Ch27 Electromagnetic Induction

Faraday's Law: $-\dfrac{d\phi_B}{dt} = \mathcal{E}$ (induced voltage)

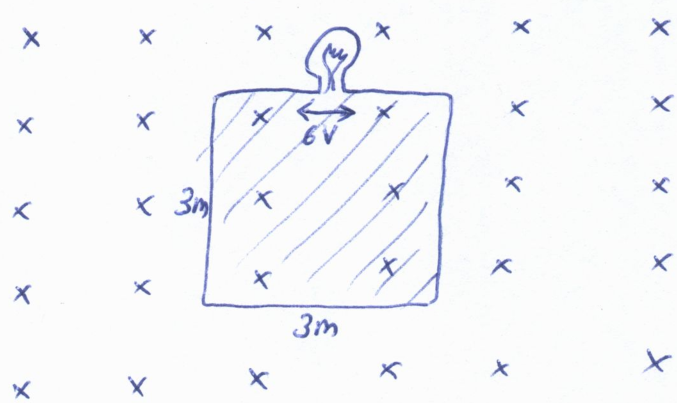
Magnetic flux: through a surface $\phi_B \equiv \int \vec{B} \cdot d\vec{A}$

→ any magnetic flux which varies over time creates an induced voltage

→ Conservation of energy (Lenz's Law)

27.37 Region of uniform magnetic field $\vec{B}$ into the page:

B = 2T into page (reduces steadily to 0T over time Δt)



Surface enclosed by loop is $9 m^2$ constant over time, since B decreases over time → ϕ_B varies over time → induced voltage $\mathcal{E}$ in loop which turns on light bulb.

a) Find Δt so lightbulb will reach full brightness ($\mathcal{E} \to 6V$)

Faraday's Law: $\mathcal{E} = -\underbrace{\dfrac{d\phi_B}{dt}}_{\text{instantaneous}} = -\underbrace{\dfrac{\Delta\phi_B}{\Delta t}}_{\text{average}} = -\dfrac{\Delta B \cdot A}{\Delta t} = -\dfrac{(0-2)\cdot 9}{\Delta t}$

$$\boxed{\mathcal{E} = \frac{18}{\Delta t}}$$

$6 = \dfrac{18}{\Delta t} \to \Delta t = 3s$: In 3s $\mathcal{E} \to 6V$ & light bulb reaches full brightness!

b) Which way will loop current flow?

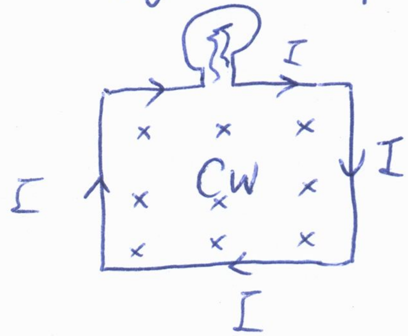
(i) Induced current flows in such a way to oppose the change in magnetic flux through the loop: due to the $\ominus$ sign in Faraday's Law:

$$\mathcal{E} = \ominus \frac{d\phi_B}{dt}$$

(ii) What is the change in ϕ_B? Since B into page decreases over time, and A = constant $\rightarrow$ ϕ_B into page decreases.

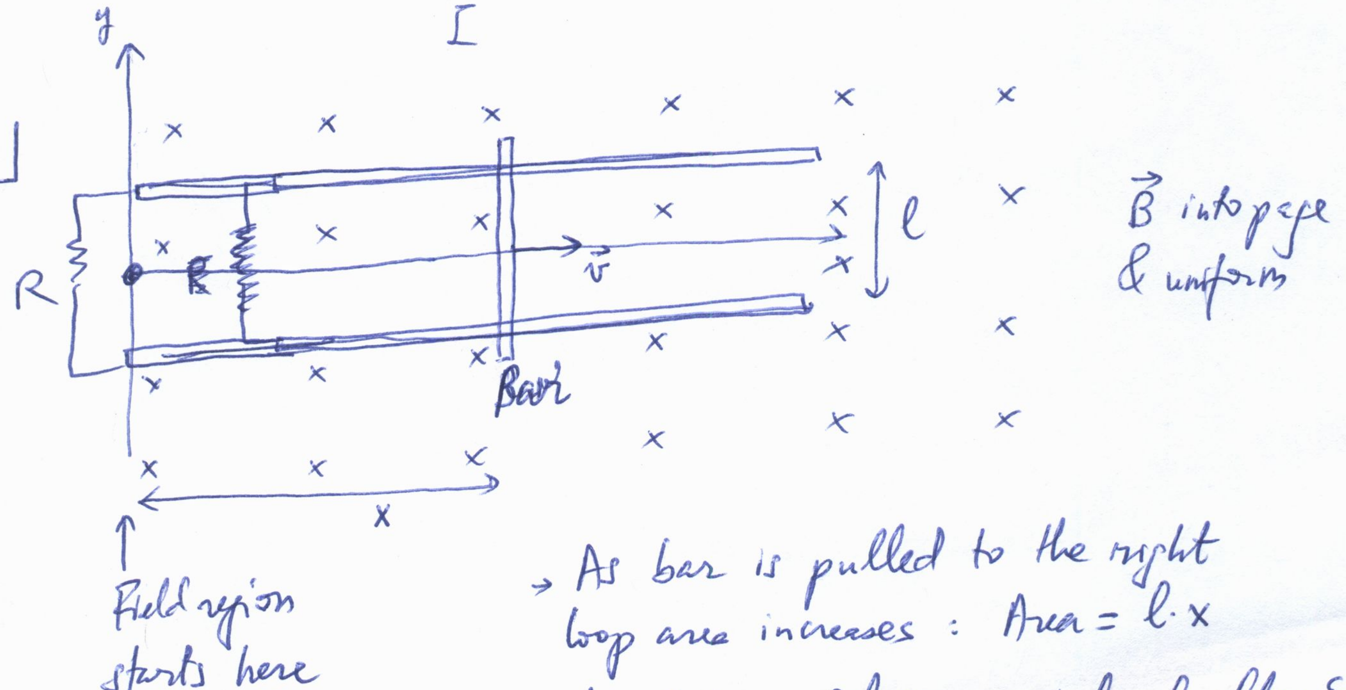
$\rightarrow$ Induced current will flow in such a way to create a magnetic field also into page to compensate for the reduction.

<u>RHR</u>:



Current will flow in CW around loop!

27.44



$\rightarrow$ As bar is pulled to the right loop area increases: Area $= l \cdot x$

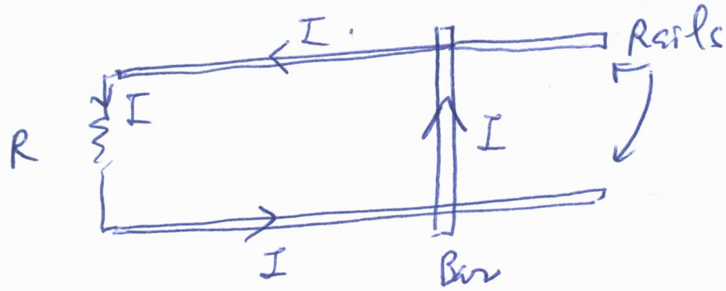
$\phi_B = B \cdot A = B \cdot l \cdot x$ $\rightarrow$ induced voltage $\mathcal{E}$

Faraday's Law: $\mathcal{E} = \ominus \frac{d\phi_B}{dt} = -B \cdot l \cdot \frac{dx}{dt} = -Blv$

a) Direction of induced current? It will flow in such a way to oppose the increase of ϕ_B into page (as A increases with bar moving to the right)

That means induced current flows in a way to create a magnetic field out of page.

RHR : current CCW to create magnetic field out of page



Current goes down @ R !

b) Rate of work (power) applied to move bar ?

$$P = \frac{W}{t} = \frac{F_B \cdot x}{t} = F_B \cdot v = Il\cancel{B}v = \frac{\mathcal{E}}{R} lBv$$

Lorentz's force : $F_B = qvB$

$$= q\frac{l}{t}B = IlB$$

$$\mathcal{E} = -Blv$$

$$P = - \frac{(Blv)^2}{R}$$

Bar does negative work
or it receives work (we need to apply work on bar to move it)

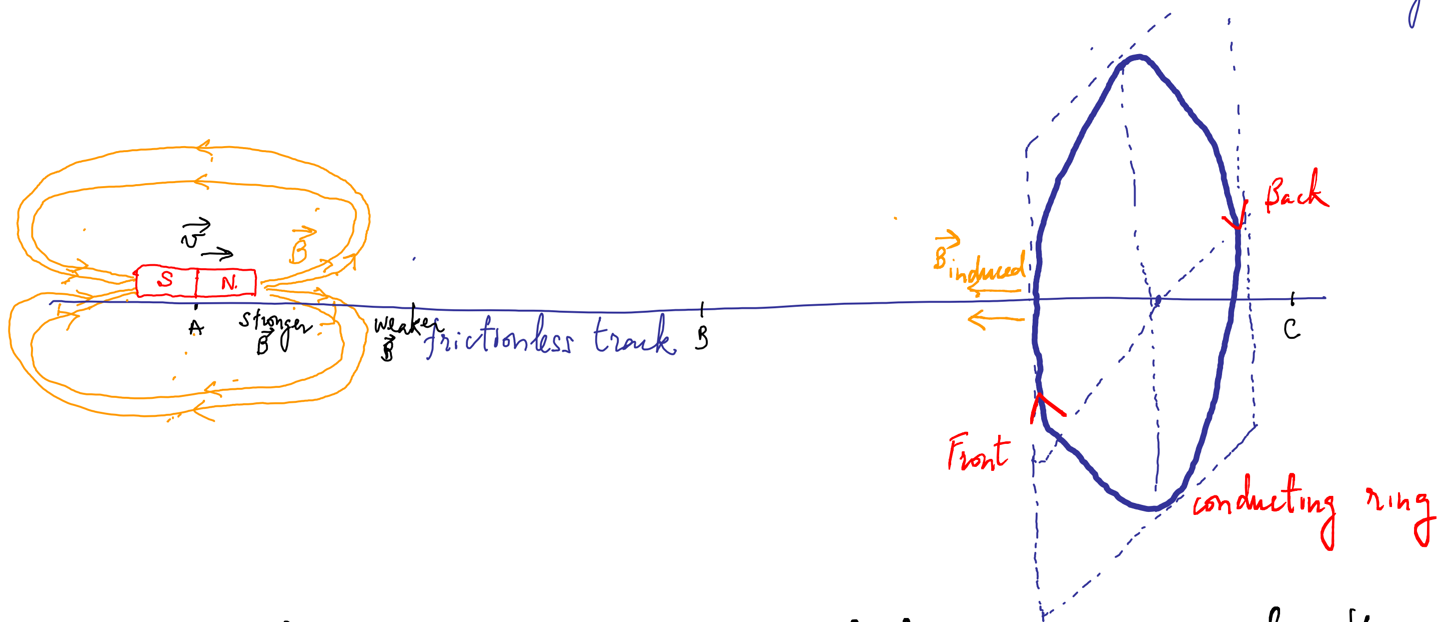
Alternative : $P = I \cdot V = \frac{\mathcal{E}^2}{R} = \frac{(Blv)^2}{R}$

$\frac{\mathcal{E}}{R} \cdot \mathcal{E}$

Ch27 (cont.): EM Induction & Conservation of Energy :

↳ Visual experiment: sliding magnet & conducting ring

↳ Can hold an induced current when there is an induced voltage.



1) As magnet slides from A to B @ constant speed v: magnetic flux Φ_B through the cross-sectional area enclosed by conducting ring increases over time. → $\frac{d\Phi_B}{dt} \neq 0$: Faraday's Law of Induction this induces a voltage $\mathcal{E} = - \frac{d\Phi_B}{dt}$ in the conducting ring

↓ Lenz's Law: the induced voltage $\mathcal{E}$ opposes the change in Φ_B: the induced current goes up in the front & down in the back to create a magnetic field in the opposite direction (so a magnetic flux in that direction) to counter the increase of the original Φ_B by the sliding magnet.

2) After the magnet is given a push, it slides from A to B @ speed v. As it approaches the ring the speed will decrease as some of its KE is transferred to the induced current in the ring. (A conducting ring has some electrical resistance, small but it's there)

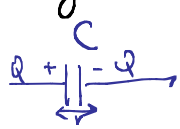
3) When magnet passes the ring toward C: Φ_B by magnet is now decreasing, the induced current in the ring decreases to 0 then reverses direction to create a counter magnetic field pointing to the right (same direction

as original field by magnet) to compensate for the reduction in ϕ_B.
(Induced $\mathcal{E}$ and I always opposes changes in ϕ_B $\Leftrightarrow$ minus sign in Faraday's Law)
The reversed current will disappear when $\frac{d\phi_B}{dt}$ goes back to 0, when energy is returned to magnet which will pick up speed.

4) This electro-mechanical energy transfer is used in many technological applications: electric motor, invisible fence for supermarket cart, etc...

Inductance & Magnetic Energy

Capacitors
(storing electric energy)

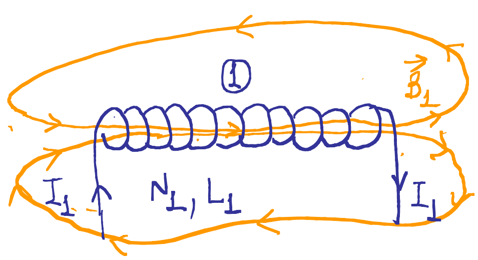


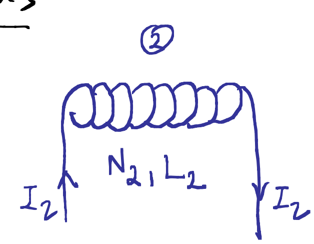
$$C = \frac{Q}{V} \quad \text{(Capacitance, F)}$$

Inductors
(storing magnetic energy)

- Self-inductance $L = \frac{\phi}{I}$ (H) Henry
  ϕ: magnetic flux
- Mutual-inductance $M = \frac{\phi_2}{I_1}$ (H)
  (new in magnetic energy)

Two solenoids





$B_1 = \mu_0 n_1 I_1$

$n_1 = \frac{N_1}{L_1}$ (# turns per unit length)

Also called an electromagnet
(Superconducting electromagnet to output stronger B)

$B_2 = \mu_0 n_2 I_2$

$n_2 = \frac{N_2}{L_2}$ (# turns per unit length)

1) B_1 goes through cross-sectional area of solenoid 2 creating a magnetic flux

$\boxed{\Phi_2} = B_1 \cdot A_2 \cdot N_2 = \mu_0 n_1 \boxed{I_1} A_2 N_2$

So if I_1 changes over time → so does ϕ_2 → Faraday's Law: $\mathcal{E}_2 = -\frac{d\Phi_2}{dt}$
($\mathcal{E}_2$ is induced in #2 if current I_1 changes in #1 !)

$$-\mathcal{E}_2 = \frac{d\Phi_2}{dt} = \underbrace{\mu_0 n_1 N_2 A_2}_{\equiv M} \frac{dI_1}{dt}$$

$\equiv M$ (mutual inductance: relates induced voltage in #2 with a changing current in #1)

$$\boxed{\phi_2 = M I_1 \text{ or } M = \frac{\phi_2}{I_1}}$$

2) And viceversa: B_2 by solenoid #2 goes through cross-sectional area A_1 of solenoid #1 → creates a magnetic flux through #1 or Φ_1

$\boxed{\Phi_1} = B_2 \cdot A_1 \cdot N_1 = \mu_0 n_2 \boxed{I_2} A_1 N_1$

If I_2 varies over time → induces a voltage $\mathcal{E}_1$ in #1

$$-\mathcal{E}_1 = \frac{d\phi_1}{dt} = \underbrace{\mu_0 n_2 N_1 A_1}_{M} \frac{dI_2}{dt}$$

M (mutual inductance: relates induced voltage in #1 with a changing current in #2)

SI Unit: $M = \frac{\text{Voltage}}{\frac{\text{Current}}{\text{time}}} \xrightarrow{\text{Units}} \frac{V}{\frac{A}{s}} = \frac{V \cdot s}{A} \equiv H$ (Henry)

3) Also $\vec{B}_1$ goes through its own cross-sectional area A_1 (& $\vec{B}_2$ through A_2)

↳ $\boxed{\phi} = B_1 \cdot A_1 \cdot N_1 = \mu_0 n_1 \boxed{I_1} A_1 N_1$

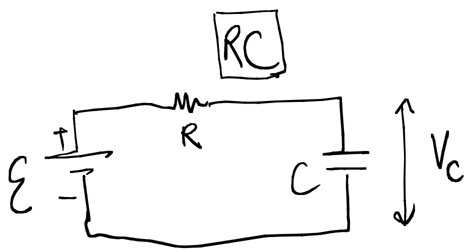
If I_1 varies over time, it will induce a voltage in the solenoid #1 (in addition to inducing an $\mathcal{E}_2$ in solenoid #2!): self-induced voltage $\mathcal{E}$

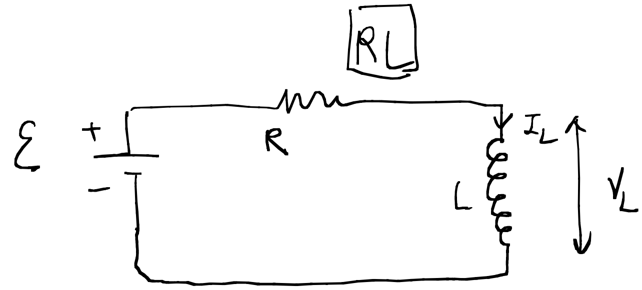
$$-\mathcal{E} = \frac{d\phi}{dt} = \underbrace{\mu_0 n_1 N_1 A_1}_{L} \frac{dI_1}{dt}$$

L (self-inductance)

$\phi = L I_1$ → $\boxed{L = \frac{\phi}{I}}$ (SI unit: H)

Electric & Magnetic Circuits





$t=0$ (switch is just closed)

C initially uncharged:
$V_C = 0 \to$ C acts like a short circuit

L initially uncharged
$I_L = 0 \to$ L acts like an open circuit

$t \to \infty$ (switch is closed sufficiently long)

C fully charged
$I_C = 0 \to$ C acts like an open-circuit

L fully charged
$I_L = max \to V_L = 0 \to$ L acts like a short-circuit

$0 < t < \infty$ (in between)

$$I(t) = \underbrace{\frac{\mathcal{E}}{R}}_{I(0)} e^{-\frac{t}{RC}}$$

max when $t=0$

$$V_C(t) = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

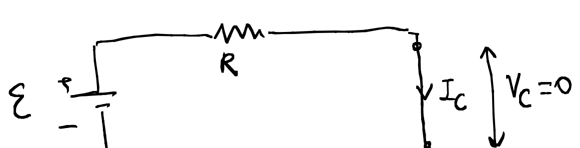
$\tau = RC$ time constant

$$V_L(t) = \mathcal{E} e^{-\frac{t}{(L/R)}}$$

$$I_L(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{(L/R)}}\right)$$

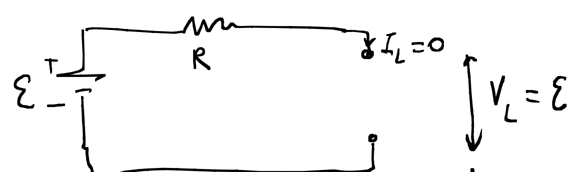
$\tau = \frac{L}{R}$ time constant

$t=0$



C acts like short-circuit

$t=0$



L acts like open-circuit

(i) As switch is closed: V_C which was 0 (C initially uncharged) stays at 0. Because of C, V_C doesn't change instantaneously (unlike V_R across R, which turns on instantaneously as switch is closed)

Capacitance = electric inertia to changes in voltage V_C

(i) As switch is closed, I_L which was 0, stays at 0. Because of L I_L doesn't change instantaneously (unlike current across R, which turns on instantaneously as switch is closed)

Inductance: magnetic inertia to changes in current I_L

(ii) $U = \frac{1}{2}\boxed{C}V^2$

(ii) $U = \frac{1}{2}\boxed{L}I^2$

$\frac{1}{2}\boxed{m}v^2$

Mass m: gravitational inertia to changes in velocity v

Something in common among these 3 interactions

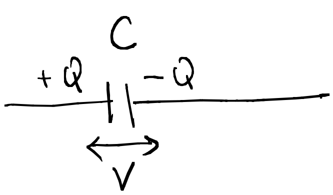
- Grav. ⟷ Newton's Univ. Gravitation Law
- Electric ⟷ Coulomb's Law
- Magnetic ⟷ Biot-Savart's Law

$\underbrace{\text{Newton's Univ. Gravitation Law, Coulomb's Law, Biot-Savart's Law}}_{\propto \frac{1}{r^2}}$

$\propto \frac{1}{r^2}$ inverse square law

Magnetic Energy

Electric

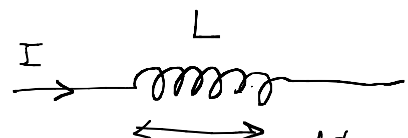


$U_C = \frac{1}{2}CV^2$ (J)

C (F)

V (V)

Magnetic



$$\mathcal{E}_L = -\frac{d\phi_B}{dt} \underset{\uparrow}{=} -L\frac{dI}{dt}$$

$\phi_B = LI$

$$U_L = \int_0^t \underbrace{P_L}_{\text{energy stored per unit time}}\, dt = \int_0^t I\cdot|\mathcal{E}_L|\,dt = L\int_0^t I\frac{dI}{\cancel{dt}}\cancel{dt}$$

$$= \frac{1}{2}L\left[I^2\right]_0^t \underset{\uparrow}{=} \frac{1}{2}LI^2 \quad \text{(J)}$$

$I(0) = 0$

Electric energy density: $u_C = \frac{U_C}{\text{vol}}$

(i) vol: vol b/w parallel plates : A.d
(A cross-sectional area of plates, d: spacing b/w plates)

(ii) $C = \frac{A\varepsilon_0}{d}$

Magnetic energy density $u_L = \frac{U_L}{\text{vol}}$

(i) vol: vol. inside solenoid : $A\cdot\ell$
(A cross-sectional area of solenoid, ℓ its length)

(ii) $L = \mu_0 \frac{N^2}{\ell} A$

$\rightarrow u_C = \frac{1}{2}\varepsilon_0 E^2 \quad \left(\frac{J}{m^3}\right)$

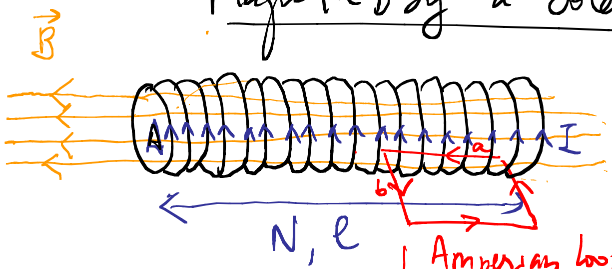
$\rightarrow u_L = \dfrac{\frac{1}{2} L I^2}{A \cdot \ell} = \dfrac{\frac{1}{2}\mu_0 \frac{N^2}{\ell}\cancel{A} I^2}{\cancel{A}\cdot \ell} = \frac{1}{2}\mu_0 \frac{N^2}{\ell^2} I^2$

(iii) $B = \mu_0 n I$

$= \mu_0 \frac{N}{\ell} I$ or $\frac{NI}{\ell} = \frac{B}{\mu_0}$

$$\rightarrow \boxed{u_L = \frac{1}{2}\mu_0 \frac{B^2}{\mu_0^2} = \frac{1}{2\mu_0} B^2}$$

Magnetic B by a Solenoid



{ Amperian loop
{ Rectangle of sides a, b

Ampere's Law:

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

Amperian Loop

by Amperian loop

$B \cdot \cancel{a} = \mu_0 \overbrace{\frac{NI\cancel{a}}{\ell}}$

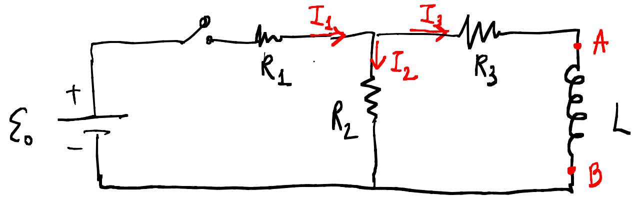
$$\boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I}$$

Self-inductance L by a solenoid:

$$\boxed{L = \frac{\Phi}{I} = \frac{B\cdot A\cdot N}{I} = \frac{\mu_0 n \cancel{I} A N}{\cancel{I}} = \mu_0 n N A = \mu_0 \frac{N^2}{\ell} A}$$

$n = \frac{N}{\ell}$

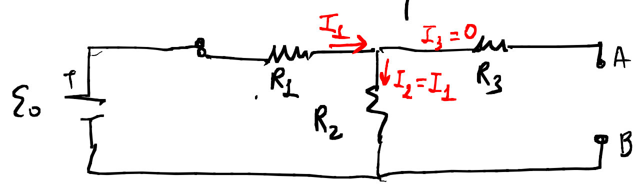
27.59



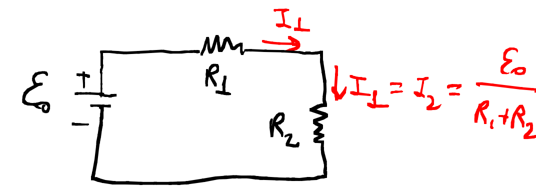
$$\begin{cases} \mathcal{E}_0 = 12V \\ R_1 = 4\Omega; \; R_2 = 8\Omega; \; R_3 = 2\Omega \\ L = 2H \end{cases}$$

a) I_2? right after switch is closed is $I_1 = I_2 = \frac{\mathcal{E}_0}{R_1+R_2} = \frac{12}{4+8} = 1A$

Inductor L : ($U = \frac{1}{2}LI^2$: L inertia to changes in I) before switch is closed Current I_3 through inductor was 0, it stays at 0 right after switch is closed. Inductor acts like open-circuit at time $t=0$.

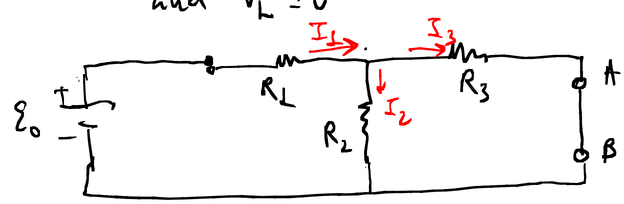


=

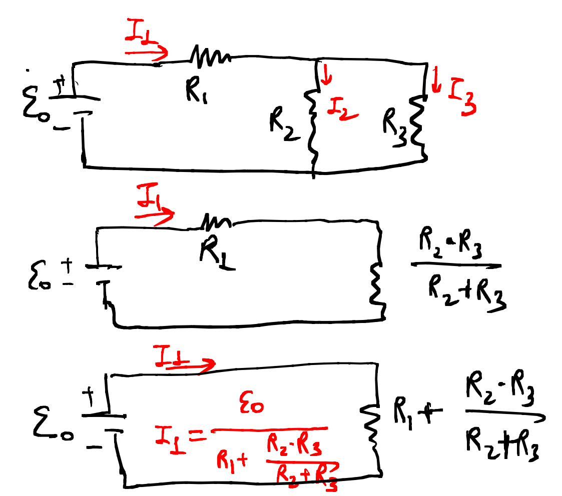


b) I_2? long after switch is closed

Inductor L: $t \to \infty$ acts like short-circuit, or current I_3 reaches max. value and $V_L = 0$



=



I_2 & I_3 are current divisions off I_1.

$$I_2 = I_1 \cdot \frac{R_3}{R_2+R_3} = \frac{\mathcal{E}_0}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}} \cdot \frac{R_3}{R_2+R_3}$$

$$I_2 = \frac{12}{4 + \frac{8 \cdot 2}{8+2}} \cdot \frac{2}{8+2}$$

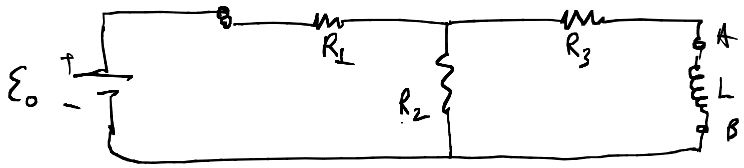
$$I_2 = \underbrace{\frac{12}{5.6}}_{I_1 = 2.14A} \cdot \frac{2}{10} = 0.429\ A$$

Curiosity: current division : $I_3 = I_1 \cdot \frac{R_2}{R_2+R_3} = 2.14 \cdot \frac{8}{8+2} = 1.71\ A$

check: $I_2 + I_3 = 0.429 + 1.71 = 2.14A = I_1$.

c) I_2? Long after switch is closed it is now reopened.

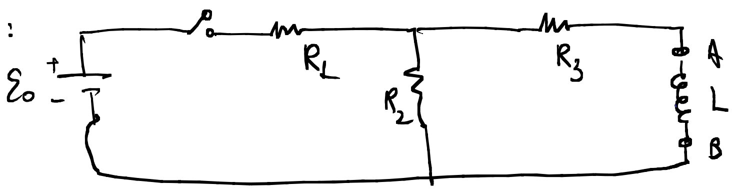
L: inertia to changes in current: at $t \to \infty$ I_L or I_3 was max., when switch is now reopened it will stay at that value using magnetic energy stored in the inductor. When U_L is depleted I_3 decreases back to 0.



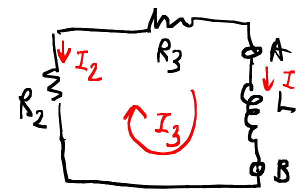
$t \to \infty$ L acts like short-circuit but it is physically there.

$I_2 = 0.429A$ (b)

Switch is reopened:



=

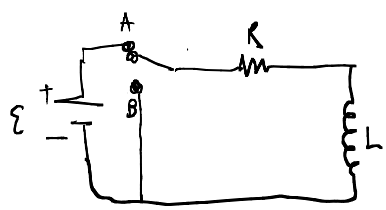


Inductor tries to maintain current $I_3 = 1.71A$ from any sudden change $\Rightarrow$ $I_2 = -1.71A$ until magnetic energy is depleted.

Application: this circuit is a delayed switch for R_3

If R_3 is a light bulb it will stay on a little bit longer after switch is open.

27.66

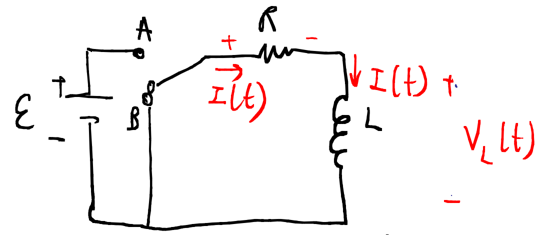


Switch to A: charging inductor:

$$I(t) = I_0\left(1 - e^{-\frac{t}{L/R}}\right)$$

$$I_0 = \frac{\mathcal{E}}{R}$$

$$\tau = \frac{L}{R}$$



Switch to B: discharging inductor

$$I(t) = I_0\, e^{-\frac{t}{L/R}}$$

$$I_0 = \frac{\mathcal{E}}{R}$$

$$\tau = \frac{L}{R}$$

a) Power dissipation at R over time:

$$P(t) = I(t) \cdot V_R(t)$$

Loop: $-I(t) \cdot R - V_L(t) = 0$

or $V_L(t) = -I(t) \cdot R$

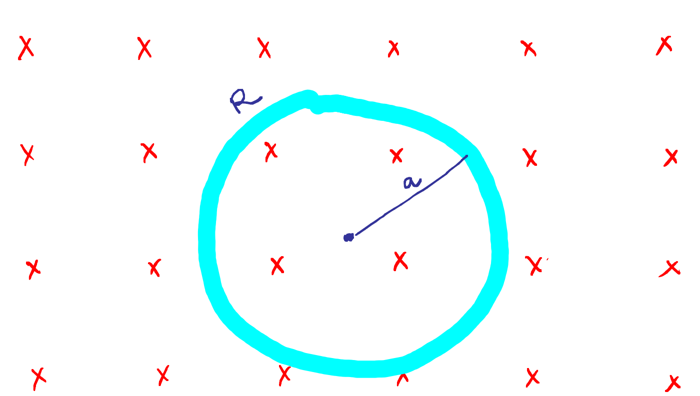
$V_R(t) = -V_L(t) = I(t) \cdot R$

$$\Rightarrow \quad P_R(t) = I^2(t)\cdot R = R\cdot I_0^2\, e^{-\frac{2t}{L/R}} = \frac{\mathcal{E}^2}{R}\, e^{-\frac{2t}{L/R}}$$

b) $$U_R = \int_0^\infty P_R(t)\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-\frac{2t}{L/R}}\,dt = \frac{\mathcal{E}^2}{R}\left[\frac{e^{-\frac{2t}{L/R}}}{-\frac{2R}{L}}\right]_0^\infty = -\frac{\mathcal{E}^2}{R}\cdot\frac{L}{2R}\left[0-1\right]$$

$$U_R = \frac{1}{2}\,\frac{\mathcal{E}^2}{R^2}\,L \underset{\frac{\mathcal{E}}{R}=I_0}{=} \frac{1}{2}L I_0^2$$ which was energy initially stored in inductor.

27.50 Circular wire loop resistance R, radius a, perpendicular to a uniform magnetic field $\vec{B}$ (going into page, for example)



$\vec{B}$ uniform, into page
Field strength to be increased from B_1 to B_2 $(B_2 > B_1)$

(i) Magnetic flux through wire loop: $\Phi_B = \oint \vec{B}\cdot d\vec{A} = B\oint dA = BA = B\pi a^2$

If B stays constant over time $\rightarrow$ Φ_B is constant over time: $\frac{d\Phi_B}{dt} = 0 = \mathcal{E}$

No induced voltage in loop $\rightarrow$ no moving charges in loop

(ii) Lab: increase B from $B_1 \rightarrow B_2$: $\frac{d\Phi_B}{dt} \neq 0 \rightarrow \mathcal{E} \rightarrow$ moving charges in loop.

<u>Faraday's Law</u>: $$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Ohm's law $\downarrow$

$$I\cdot R = -\frac{d}{dt}\left(B\pi a^2\right)$$

$\downarrow$

$$\frac{dq}{\cancel{dt}}\cdot R = -\pi a^2\,\frac{dB}{\cancel{dt}}$$

$$\boxed{dq = -\frac{\pi a^2}{R}\,dB}$$

Total charge moved: $\displaystyle\int_1^2 dq = -\frac{\pi a^2}{R}\int_1^2 dB$

$$Q_2 - Q_1 = -\frac{\pi a^2}{R}\left(B_2 - B_1\right)$$

$$\boxed{\Delta Q = Q_2 - Q_1 = \frac{\pi a^2}{R}\left(B_1 - B_2\right)}$$

(iii) Does it matter how did you increase field strength b/w 1 & 2 ? (e.g. quickly or slowly)

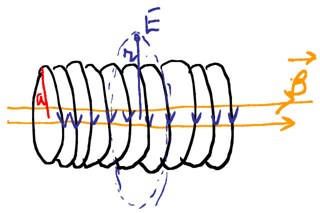
No! ΔQ only depends on the difference $B_1 - B_2$!

27.30

$a = 0.1\,m$

$r = 0.12\,m$

$E = 45\,\frac{V}{m}$



Find $\dfrac{\partial B}{\partial t}$?

Faraday's Law: $\displaystyle \mathcal{E} = -\frac{d\Phi_B}{dt}$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint \vec{B}\cdot d\vec{A}$$

$\vec{E}$ wraps around a circular loop of radius r centered at axis of solenoid!

$(\vec{E} \parallel d\vec{l})$ $\qquad \displaystyle\oint E\cdot dl = -\oint \frac{\partial B}{\partial t}\cdot dA$ $\qquad (\vec{B} \parallel d\vec{A})$

($\vec{E}$ constant along this loop) $\qquad \displaystyle E\oint dl = -\frac{\partial B}{\partial t}\oint dA$ $\qquad$ ($\vec{B}$ uniform over cross-sectional area of solenoid)

$$E\cdot 2\pi r = -\frac{\partial B}{\partial t}\cdot A \qquad (A = \pi a^2)$$

$$\frac{\partial B}{\partial t} = -\frac{E\,2\cancel{\pi} r}{\cancel{\pi} a^2} = -\frac{45\cdot 2\cdot 0.12}{0.1^2} = -1080\,\frac{T}{s}$$

Ch 29 Maxwell's Equation:

Maxwell's equations

1) Gauss' Law (Ch 21): $\oint_{\text{Gaussian surface}} \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

2) "Magnetic Gauss' Law": $\oint_{\text{Closed surface}} \vec{B}\cdot d\vec{A} = 0$ (no magnetic monopole has been found)

3) Ampere's Law (Ch 26): $\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enclosed}$ $+\mu_0 I_{displacement}$

4) Faraday's Law (Ch 27): $\oint \vec{E}\cdot d\vec{\ell} = \mathcal{E} = -\frac{d\Phi_B}{dt} = -\oint \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$

$\Phi_B = \int \vec{B}\cdot d\vec{A}$ magnetic flux

Observations: (i) A time-varying magnetic field $\left(\frac{\partial B}{\partial t}\right)$ can create an electric field $\vec{E}$

↳ Important hint about connection b/w $\vec{E}$ & $\vec{B}$

(ii) Is there a viceversa to (i)? Can a time-varying electric field $\left(\frac{\partial \vec{E}}{\partial t}\right)$ create a magnetic field $\vec{B}$?

Maxwell researched about this & demonstrated this was possible if an extra term is added in Ampere's Law: "the displacement current term".

$$I_{displacement} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot dA$$

$\Phi_E =$ electric flux $= \int \vec{E}\cdot d\vec{A}$

Modified Ampere's Law: $\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$

↳ gives the viceversa connection b/w $\vec{E}$ & $\vec{B}$ to Faraday's Law

Maxwell's displacement current term implications:

(i) Ampere's Law is more symmetric to Faraday's Law

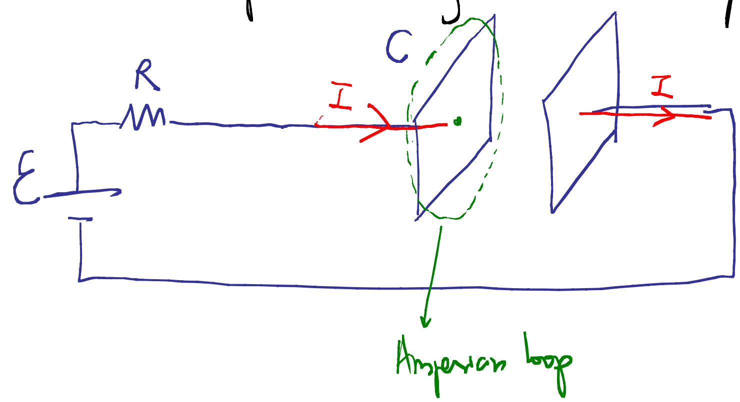
(ii) Two-way connection b/w $\vec{E}$ & $\vec{B}$ $\begin{cases} 1) \text{ Time-varying } \vec{B} \text{ creates } \vec{E} \\ 2) \text{ Time-varying } \vec{E} \text{ creates } \vec{B} \end{cases}$

$\vec{B}(t) \to \vec{E}(t) \to \vec{B}(t) \to \vec{E}(t) \to \cdots$ allows propagation of EM in vacuum

EM wave propagation w/o medium $\left\{\begin{array}{l}\text{Sun light}\\ \text{Space probe signals}\\ \text{Astronomy observations}\\ \text{Cell phone signals}\\ \text{etc..}\end{array}\right.$

wave propagation w/ medium $\left\{\begin{array}{l}\text{Sound waves (medium : air molecules)}\\ \text{water waves (medium : water molecules)}\\ \text{etc...}\end{array}\right.$

Technicality: a magnetic field can be measured around the plates of a capacitor when it is charging. This could not be explained using the old Ampere's Law!



Current I does not cross the gap b/w the plates

So it does not cross the Amperian loop in plane with left plate as shown $\to I_{enclosed} = 0$

Old Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed} = 0 \Rightarrow B\ell = 0$ or $B = 0$

↳ contradicts measurement of $\vec{B}$ around plates.

Maxwell's displacement current explains this magnetic field:

$$\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot dA$$

↳ $\frac{\partial \vec{E}}{\partial t}$ creates $\vec{B}$ happens in a parallel plate capacitor!

part of EM propagation in vacuum!

Summary:

Maxwell's equations
1) Propagation of EM waves in vacuum $\vec{B}(t) \to \vec{E}(t) \to \vec{B}(t) \to \vec{E}(t) \cdots$
2) Both $\vec{E}$ & $\vec{B}$ are vectors $\to$ polarization of EM waves (Sun glasses!)

Maxwell's equations in vacuum:

↳ No charges ($q_{enclosed} = 0$); no currents ($I_{enclosed} = 0$)

1) Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = 0$
2) "Magnetic Gauss' Law" $\oint \vec{B} \cdot d\vec{A} = 0$

} Trivial equations

3) Ampere's Law (Modified): $\oint \vec{B} \cdot d\vec{l} = \underbrace{\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}}$

doesn't involve materials → applies in vacuum!

4) Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

Completely symmetric in vacuum (except for a constant $\mu_0 \varepsilon_0$ & a minus sign)

	Integral forms		Differential forms
3) Ampere's Law:	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$	→	3) $\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$
4) Faraday's Law:	$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$	→	4) $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

$\frac{\partial}{\partial t}$ 3) → $\frac{\partial^2 B}{\partial x \partial t} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$

$\frac{\partial}{\partial x}$ 4) → $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$

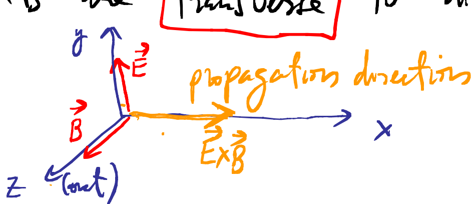
$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Electric field wave equation
(~ mechanical $_{\wedge}$ wave equations in Physic 113)
transverse

$\frac{\partial}{\partial x}$ 3), $\frac{\partial}{\partial t}$ 4) } → $\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$

Magnetic field wave equation

(i) They are both similar to transverse wave equations in a string
↳ Both $\vec{E}$ & $\vec{B}$ are transverse to direction of propagation



However $\vec{E}(t) \to \vec{B}(t) \to \vec{E}(t) \to \dots$ makes EM waves propagate in vacuum unlike mechanical wave in the string!

(ii) Mechanical wave equation: $\dfrac{\partial^2 y}{\partial x^2} = k \dfrac{\partial^2 y}{\partial t^2}$ $\begin{cases} y: \text{transverse perturbation} \\ x: \text{direction of propagation} \end{cases}$

↳ perturbation y was a scalar

EM wave equation → perturbations $\vec{E}$ & $\vec{B}$ are vectors! → polarization!

(iii) Similar solutions to mechanical wave equations:

$$\begin{cases} \vec{E}(x,t) = E_p \sin(kx - \omega t)\ \hat{j} \\ \vec{B}(x,t) = B_p \sin(kx - \omega t)\ \hat{k} \end{cases}$$

E_p & B_p : amplitudes

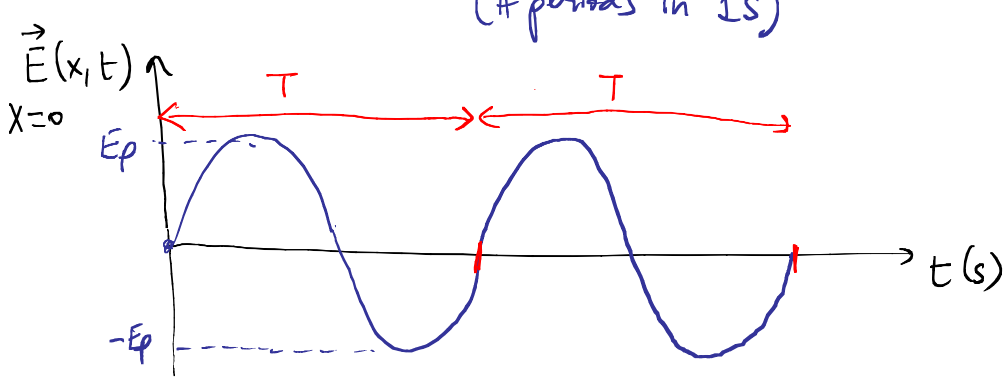
k : wave number $= \dfrac{2\pi}{\lambda}$ (m^{-1})
(# wavelengths in 2π)

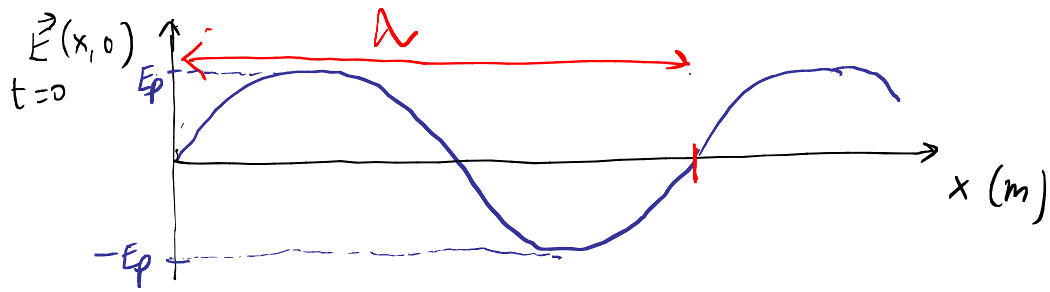
λ : wavelength (m)

ω : angular frequency $= \dfrac{2\pi}{T}$ (s^{-1})
(# periods in 2π)

T : wave period (s)

f : linear frequency $= \dfrac{1}{T}$ (Hz for Hertz)
(# periods in 1s)





Electromagnetic Wave Speed in Vacuum:

(i) Wave speed = (for any wave mechanical or EM) $v = \frac{\lambda}{T} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\omega}{k}$

(ii) What is v for EM waves? From Faraday's 4) & Ampere's 3) Laws in differential forms (in vacuum)

4) $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ $\left\{ \begin{array}{l} E(x,t) = E_p \sin(kx - \omega t) \rightarrow \frac{\partial E}{\partial x} = kE_p \cos(kx - \omega t) \\ B(x,t) = B_p \sin(kx - \omega t) \rightarrow \frac{\partial B}{\partial t} = -\omega B_p \cos(kx - \omega t) \\ \text{Faraday's Law: } kE_p = \omega B_p \rightarrow \boxed{v = \frac{\omega}{k} = \frac{E_p}{B_p}} \text{ Faraday's} \end{array} \right.$

3) $\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ $\left\{ \begin{array}{l} \frac{\partial B}{\partial x} = kB_p \cos(kx - \omega t) \\ -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = +\mu_0 \varepsilon_0 \omega E_p \cos(kx - \omega t) \end{array} \right\}$ $kB_p = \mu_0 \varepsilon_0 \omega E_p$ $\boxed{\frac{E_p}{B_p} = \frac{k}{\mu_0 \varepsilon_0 \omega}}$ Ampere's

$$\frac{\omega}{k} = \frac{k}{\mu_0 \varepsilon_0 \omega} \Rightarrow \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0} \rightarrow v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \cdot 10^{-7} \cdot 8.85 \cdot 10^{-12}}}$$

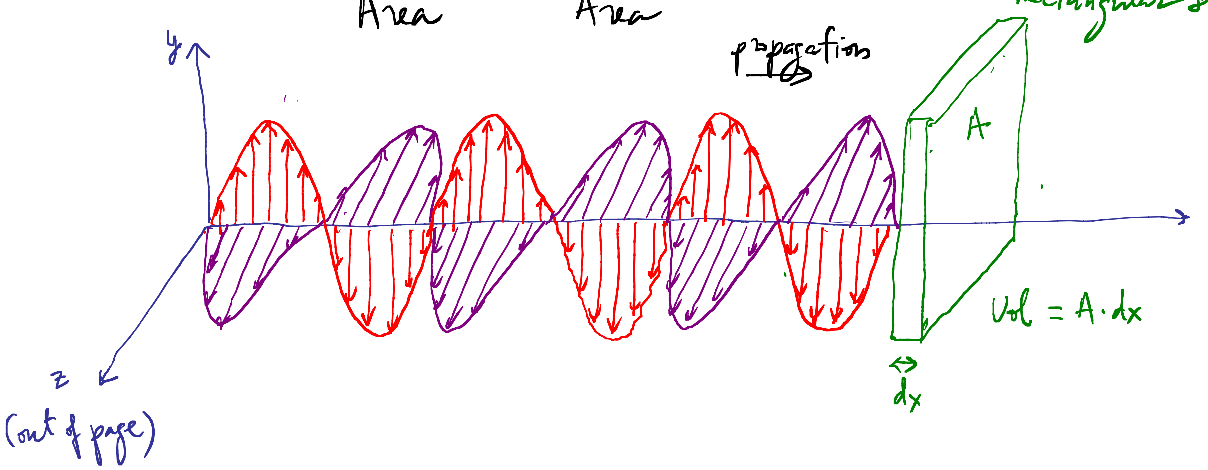
$$= 2.999 \cdot 10^8 \ \frac{m}{s} = C$$

C = speed of light (and of any EM wave in vacuum)
Max speed for any object including EM waves: Einstein Special Theory of Relativity.

Intensity of EM waves:

$\rightarrow$ $\vec{E} = E\hat{j}$; $\vec{B} = B\hat{k}$; propagation along $\vec{E} \times \vec{B}$ ($\hat{i}$) by RHR

$$\hookrightarrow S \equiv \frac{P}{\text{Area}} = \frac{\frac{dU}{dt}}{\text{Area}}$$



S through the rectangular slab of cross-sectional area A, thickness dx

Rate of change of energy through slab:

$$\frac{dU}{dt} = \frac{d(u \cdot vol)}{dt} = \frac{d}{dt}(u \cdot A \cdot dx) = uA\underbrace{\frac{dx}{dt}}_{\text{wave speed} \to EM \to v = c}$$

U: total energy

u: energy per unit volume or energy density

$$S = \frac{\frac{dU}{dt}}{Area} = \frac{u\cancel{A}c}{\cancel{A}} = uc = \Big(\underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\text{electric}} + \underbrace{\frac{1}{2\mu_0}B^2}_{\text{magnetic}}\Big)c$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0} \to \varepsilon_0 = \frac{1}{c^2 \mu_0}$$

or $\boxed{\varepsilon_0 c^2 = \frac{1}{\mu_0}}$

$$\frac{\omega}{k} = \frac{E}{B} \to \boxed{B = \frac{E}{c}}$$

($\frac{\omega}{k} = c$)

$$S = \Big(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 c^2 B^2\Big)c$$

$$= \frac{1}{2}\varepsilon_0\Big(E^2 + \cancel{c^2}\frac{E^2}{\cancel{c^2}}\Big)c = \varepsilon_0 E^2 c$$

$$= \frac{1}{\mu_0 c^{\cancel{2}}}E^2\cancel{c}$$

$$\boxed{S = \frac{1}{\mu_0}E\frac{E}{c} = \frac{1}{\mu_0}EB}$$

(SI unit: $\frac{W}{m^2}$)

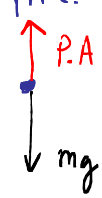
more generally: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

Radiation Pressure: P

What laser power is need to hold a piece of Aluminum foil of $m = 30\,\mu g$ in the air?

Hovering piece of Al foil:

Free-body diagram for piece of Al foil:



$\begin{cases} P: \text{radiation pressure} \\ A: \text{foil area} \end{cases}$

$$\boxed{P \cdot A = mg}$$

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{F \cdot x}{t} = F \cdot v$$

$$S = \frac{\text{Power}}{\text{Area}} = \frac{F}{A}v = \underset{\uparrow \text{Pressure}}{P} \cdot v \to \text{Radiation: } S = \underset{\uparrow \text{Pressure}}{P} \cdot \underset{\uparrow \text{wave speed}}{c}$$

$$\boxed{2}\frac{S}{c} \cdot A = mg$$

⇓ b/c Al foil reflects all lights: light comes back at same speed c but in opposite direction after reflecting off foil

↓ Similar situation: gas molecule reflecting off container wall

↓ bouncing off ⟺ elastic collision

↓ momentum transfer was $\boxed{2}mv$ to the wall

$$\rightarrow 2\frac{S}{c}A = mg \quad \rightarrow \quad S\cdot A = \frac{mgc}{2}$$

$$\text{Power} = \frac{mgc}{2} = \frac{30\cdot 10^{-6}\cdot 10^{-3}\cdot 9.81\cdot 3\cdot 10^{8}}{2} = 44.1\,W$$

29.43

EM waves: 2 important properties

(i) Can create one another: $E(t) \rightarrow B(t) \rightarrow E(t) \rightarrow \dots$ to propagate in vacuum

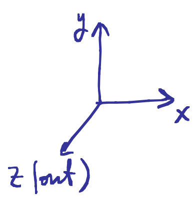
(ii) Vector nature of $\vec{E}$ & $\vec{B}$ → Polarization

Polarizer: a type of material that only allows EM fields along certain directions to pass through.

↳ set by the polarizer's axis

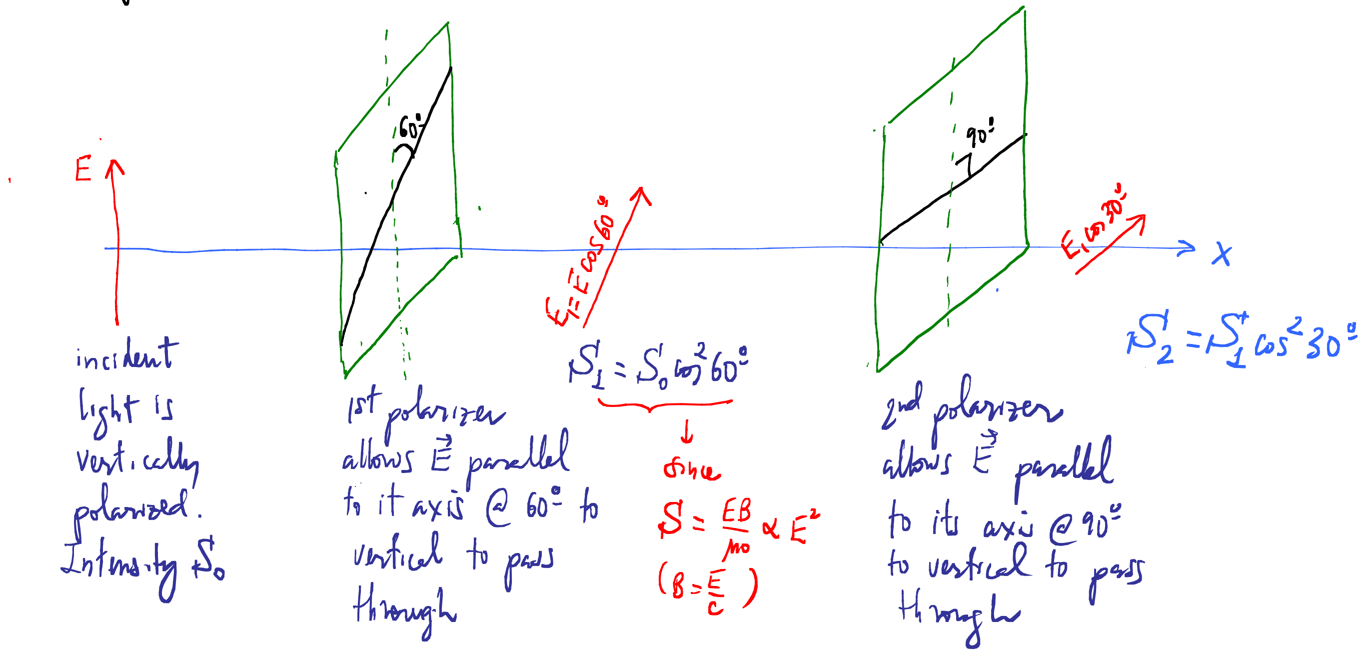
Vertically polarized light: where $\vec{E}$ points along the vertical axis

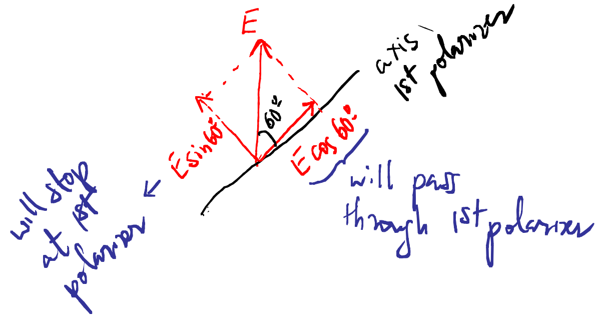
(Sun light is unpolarized: $\vec{E}$ points along all possible directions)

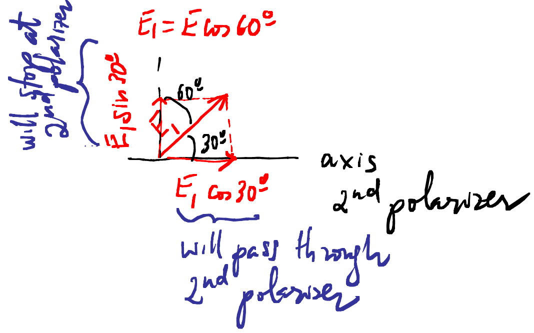


1st polarizer: axis 60° to vertical

2nd polarizer: axis 90° to vertical







Summary
- Incident: S_0
- After 1st polarizer: $S_1 = S_0 \cos^2 60°$
- After 2nd polarizer: $S_2 = S_1 \cos^2 30 = S_0 \cos^2 60 \cos^2 30$

Fraction of light that passes through both is $\frac{S_2}{S_0} = \cos^2 60 \cdot \cos^2 30 = 0.1875$ or 18.75%

Curiosity: (i) with only one polarizer would we get more light through?
→ Not necessarily! With only the 2nd polarizer we get no light through or 0%!

(ii) If we started out with only polarizer #2 (no light through): "illusion of creating light" by inserting polarizer #1!