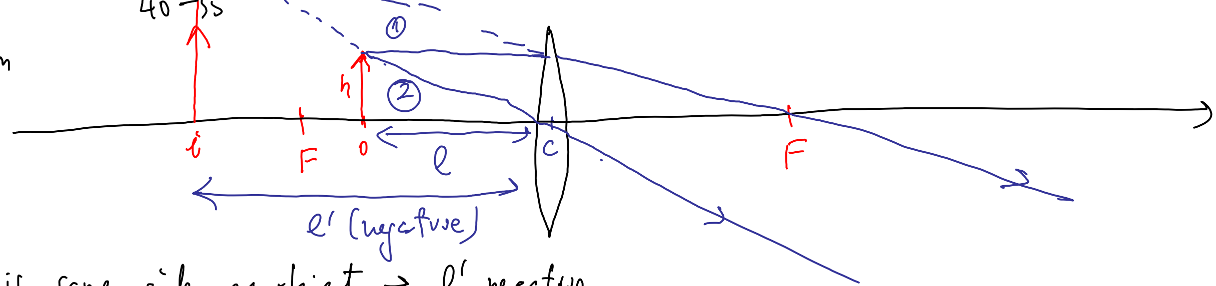


image is same side as object $\rightarrow$ l' negative

image is formed by extension rays $\rightarrow$ virtual & upright

$l' = \frac{30 \cdot 35}{30-35} = -210\,cm \rightarrow$ sep. b/w o & i $|30 - 210| = 180\,cm$

Ch 27 Faraday's Law : 1) time-varying magnetic flux induces a voltage along a closed loop where the flux goes through, that opposes the change of magnetic flux:

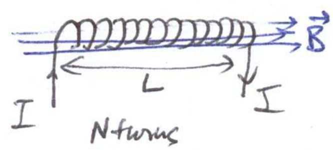
$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

2) w/o a wire the induced voltage or electric field is there. If there is a wire on that loop, the induced $\vec{E}$ drives a current in that wire

Inductance
- self $L = \frac{\Phi}{I}$ (one solenoid)
- mutual $M = \frac{\Phi_2}{I_1}$ (two solenoids)

solenoid of { N turns, length L, I in each turn } needs $B = \mu_0 \underbrace{\frac{N}{L}}_{n} I$

n: # turns per unit length.

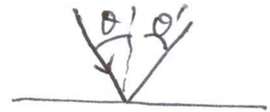


Energy
- electric : $U_E = \frac{1}{2} C V^2 \longleftrightarrow$ C inertia to change in voltage
  $u_E = \frac{1}{2} \varepsilon_0 E^2$
- magnetic : $U_M = \frac{1}{2} L I^2 \longleftrightarrow$ L inertia to changes in current
  $u_M = \frac{1}{2\mu_0} B^2$

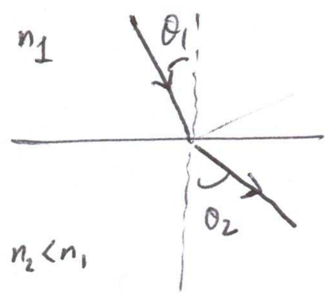
Ch 29 Maxwell's equation:
1) Displacement current $\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$ allows complete symmetry b/w equations relating $\vec{E}$ & $\vec{B}$ in vacuums ; $v = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} =$
Ampere's & Faraday's law
$E(t) \to B(t) \to E(t) \dots \to$ propagates in vacuum !
2) Vector nature of $\vec{E}$ & $\vec{B}$ $\to$ polarization
Intensity $S \propto E^2$

Radiation pressure: $\frac{S}{c}$ (pressure comes from transfer of momentum to the reflecting surface)

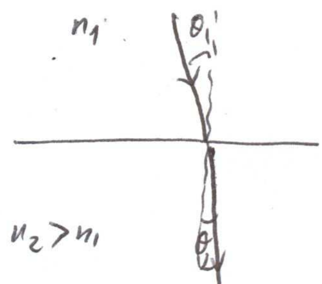
Ch 30 Reflection & Refraction



- Law of reflection $\theta' = \theta$
- Law of refraction or Snell's Law : $n_1 \sin\theta_1 = n_2 \sin\theta_2$



wave front is rotated CCW



→ wave front rotated CW

→ if $\theta_1 \geq \theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

→ Total internal reflection

- Brewster's angle: $\vec{E}$ in plane of page & $\theta = \theta_B$ or $\theta_P = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

  → No reflection, all refraction.

- Multiple mirrors → use triangular geometry to relate outgoing ray direction to incident ray direction
- Dispersion occurs when the index of refraction n varies for different wavelengths that come in the incident light.

ch 31 Lenses & Mirrors

Mirror equation : $\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$

$M = \frac{h'}{h} = -\frac{\ell'}{\ell}$

f { + concave, − convex

ℓ' { + i same side as o, − i other side as o

Lenses

- Thin : $\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$

  f { − concave or diverging lenses, + convex or converging lenses

  ℓ' { + i opposite side as o, − i same side as o

- Thick 1 boundary : $\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}$ R { + convex, − concave From incident side

- Thick 2 boundaries $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ R { + convex, − concave

Ch 32

Interference & diffraction

Interference

spacing b/w slits is d

two-slit:
- Bright spot $\Delta path = d\sin\theta = m\lambda$ $(m = 0, 1, 2, 3\ etc.)$
  $\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$; $y_m = L\tan\theta_m$
- Dark spot $\Delta path = d\sin\theta = (2m+1)\frac{\lambda}{2}$ $(m = 0, 1, 2, 3, etc.)$
  $\theta_m = \sin^{-1}\left(\frac{(2m+1)\lambda}{2d}\right)$; $y_m = L\tan\theta_m$

three-slit:
- Bright spot: $\Delta path = d\sin\theta = m\lambda$
- Dark spot: $\Delta path = d\sin\theta = (m+\frac{1}{3})\lambda$ $(m = 0, 1, 2, 3\ etc$
  $\theta_m = \sin^{-1}\left[\frac{(m+\frac{1}{3})\lambda}{d}\right]$; $y_m = L\tan\theta_m$

thin film (thickness is d and index is n):
- constructive: $2d = (2n+1)\frac{\lambda}{2}$ $(n = 0, 1, 2, 3, etc.)$
- destructive: $2d = (n+1)\lambda$ $(n = 0, 1, 2, 3\ etc.)$

This λ wavelength in the film: $\lambda = \frac{\lambda_0}{n}$

Diffraction: dark spots: $a\sin\theta = n\lambda$ $(n = 1, 2, 3, etc...)$

↳ single slit of width a.

↳
- → overlap of ~~order~~ dark spots for red & violet of consecutive order
- → Optical instrument resolution: $\theta_{min} = \frac{1.22\lambda}{D}$; D: diameter of of lense or slit