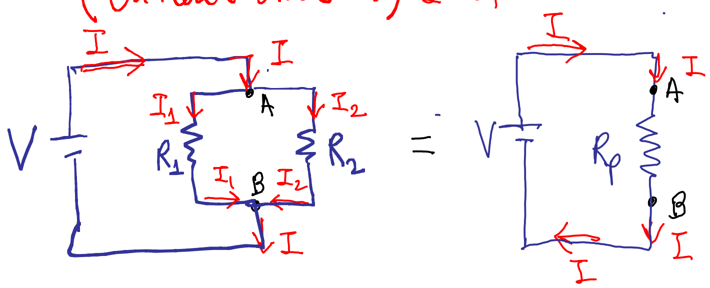
Connecting resistors

Parallel connection

(Current division) $I = I_1 + I_2$



$I = I_1 + I_2$

R_p : equivalent resistance for a parallel connection of R_1 & R_2

Ohm's Law on R_1 & R_2:

original: $I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

equivalent $I = \frac{V}{R_p}$

Conclusion: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

or $R_p = \frac{R_1 \cdot R_2}{R_1 + R_2}$

Current division:

$I_1 = \frac{V}{R_1} = I \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \cdot \frac{1}{R_1}$

$= I \frac{1}{\frac{R_1+R_2}{\cancel{R_1} \cdot R_2}} \cdot \frac{1}{\cancel{R_1}}$

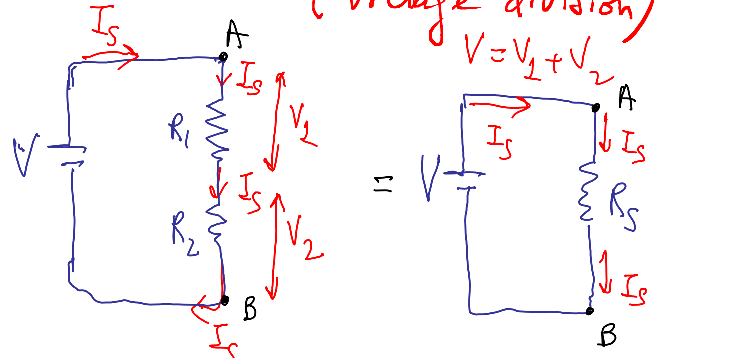
$= I \cdot \frac{R_2}{R_1+R_2} < I$

$I_2 = I \cdot \frac{R_1}{R_1+R_2} < I$

$I_1 + I_2 = I\left(\frac{R_2}{R_1+R_2} + \frac{R_1}{R_1+R_2}\right)$
$= I$

Series Connection

(Voltage division) $V = V_1 + V_2$



$V = V_1 + V_2$

R_s : equivalent resistance for a series connection of R_1 & R_2

Ohm's Law:

original: $V = V_1 + V_2 = I_s \cdot R_1 + I_s \cdot R_2$
$= I_s(R_1 + R_2)$

equivalent: $V = I_s \cdot R_s$

Conclusion: $R_s = R_1 + R_2$

Voltage division:

$V_1 = I_s \cdot R_1 = \frac{V}{R_1+R_2} \cdot R_1$

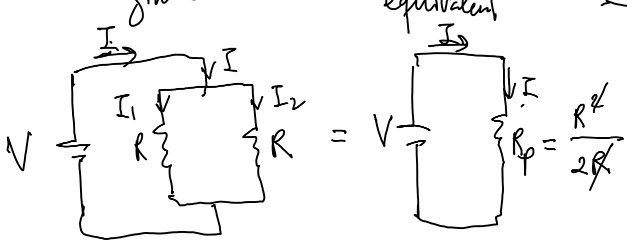
or $V_1 = V \cdot \frac{R_1}{R_1+R_2} < V$

$V_2 = I_s \cdot R_2 = V \cdot \frac{R_2}{R_1+R_2} < V$

$V_1 + V_2 = V \cdot \frac{R_1}{R_1+R_2} + V \cdot \frac{R_2}{R_1+R_2} = V$

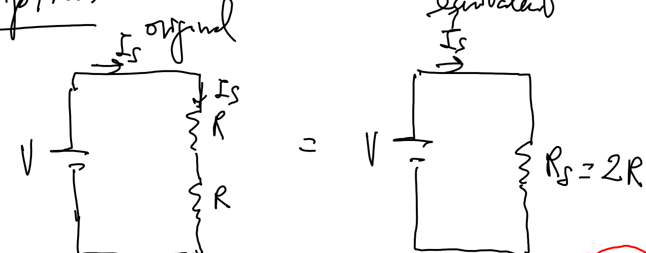
Power Consumption

$P = I \cdot V$



$I_1 = I_2 = \frac{I}{2} \rightarrow P = I \cdot V = \frac{V}{R_p} \cdot V$

$= \frac{V^2}{R_p} = \frac{2V^2}{R}$ more



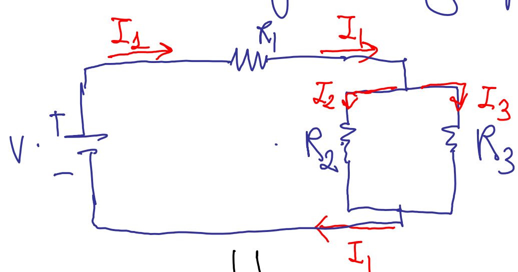
$P = I_s \cdot V = \frac{V^2}{R_s} = \frac{V^2}{2R}$ less $\left(\frac{1}{4}\right)$

Ch 25 Electrical Circuits

↳ linear circuits
↳ linear relationship b/w voltage & current ($V = I \cdot R$ or Ohm's law)
↳ R, C, L
R, C: electric; L: magnetic (inductors)

Electric circuits, 2 types
- 1) R's only
  - (i) solve using parallel & series connections ✓
  - (ii) solve using loop or node analysis ✓
- 2) R's & C's: solve using asymptotic analysis ($t \to 0$ & $t \to \infty$) or solution to differential equations.

1) Circuit analysis using parallel & series connection:

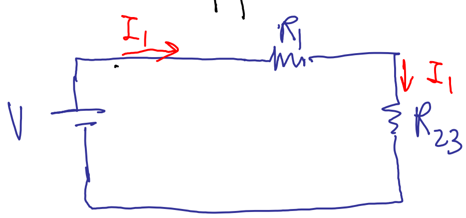


a) Analyze this circuit = provide I_1, I_2, I_3

b) R_2 is parallel w/ R_3 $\Rightarrow R_{23} = \dfrac{R_2 \cdot R_3}{R_2 + R_3}$

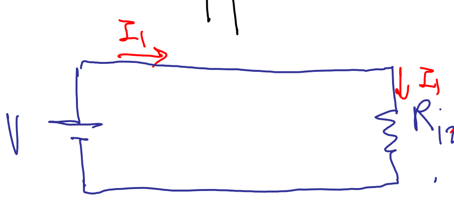
c) R_{23} is in series w/ R_1 $\Rightarrow R_{123} = R_1 + R_{23}$

||



$R_{23} = \dfrac{R_2 \cdot R_3}{R_2 + R_3}$

||



$R_{123} = R_1 + \dfrac{R_2 \cdot R_3}{R_2 + R_3}$

Observations:

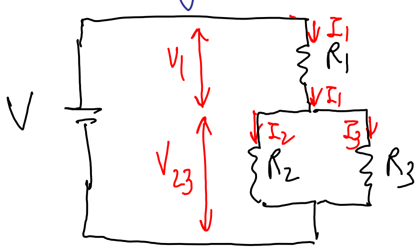
(i) Reduce original circuit to a simplest one with one battery & one resistor

(ii) Then use Ohm's law to calculate $I_1 = \dfrac{V}{R_{123}} = \dfrac{V}{R_1 + \dfrac{R_2 \cdot R_3}{R_2 + R_3}}$

(iii) Next calculate I_2 & I_3: 2 alternatives:

1) Use current division
$$\begin{cases} I_2 = I_1 \cdot \dfrac{R_3}{R_2 + R_3} = \underbrace{\dfrac{V}{R_1 + \dfrac{R_2 \cdot R_3}{R_2 + R_3}}}_{\text{same result}} \cdot \dfrac{R_3}{R_2 + R_3} \\ I_3 = I_1 \dfrac{R_2}{R_2 + R_3} \end{cases}$$

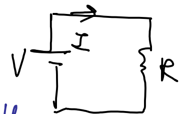
2) Use voltage division:

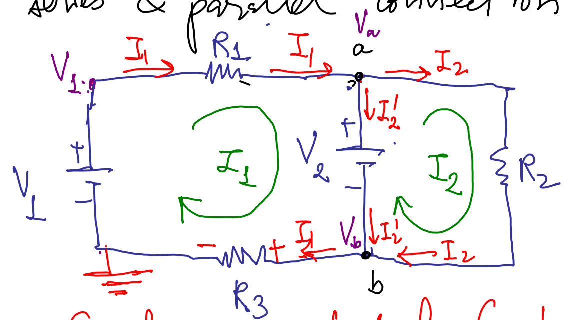


$$I_2 = \frac{V_{23}}{R_2} = \frac{V \cdot \frac{\cancel{R_2} \cdot R_3}{R_2+R_3}}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}} \cdot \frac{1}{\cancel{R_2}}$$

$$I_3 = \frac{V_{23}}{R_3} = \dots$$

2)' Circuit analysis using Loop or Node analysis.

For some circuits reduction to a simple  is not possible using only series & parallel connection; (when there are more than one battery or voltage source)



Observations: (i) R_1 & R_2 are not in series (not same currents going through both of them: $I_2 \neq I_1$!)

(ii) R_1 & R_3 are not in parallel (not same voltage across both of them!)

Ground or zero potential (Critical when using loop or node analysis!)

Loop Analysis (2 loops in this example)

Kirchoff's Law: total voltage difference across elements in a closed loop is 0

$$\sum_i V_i = 0$$

↳ Some V_i will be + some −

↳ Sign convention

↳ Assume a direction for the current in each loop! (CW I_1 & I_2 in left & right loops, respectively)

1) If this current goes through battery from − to + → battery voltage is positive. Negative if this current goes from + to −

2) Voltage difference across any resistor is always negative

Node Analysis

Kirchoff's Law: total current in & out of any node in a circuit is 0

$$\sum_i I_i = 0$$

↳ Some I_i will be + some −

↳ Sign convention

1) Any current going into node is positive

2) Any current leaving a node is negative

Assign a name for the voltage at each node (in this example: V_a & V_b)

Loop Analysis

- 2 loops in this circuit
- We have assumed clockwise (CW) currents in each loop : I_1 & I_2

Kirchoff's Laws:

Loop 1 : $+V_1 - I_1 \cdot R_1 - V_2 - I_1 \cdot R_3 = 0$ (1)

Loop 2 : $+V_2 - I_2 \cdot R_2 = 0$ (2)

Set of 2 equations with 2 unknowns : I_1 & I_2 (provided we are given V_1, V_2, R_1, R_2, R_3)

(2) $\rightarrow I_2 = \dfrac{V_2}{R_2}$

(1) $V_1 - V_2 = I_1 (R_1 + R_3) \rightarrow I_1 = \dfrac{V_1 - V_2}{R_1 + R_3}$

Node Analysis

- 2 nodes in this circuit (a, b)
- We called voltages there V_a, V_b

Kirchoff's Laws:

Node a : $I_1 - I_2 - I_2' = 0$

Node b : $I_2 + I_2' - I_1 = 0$

(Same equation as for node a!)

↳ There is only one independent node in this circuit!

Write currents in terms of voltages V_a & V_b or V_1 & V_2

$$I_1 = \frac{V_1 - V_a}{R_1}$$

$$V_a = V_2 + I_1 \cdot R_3 \quad \text{(from node a to the ground)}$$

$$\rightarrow I_1 = \frac{V_1 - (V_2 + I_1 \cdot R_3)}{R_1}$$

$$I_1 R_1 + I_1 R_3 = V_1 - V_2$$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

also $\boxed{I_2 = \dfrac{V_2}{R_2}}$

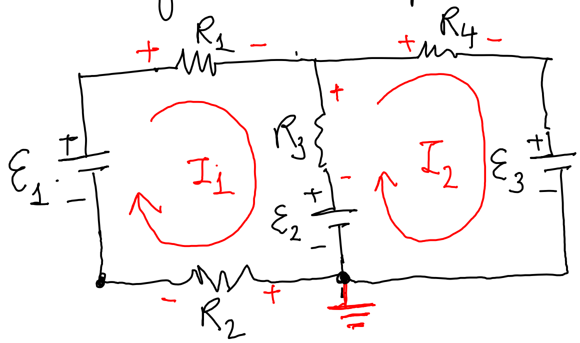
Node equation: a : $I_1 - I_2 - I_2' = 0$

$$\frac{V_1 - V_2}{R_1 + R_3} - \frac{V_2}{R_2} - I_2' = 0$$

Provided we are given V_1, V_2, R_1, R_2, R_3

$$\rightarrow I_2' = \frac{V_1 - V_2}{R_1 + R_3} - \frac{V_2}{R_2}$$

Circuit Analysis Example:



$\mathcal{E}_1 = 6V$; $\mathcal{E}_2 = 1.5V$; $\mathcal{E}_3 = 4.5V$

$R_1 = 270\,\Omega$; $R_2 = 150\,\Omega$;

$R_3 = 560\,\Omega$; $R_4 = 820\,\Omega$

Find current through R_3 with direction (up or down)

1) Using Loop Analysis:

How many loops? 2 loops → assume CW directions I_1 & I_2 (needed to apply sign conventions for Kirchhoff's Law!)

Loop 1: $+\mathcal{E}_1 - I_1 \cdot R_1 - (I_1 - I_2)\cdot R_3 - \mathcal{E}_2 - I_1 \cdot R_2 = 0$ (1)

Loop 2: $+\mathcal{E}_2 - (I_2 - I_1)\cdot R_3 - I_2 \cdot R_4 - \mathcal{E}_3 = 0$ (2)

System of 2 equations with 2 unknowns I_1, I_2

Find current through R_3 = find I_1 & I_2!

Algebraic manipulations:

(1)+(2) ⇒ $\mathcal{E}_1 - \mathcal{E}_3 - I_1\cdot(R_1+R_2) - I_2 \cdot R_4 = 0$

↳ solve for I_1: $I_1 = \dfrac{\mathcal{E}_1 - \mathcal{E}_3 - I_2 \cdot R_4}{R_1 + R_2}$ (3)

(2) $\mathcal{E}_2 - \mathcal{E}_3 - I_2 \cdot (R_3 + R_4) + I_1 \cdot R_3 = 0$

$$\mathcal{E}_2 - \mathcal{E}_3 - I_2 (R_3 + R_4) + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} R_3 - I_2 \frac{R_3 R_4}{R_1 + R_2} = 0$$

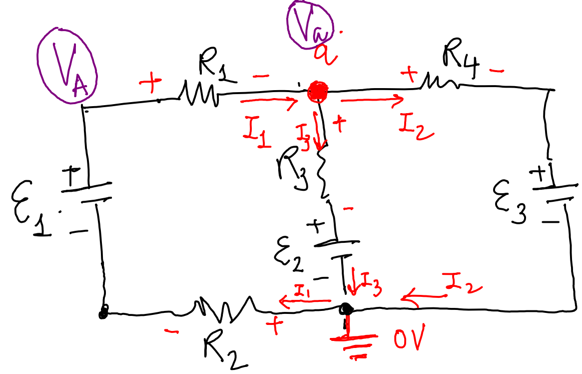
$$\mathcal{E}_2 - \mathcal{E}_3 + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} R_3 = I_2 \cdot \left(R_3 + R_4 + \frac{R_3 \cdot R_4}{R_1 + R_2} \right)$$

$$\boxed{I_2 = \frac{\mathcal{E}_2 - \mathcal{E}_3 + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} R_3}{\left(R_3 + R_4 + \frac{R_3 \cdot R_4}{R_1 + R_2} \right)}}$$

Plug in given values: $I_2 = \dfrac{-3 + \frac{1.5}{420} 560}{\left(1380 + \frac{560 \cdot 820}{420}\right)} = -0.4 \cdot 10^{-3}$ A or -0.4 mA

Use (3) for $I_1 = \dfrac{1.5 - (-0.4 \cdot 10^{-3}) \cdot 820}{420} = 4.36 \cdot 10^{-3}$ A or 4.36 mA

⇒ Current through R_3 is $I_1 - I_2 = 4.36\,\text{mA} - (-0.4\,\text{mA}) = \boxed{4.76\,\text{mA}}$ positive or downward!



$\mathcal{E}_1 = 6V$; $\mathcal{E}_2 = 1.5V$; $\mathcal{E}_3 = 4.5V$

$R_1 = 270\,\Omega$; $R_2 = 150\,\Omega$;

$R_3 = 560\,\Omega$; $R_4 = 820\,\Omega$

Find current through R_3 with direction (up or down)

2) Now using Node Analysis:

How many nodes? Visual inspection: one independent node a

(i) Assign directions (assume, choose) for the 3 currents that meet at this node! I_1 (in); I_2 (out); I_3 (out)

(ii) Ground location is essential in node analysis

↳ To write the currents in terms of voltages (battery voltages and some other voltage)

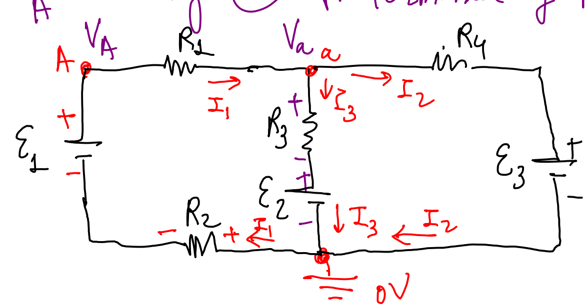
Kirchoff's Law for current at node a: $I_1 - I_2 - I_3 = 0$

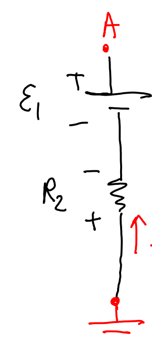
We don't know any of these currents but we know the battery voltages: we need to write these currents in terms of $\mathcal{E}$'s & R's:

(i) Ohm's Law @ R_1: $I_1 = \dfrac{\Delta V_1}{R_1} = \dfrac{V_A - V_a}{R_1} = \dfrac{\overbrace{\mathcal{E}_1 - I_1 R_2}^{(a)} - \overbrace{(I_3 R_3 + \mathcal{E}_2)}^{(b)}}{R_1}$

Important observations

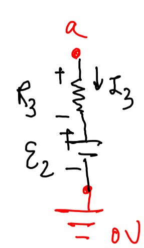
V_A: voltage @ left terminal of R_1:





$V_A = \mathcal{E}_1 - I_1 \cdot R_2$ (a)

V_a: voltage @ right terminal of R_1:



$V_a = I_3 R_3 + \mathcal{E}_2$ (b)

$I_1 R_1 = \mathcal{E}_1 - I_1 R_2 - I_3 R_3 - \mathcal{E}_2$

$I_1 (R_1 + R_2) = \mathcal{E}_1 - \mathcal{E}_2 - I_3 R_3$ or $\boxed{I_1 = \dfrac{\mathcal{E}_1 - \mathcal{E}_2 - I_3 R_3}{R_1 + R_2}}$

(ii) Ohm's Law @ R_4: $\boxed{I_2 = \frac{V_a - \mathcal{E}_3}{R_4} \overset{(b)}{=} \frac{I_3 R_3 + \mathcal{E}_2 - \mathcal{E}_3}{R_4}}$

Kirchoff's Law for currents @ node a:

$$I_1 - I_2 - I_3 = 0 \text{ or } \frac{\mathcal{E}_1 - \mathcal{E}_2 - I_3 R_3}{R_1 + R_2} - \frac{I_3 R_3 + \mathcal{E}_2 - \mathcal{E}_3}{R_4} - I_3 = 0$$

Algebraic manipulation to solve for I_3:

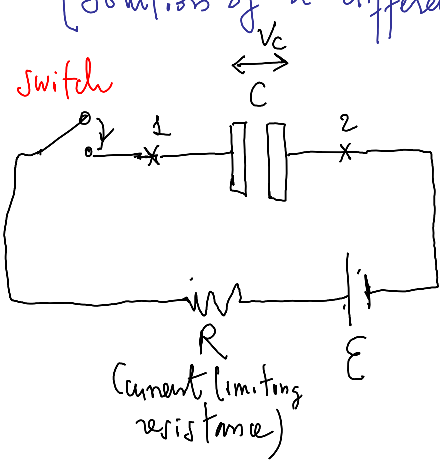
$$\frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} - \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4} - I_3\left(\frac{R_3}{R_1 + R_2} + \frac{R_3}{R_4} + 1\right) = 0$$

$$I_3 = \frac{\frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} - \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4}}{\left(\frac{R_3}{R_1 + R_2} + \frac{R_3}{R_4} + 1\right)} = \frac{\frac{4.5}{420} - \frac{-3}{820}}{\left(\frac{560}{420} + \frac{560}{820} + 1\right)}$$

$\boxed{I_3 = +4.76\,mA}$ positive or downward @ R_3 !

Analysis of Circuits with Resistors & Capacitors:

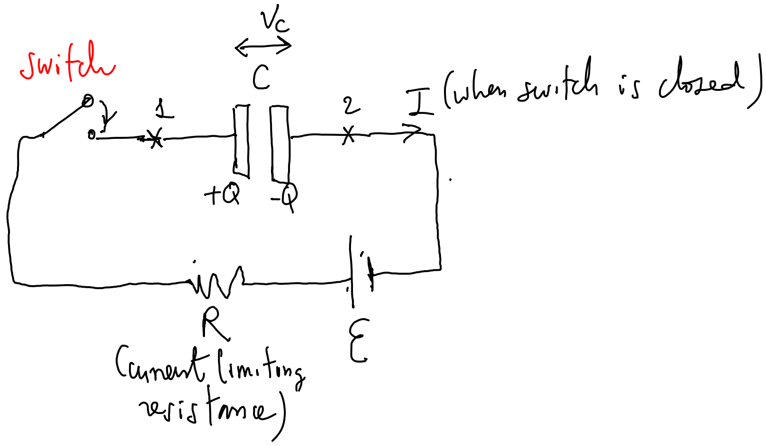
- Asymptotic analysis
  - $t \to 0$ when switch is closed or circuit starts
  - $t \to \infty$ when switch has been closed sufficiently long
- Solution of a differential equation

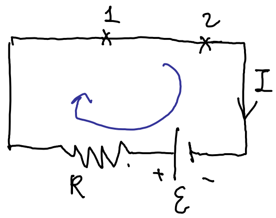


What happens when switch is closed? or capacitor C is connected to circuit.

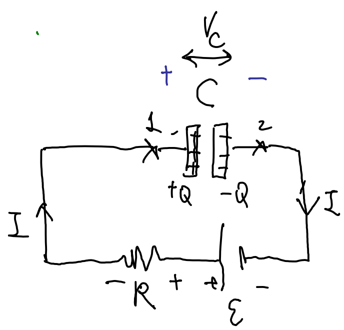
$\mathcal{E}$ for e.m.f (electromotive force)

Assumption:
Capacitor initially uncharged!



time t	Circuit	Q	V_C	Capacitor behaves like	I
0 (switch is closed)		0	$V_C = 0$	a wire or "short circuit"	$I = I_0 = \frac{\mathcal{E}}{R}$ max since it gets harder to move charges to charge a capacitor

$t > 0$



$0 < I < \frac{\mathcal{E}}{R}$

$0 < V_C < \mathcal{E}$

Q changes over time as charges are being moved to charge the capacitor.

Changes decrease over time as it is harder to move later charges against $\vec{E}$ b/w plates)

$0 < V_C < \mathcal{E}$

$$\frac{d}{dt}\left(\mathcal{E} - I \cdot R - V_C = 0\right) \longrightarrow -R\frac{dI}{dt} - \frac{I}{C} = 0$$

$\longrightarrow V_C = \mathcal{E} - I \cdot R$

$V_C(t) = \mathcal{E} - \mathcal{E}e^{-\frac{t}{RC}}$

$V_C(t) = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$

Note (i) $\frac{d\mathcal{E}}{dt} = 0$

(ii) $C = \frac{Q}{V_C}$

$\Rightarrow V_C = \frac{Q}{C}$

$\Rightarrow \frac{dV_C}{dt} = \frac{1}{C}\frac{dQ}{dt}$

$\frac{dV_C}{dt} = \frac{I}{C}$

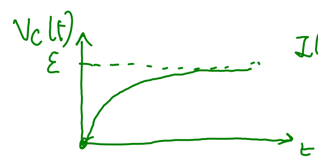
$\frac{dI}{dt} = -\frac{I}{RC}$

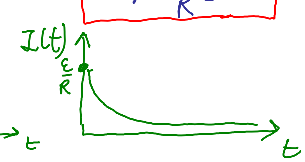
$\int\left(\frac{dI}{I} = -\frac{dt}{RC}\right)$

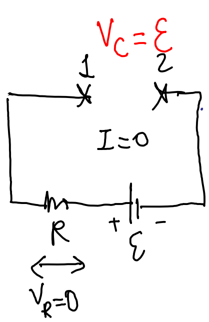
$\ln I = -\frac{t}{RC} + \text{const.}$

$I(t) = I_0 e^{-\frac{t}{RC}}$

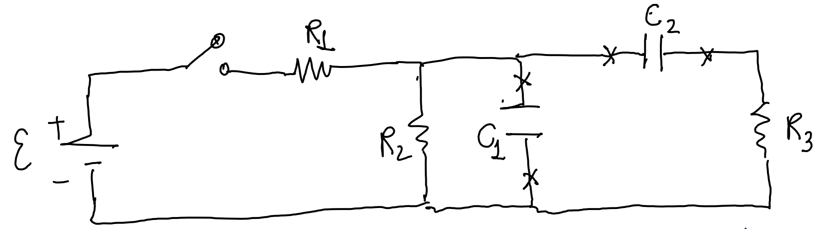
$\boxed{I(t) = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}}$





time t	Circuit	Q	V_C	Capacitor behaves like	I
$t \to \infty$ (switch is closed long enough!)		Capacitor is fully charged ↓ No more motion of charges → $I = 0$	Electric field is max. ↓ V_C is max $V_C = \mathcal{E}$	"open circuit" (doesn't allow any more current!)	$I = 0$

25.60



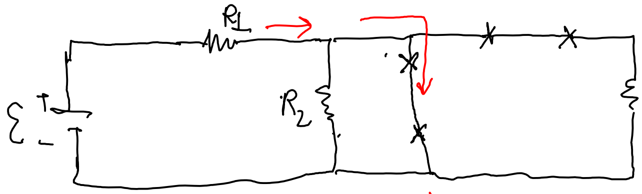
$R_1 = R_2 = R$

Capacitors are initially uncharged.

Determine current in R_2 when
- switch is closed $t=0$
- after it is closed sufficiently long $t \to \infty$

(i) @ $t=0$ switch is just closed: Q still 0, $\vec{E}=0$, $V_c=0$ → both capacitors C_1 & C_2 act like short-circuit! Between their terminals → like a wire

@ $t=0$



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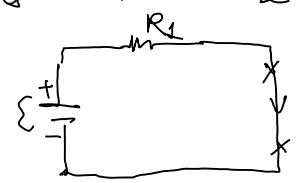


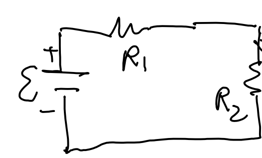
$I = \frac{\varepsilon}{R_1}$

$\boxed{I_2 = 0}$

this 'wire' shortcircuits R_2 & R_3
wires offer very low or zero resistance to currents. ⇒ R_2 & R_3 act like they are not there! (no current through them or $I_2 = I_3 = 0$)

(ii) @ $t \to \infty$: C_1 & C_2 are fully charged → no further current through them
they act like open circuits:

=

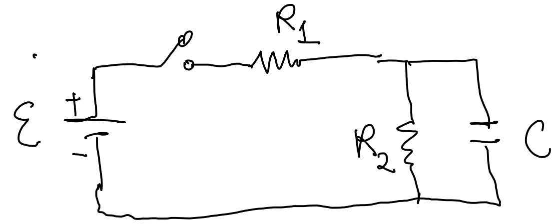


$\boxed{I_2 = \frac{\varepsilon}{R_1 + R_2}}$

HW 4 updated: Ch 27: Questions: 1, 8

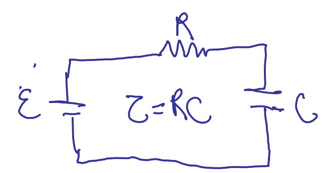
Problems: 14, 37, 39, 42, 44, 68

25.75



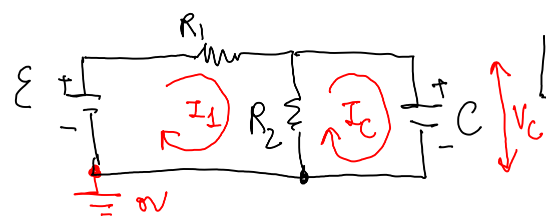
Write loop & node equations & find the time constant

Time constant τ (tau): time for current across the capacitor to decay by a factor of e



$$\Rightarrow \begin{cases} I(t) = I_0 e^{-\frac{t}{RC}} \\ I(t=RC) = I_0 e^{-1} = \frac{I_0}{e} \Rightarrow \tau = RC \end{cases}$$

In 25.75 the circuit has 2 resistors R_1 & R_2 $\Rightarrow$ τ is different!



Loop equations

$$\begin{cases} 1)\ \varepsilon - I_1 \cdot R_1 - (I_1 - I_C) \cdot R_2 = 0 \\ 2)\ -(I_C - I_1) \cdot R_2 - V_C = 0 \end{cases}$$

Solve for $I_c(t)$ to find τ

$$2) \Rightarrow \quad -V_C = (I_C - I_1) \cdot R_2$$

$$C = \frac{Q}{V_C} \Rightarrow \quad -\frac{Q}{C} = -(I_1 - I_C) \cdot R_2 \quad (a)$$

$$1) \& a) \Rightarrow \varepsilon - I_1 \cdot R_1 - \frac{Q}{C} = 0 \Rightarrow I_1 = \frac{\varepsilon - \frac{Q}{C}}{R_1} \quad (b)$$

$$b) \text{ in } a) \quad \frac{Q}{C} = I_1 \cdot R_2 - I_C \cdot R_2 \overset{(b)}{=} \left(\varepsilon - \frac{Q}{C}\right) \cdot \frac{R_2}{R_1} - I_C \cdot R_2$$

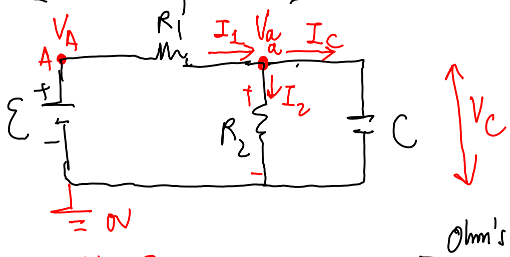
$$\frac{d}{dt}\left[\frac{Q}{C}\left(1 + \frac{R_2}{R_1}\right) = \varepsilon \frac{R_2}{R_1} - I_C \cdot R_2\right] \Rightarrow I_C \frac{1}{C}\left(1 + \frac{R_2}{R_1}\right) = -R_2 \frac{dI_C}{dt} \Rightarrow \int \left[-\frac{dt}{R_2 C}\left(1 + \frac{R_2}{R_1}\right) = \frac{dI_C}{I_C}\right]$$

$$\Rightarrow \ln I_C = -\frac{\left(1 + \frac{R_2}{R_1}\right)}{R_2 C} t + \text{const} = -\frac{\frac{R_1 + R_2}{R_1}}{R_2 C} t + \text{const} = -\frac{R_1 + R_2}{R_1 R_2 C} t + \text{const} = \frac{-t}{\frac{R_1 R_2}{R_1 + R_2} \cdot C} + \text{const.}$$

$$\boxed{I_C(t) = I_0 e^{-\frac{t}{\frac{R_1 R_2}{R_1 + R_2} \cdot C}}} \Rightarrow \text{Time constant: } \boxed{\tau = \frac{R_1 R_2}{R_1 + R_2} \cdot C}$$

Node equation:

one: $I_1 - I_2 - I_C = 0$ (1)



Write I_1 & I_2 in terms of voltages $\mathcal{E}$ & V_C, solve for $I_C(t)$ to find τ

Note: $\begin{cases} V_A = \mathcal{E} \\ V_a = I_2 \cdot R_2 = V_C \\ V_C = \dfrac{Q}{C} \\ I_C = \dfrac{dQ}{dt} \end{cases}$

Ohm's law

$$\left\{ \begin{aligned} I_1 &= \frac{V_A - V_a}{R_1} = \frac{\mathcal{E} - V_C}{R_1} \\ I_2 &= \frac{V_C}{R_2} \end{aligned} \right\} \overset{(1)}{\Rightarrow} \frac{\mathcal{E} - V_C}{R_1} - \frac{V_C}{R_2} - I_C = 0$$

$$\frac{\mathcal{E}}{R_1} - V_C\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - I_C = 0$$

$$\frac{d}{dt}\left[\frac{\mathcal{E}}{R_1} - \frac{Q}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - I_C = 0\right] \Rightarrow -\frac{I_C}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{dI_C}{dt} = 0 \Rightarrow \left[\frac{dI_C}{I_C} = -\frac{dt}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]$$

$$\ln I_C = -\frac{t}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \text{Const} \Rightarrow \boxed{I_C(t) = I_0\, e^{-\frac{t}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

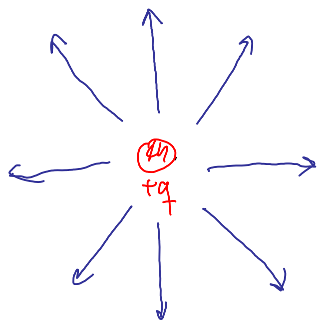
time constant: $\boxed{\tau = C\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$

or $\tau = C\left(\frac{R_1 + R_2}{R_1 \cdot R_2}\right)^{-1}$

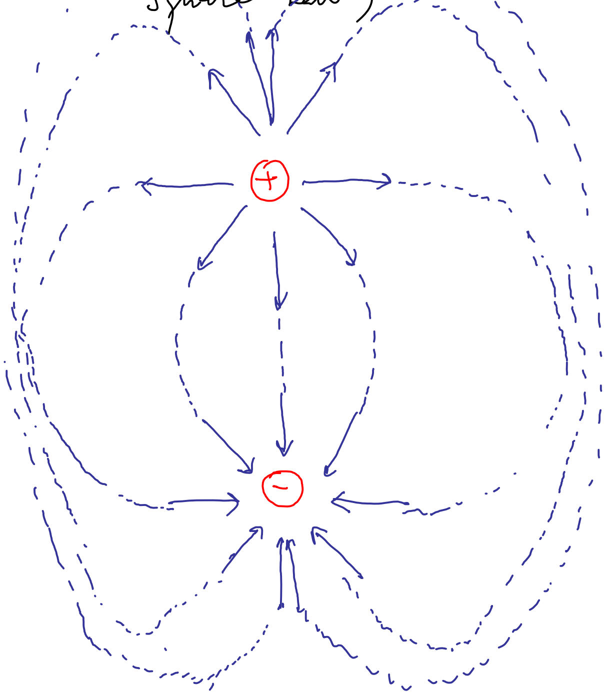
$\boxed{\tau = C\,\frac{R_1 R_2}{R_1 + R_2}}$ Same as with loop equations!

Ch26 Magnetic Field:

Electric field



Density of electric field lines is higher closer to charge ~ field strength decreases away from charge (inverse-square law)



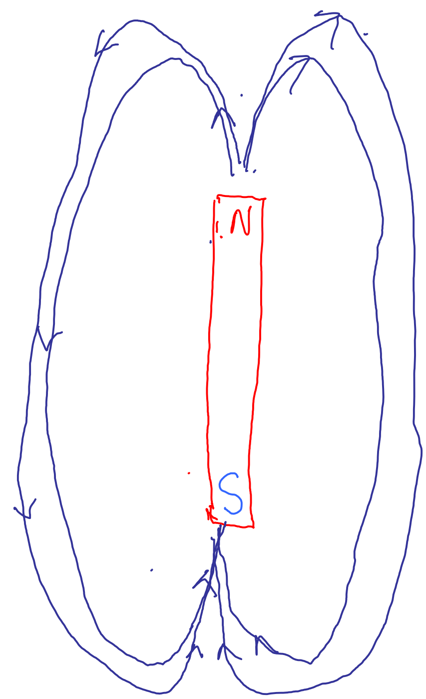
Electric dipole: all field lines are closed

Effect: change direction of (moving) charges (ink droplets in printers)

$\vec{F}_E = q \cdot \vec{E}$ ($\vec{E}$: electric field)

Magnetic field

No equivalent: magnetic monopole has not been found or there is no single magnetic "pole"



Magnetic dipole: all field lines are closed

Effect: keep moving charges in circular trajectories (magnetic confinement for fusion energy)

$\vec{F}_B = q \vec{v} \times \vec{B}$ ($\vec{B}$: magnetic field)

A charge can feel the electric field when it is moving or NOT

A charge only feels the magnetic field when moving $\vec{v} \neq 0$

Lorentz force : $\vec{F} = q \boxed{\vec{v} \times \vec{B}}$ → a vector

Cross product b/w two vectors $\vec{v}$ & $\vec{B}$

→ Direction by RHR (right hand rule) (when right hand fingers close from $\vec{v}$ to $\vec{B}$, thumb points in direction of $\vec{v} \times \vec{B}$)

→ Magnitude : $vB \sin\theta$ (θ : angle b/w $\vec{v}$ & $\vec{B}$)

* Cross product is always $\perp$ to the multiplying vectors

→ Zero Lorentz force when:

(i) $q = 0$ Neutral particles don't feel the magnetic field

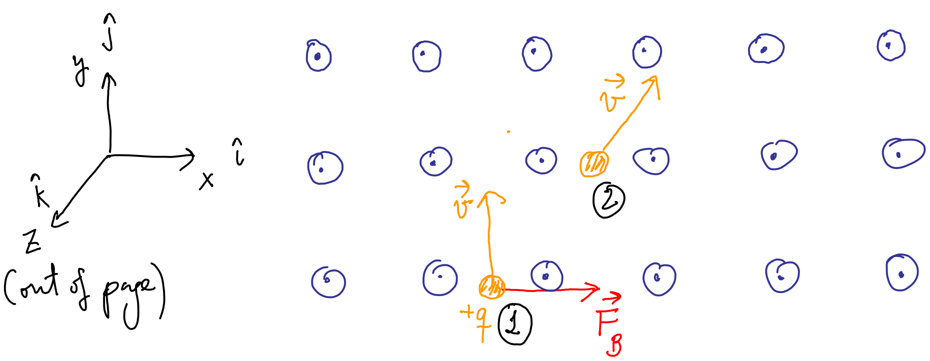
(ii) A static charged particle $q \neq 0$ but $\vec{v} = 0$

(iii) A moving charge going parallel to the magnetic field:

$\theta = 0°; 180° \rightarrow \sin\theta = 0 \Rightarrow \vec{F}_B = 0$

<u>Charged particle going through a region of uniform magnetic field:</u>

$\vec{B}$ pointing out of page ⊙



Region of uniform $\vec{B}$

$\vec{B} = B\hat{k}$

① A charge $+q$ travels in $+y$ direction:

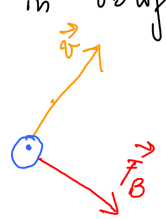
$\vec{v} = v\hat{j}$

↳ $\vec{F}_B = q\vec{v} \times \vec{B} = qvB \underbrace{\hat{j} \times \hat{k}}_{\hat{i}}$ (RHR)

Effect of $\vec{B}$: pull charged particle in $+x$ direction: adding a v_x to an originally only v_y velocity!

② $+q$ now travels in oblique direction

Effect of $\vec{B}$:



$\vec{F}_B$ is always $\perp$ $\vec{v}$ → $\left\{ \begin{array}{l} \vec{F}_B \text{ is along radial direction} \\ \vec{v} \text{ is in tangential direction} \end{array} \right\}$ UCM

UCM: particle travels on a circular trajectory at constant speed (but <u>not constant velocity</u>)

↳ requires a radial acceleration towards center of curvature

↳ $a = \frac{v^2}{R}$ (R: radius of curvature)

If a charged particle travels in a region of uniform $\vec{B}$ → F_B provides this radial acceleration

$$F_B = q\cancel{v}B = m \cdot \frac{v^{\cancel{2}}}{R} \Rightarrow \boxed{R = \frac{mv}{qB}}$$

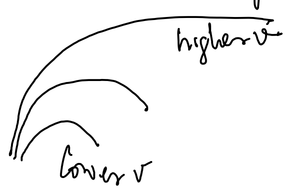
$$\vec{v} \perp \vec{B} \Rightarrow \sin\theta = \sin 90 = 1$$

Particle physics: subatomic particles: e, p, muons, neutrinos, etc.

microscopic

↳ detectors: outcomes of collisions:

- if same mass m, larger v → larger orbit (radius R)
- if same v, larger m → larger orbit.



- R: orbital radius; T: orbital period: time to complete one orbit:

$$T = \frac{2\pi R}{v} = \frac{2\pi \cancel{R}}{\frac{qB\cancel{R}}{m}} = \frac{2\pi m}{qB}$$

Calculation of Magnetic Field:

Electric field

Source → charge

$$d\vec{E} = \frac{k\,dq}{r^2}\,\hat{r}$$

Coulomb's Law
inverse-square law : $\sim \frac{1}{r^2}$
$\hat{r}$ radial unit vector

Magnetic field

Source → $\boxed{\text{moving}}$ charge
or current

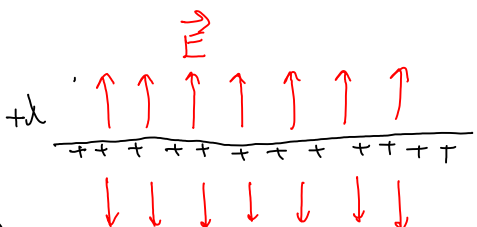
$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Biot-Savart's Law
inverse-square law : $\sim \frac{1}{r^2}$

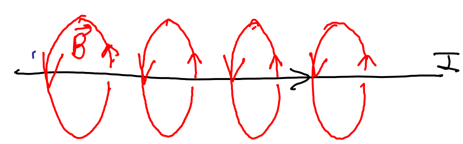
$\mu_0 = 4\pi\times10^{-7}\ \frac{N}{A^2}$
↳ permeability in vacuum

direction of field : $d\vec{l}\times\hat{r}$
↳ magnetic field due to a line of current wraps around the current (RHR)

line of charge (static charges)

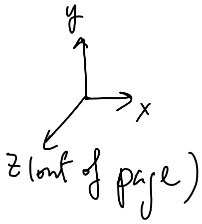


line of current I (moving charge)

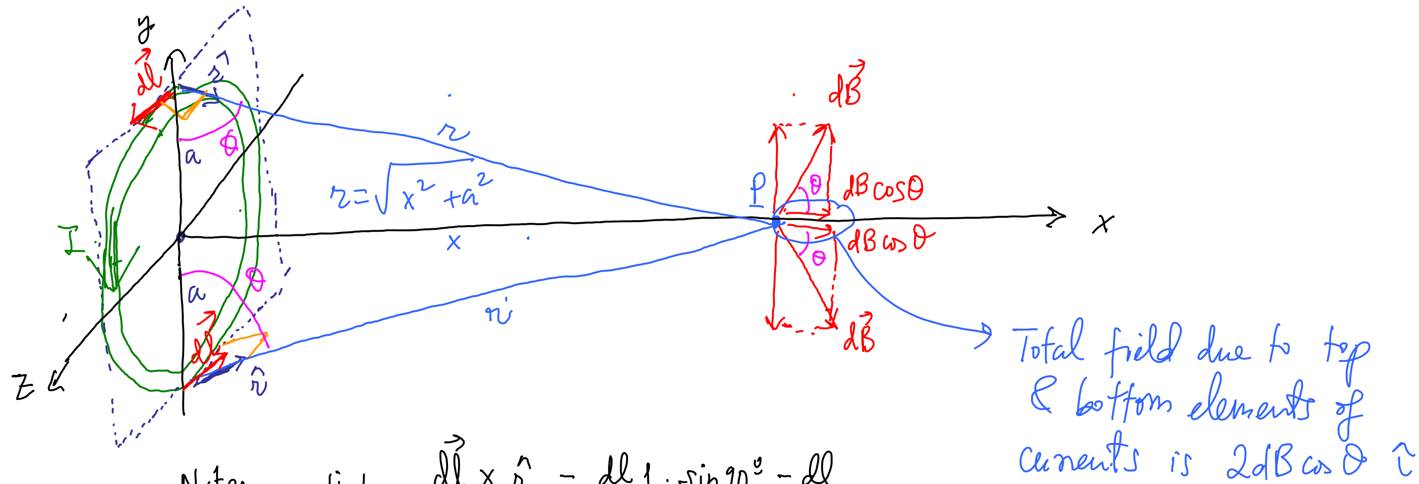


Direction of $\vec{B}$ in relation to its source I is given by RHR
if RH fingers close in direction of $\vec{B}$, thumb points in the direction of source I!

Magnetic field due to a loop of current (superposition method)



Loop of current in YZ plane
Calculate $\vec{B}$ on x-axis (through center of loop!)



Notes: (i) $\vec{dl} \times \hat{r} = dl\, 1 \cdot \sin 90° = dl$

(ii) $\cos\theta = \frac{a}{r}$

(iii) total field due to top & bottom element of current is $2\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\frac{a}{r}\hat{i}$

Total field $\vec{B}$ due to whole ring:

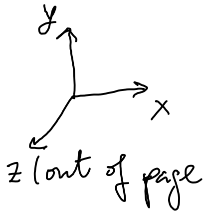
$$\vec{B} = \int_{\substack{\text{Half}\\\text{ring}}} d\vec{B} = \frac{\cancel{2}\mu_0 I a}{_2\cancel{4}\pi r^3}\hat{i}\int_{\substack{\text{Half}\\\text{ring}}} dl = \frac{\mu_0 I a}{2\cancel{\pi} r^3}\cancel{\pi} a\ \hat{i} = \frac{\mu_0 I a^2}{2r^3}\hat{i}$$

$$\vec{B} = \frac{\mu_0}{2}\frac{Ia^2}{(x^2+a^2)^{3/2}}\hat{i}$$

Very far away from ring of current: $x \gg a \rightarrow x^2+a^2 \simeq x^2$

$$\vec{B} = \frac{\mu_0}{2}\frac{Ia^2}{(x^{\cancel{2}})^{3/\cancel{2}}}\hat{i} = \frac{\mu_0}{2}\frac{Ia^2}{x^3}$$ or $B \sim \frac{1}{x^3}$ inverse-cube Law like electric dipole!

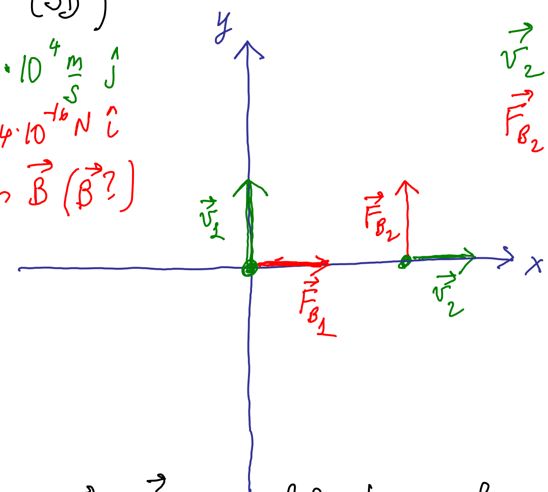
26.44 Application of Lorentz's force: $\vec{F}_B = q\vec{v}\times\vec{B}$ (3D)



$\vec{v}_1 = 3.6\cdot 10^4 \frac{m}{s}\,\hat{j}$

$\vec{F}_{B_1} = 7.4\cdot 10^{-16}\,N\,\hat{i}$

Uniform $\vec{B}$ ($\vec{B}$?)



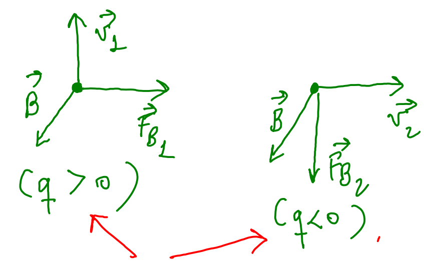
$\vec{v}_2 = \pm v_2\,\hat{i}$ (v_2?)

$\vec{F}_{B_2} = 2.8\cdot 10^{-16}\,N\,\hat{j}$

Observations: (i) $\vec{F}_B = q\vec{v}\times\vec{B}$ by definition of cross-product $\vec{F}_B$ is $\perp$ to both $\vec{v}$ & $\vec{B}$ (b/c RHR)

↳ a) $\vec{B}$ has to point along z-axis

↳ 1) If $\vec{B} = B\hat{k}$ (+z):

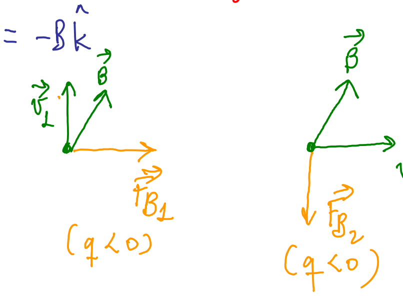


$\boxed{\vec{v}_2 = +v_2\hat{i}}$

No

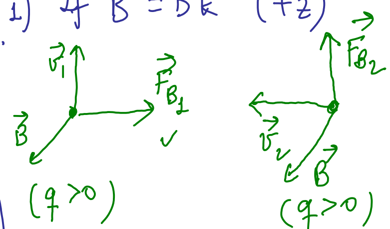
$\vec{B}$ should point in −z!

2) $\vec{B} = -B\hat{k}$



No

1) If $\vec{B} = B\hat{k}$ (+z)



$\boxed{\vec{v}_2 = -v_2\hat{i}}$

Yes

Conclusions: $\vec{v}_2 = -v_2\,\hat{i}$!

Continue with document camera

26.44 (Cont.)

Conclusions : (RHR) $\begin{cases} \vec{v}_2 = -v_2\hat{i} \\ \vec{B} = B\hat{k} \end{cases}$ ($q>0$, protons)

Proton #1: $q=+e$

$$\vec{F}_{B_1} = \underbrace{7.4\cdot10^{-16}N}\,\hat{i} = e\vec{v}_1\times\vec{B} = \underbrace{ev_1B}\,(\underbrace{+\hat{j}\times\hat{k}}_{\hat{i}})$$

$$\hookrightarrow 7.4\cdot10^{-16} = 1.6\cdot10^{-19}\cdot3.6\cdot10^{4}B$$

$$B = \frac{7.4\cdot10^{-16}}{1.6\cdot3.6\cdot10^{-15}} = \frac{0.74}{1.6\cdot3.6} = 0.128\text{T}$$

(Tesla)

$$\boxed{B = 128\text{ mT}}$$

Proton #2: $q=+e$

$$\vec{F}_{B_2} = 2.8\cdot10^{-16}N\,\hat{j} = e\vec{v}_2\times\vec{B} = ev_2B\,(\underbrace{-\hat{i}\times\hat{k}}_{\hat{j}})$$

$$2.8\cdot10^{-16} = 1.6\cdot10^{-19}v_2\,0.128$$

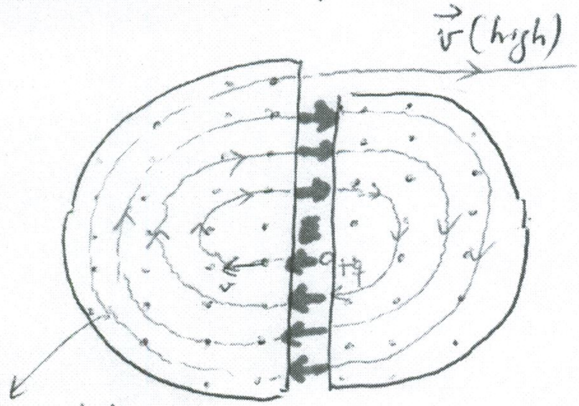
$$v_2 = \frac{2.8\cdot10^{-16}}{1.6\cdot10^{-19}\cdot0.128} = \frac{2.8}{1.6\cdot0.128}10^{3}$$

$$\boxed{v_2 = 1.37\cdot10^{4}\,\frac{m}{s}}$$

Cyclotron (accelerate charged particles to high speed by running them through circular orbits multiple times)

→ We can also accelerate charged particles to high speed using an electric field & a very long tunnel

→ But cyclotron uses $\vec{B}$ & $\vec{E}$ can do the job in a smaller area like a hospital lab.



largest orbit
highest speed

Cyclotron

Use radius of Cyclotron R

1) Two dee's with a gap b/w them, where an alternating electric field is applied: left, right, left, etc..

2) A uniform $\vec{B}$ fill the dee's pointing out of page

3) $KE_{max} = \frac{1}{2} m v_{max}^2$

$$r = \frac{mv}{qB} \rightarrow R = \frac{m v_{max}}{qB}$$

$$\rightarrow v_{max} = \frac{qBR}{m}$$

$$\rightarrow KE_{max} = \frac{1}{2} m \left(\frac{qBR}{m}\right)^2 = \frac{(qBR)^2}{2m}$$

Note: for $v \ll c = 3 \cdot 10^8 \frac{m}{s}$: Newtonian mechanics applies

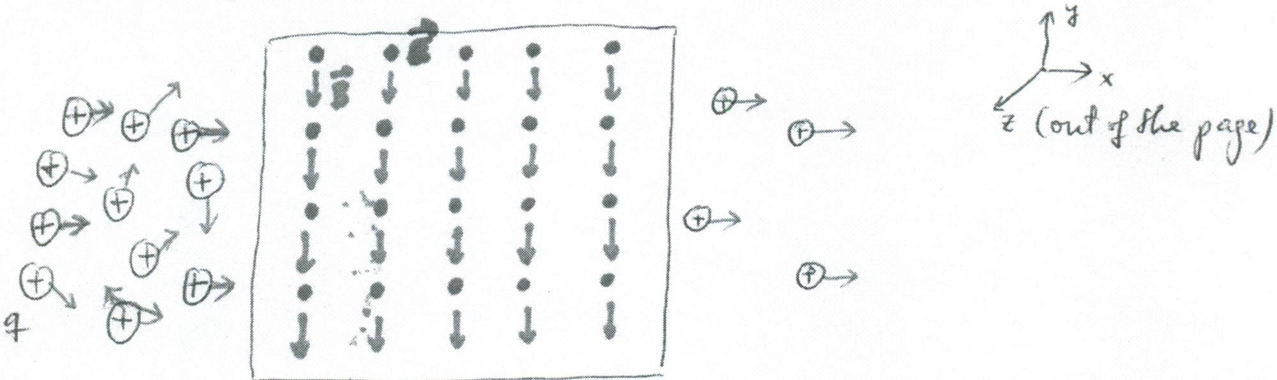
but if $v \rightarrow c$: we need relativistic corrections:

→ synchrotron

Velocity Selector:

We can pick among those ions with different velocities, particular ones with a selected velocity by running them through a region filled with both $\vec{E}$ & $\vec{B}$.

For example, to pick among ions with ~~velocities~~ velocities in random direction, those with velocities in the x-direction:



$\vec{E} = E\hat{k}$ & $\vec{B} = B(-\hat{j})$

For those ions with $\vec{v} = v\hat{i}$

$$\begin{cases} \vec{F}_E = q\vec{E} = qE\hat{k} \\ \vec{F}_B = qvB\underbrace{(-\hat{i}\times\hat{j})}_{-\hat{k}} = -qvB\hat{k} \end{cases}$$

$\to \vec{F}_{net} = \vec{F}_E + \vec{F}_B = (qE - qvB)\hat{k}$ could be zero

If $\vec{F}_{net} = 0 \to$ ions go through unaffected:

$$E - vB = 0 \quad \text{or} \quad \boxed{v = \frac{E}{B}}$$

Those ions with $\vec{v} = \frac{E}{B}\hat{i}$ they are "selected".

Ch 26 (cont.)

Calculations of fields

Electric

(i) Vector superposition & Coulomb's law

(ii) Gauss' Law

$$\underbrace{\oint \vec{E} \cdot d\vec{A}}_{\substack{\phi_E \\ \text{(electric flux)}}} = \frac{q_{enclosed}}{\varepsilon_0}$$

Gaussian surface so that $\phi_E = E \cdot A$

↳ { $\vec{E}$ constant on G-surface
$\vec{E}$ perpendicular to G-surface
or $\vec{E} \parallel d\vec{A}$

- side task: calculate charge enclosed by G-surface

(iii) Electric potential V (scalar)
(arithmetic addition)

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

Magnetic

(i) Vector superposition & Biot-Savart's Law

(ii) Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Amperian loop so that $\oint \vec{B} \cdot d\vec{l} = B \cdot l$

↳ { $\vec{B}$ constant along loop
$\vec{B}$ tangential to Amperian loop
$\vec{B} \parallel d\vec{l}$

- side task: calculate current enclosed by A-loop.

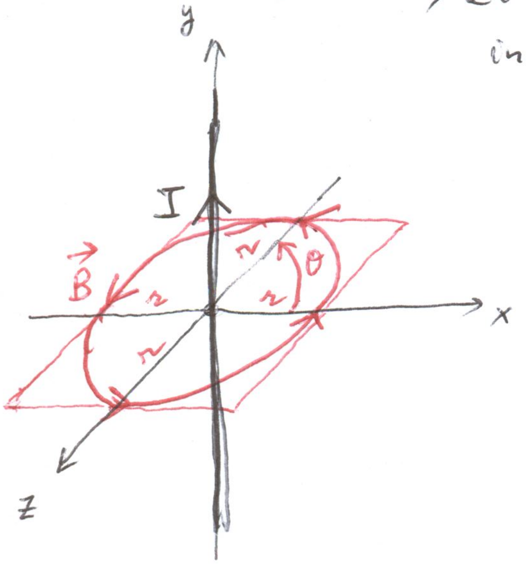
(iii) Vector potential $\vec{A}$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

↳ Rotational of $\vec{A}$
or Curl of $\vec{A}$

Magnetic field due to a long line of current I using Ampere's Law:

→ It wraps around the current following RHR as shown in red.



↳ (i) Amperian loop = $\left\{ \begin{array}{l} \vec{B} \text{ constant} \\ \vec{B} \text{ tangential} \end{array} \right\}$ along loop

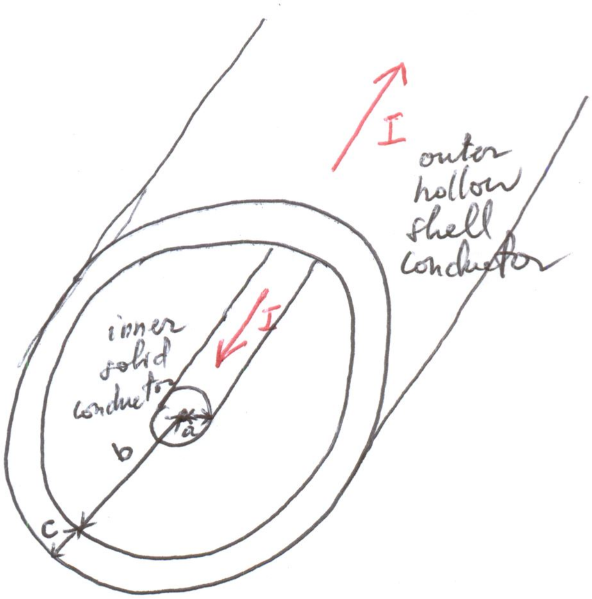
↳ Circle of radius r centered @ current

(ii) If θ is an angle on XZ plane starting from x-axis ⇒ $\hat{\theta}$ is a unit vector to indicate direction of magnetic field: $\vec{B} = B(r)\,\hat{\theta}$

(iii) Ampere's Law $\left\{ \begin{array}{l} \text{a) LHS: } \underbrace{\oint}_{\text{Amperian loop}} \vec{B}\cdot d\vec{l} = B \underbrace{\oint dl}_{\text{circumference}} = B\cdot 2\pi r \\ \text{b) RHS: } \mu_0 I_{enclosed} = \mu_0 I \end{array} \right\}$ $B 2\pi r = \mu_0 I \;\to\; B = \dfrac{\mu_0 I}{2\pi r}$

$$\rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}}$$

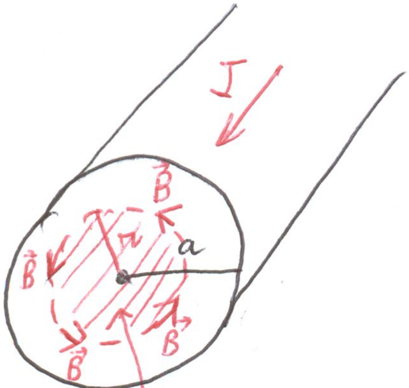
26.66 Coaxial cable:



Current I → $\vec{B}$? $\left\{ \begin{array}{l} 0 < r < a \\ a < r < b \\ r > b+c \end{array} \right.$

↳ place an Amperian loop in each of these 3 regions to find magnetic field!

a) Inside inner conductor: $0<r<a$



Amperian loop: circle of radius r centered at axis of coaxial cable (shown in red)

↳ $\vec{B}$ is constant & $\boxed{\text{tangential}}$ to this loop! ⇒ $\oint \vec{B}\cdot d\vec{l} = B\cdot 2\pi r$

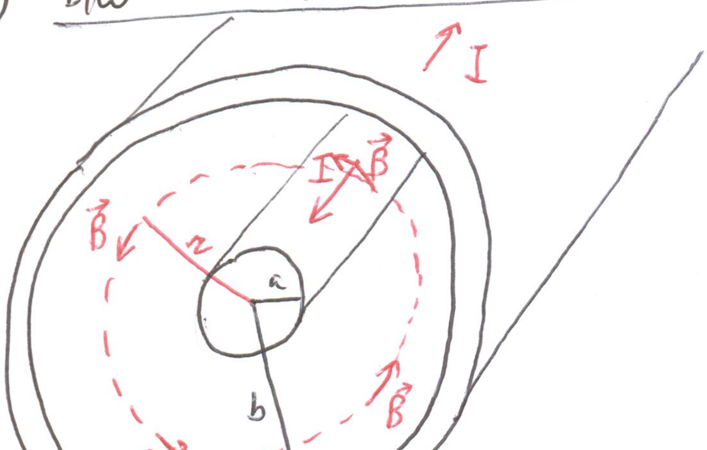
reduces to arithmetic

RHS of Ampere's Law: $\mu_0 \boxed{I_{enclosed}}$ $= \mu_0 I \cdot \frac{\pi r^2}{\pi a^2}$

↳ proportional to area enclosed by Amperian loop

→ $B\cdot 2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{a^2}$ → $\boxed{B = \frac{\mu_0 I}{2\pi a^2} r}$ B linearly increases with r!

b) B/w inner solid conductor & outer hollow shell conductor: $a<r<b$



Amperian loop: circle of radius r centered at axis of coaxial cable (shown in red)

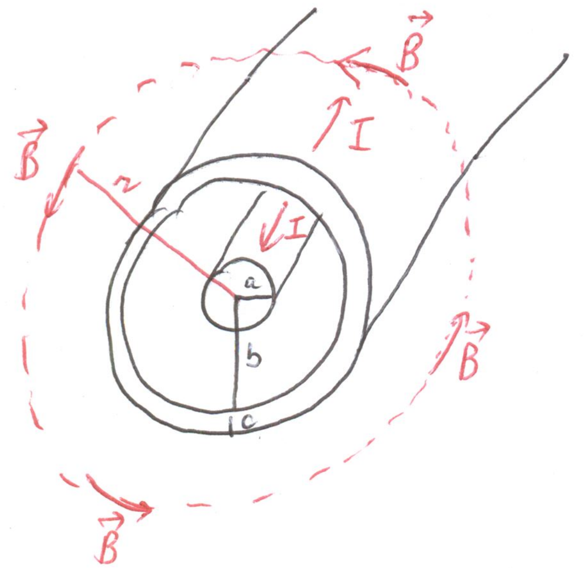
↳ $\vec{B}$ constant & tangential to this loop → $\oint \vec{B}\cdot d\vec{l} = B\cdot 2\pi r$

RHS of Ampere's Law: $\mu_0 I_{enclosed} = \mu_0 I$

→ $B\cdot 2\pi r = \mu_0 I$ ⇒ $\boxed{B = \frac{\mu_0 I}{2\pi r}}$ (field due to long line of current)

↳ Current in the outer conducting shell doesn't affect this interior region!

c) $\vec{B}$ outside outer conducting shell $r > b+c$:



Amperian loop: circle of radius r centered @ coaxial cable axis

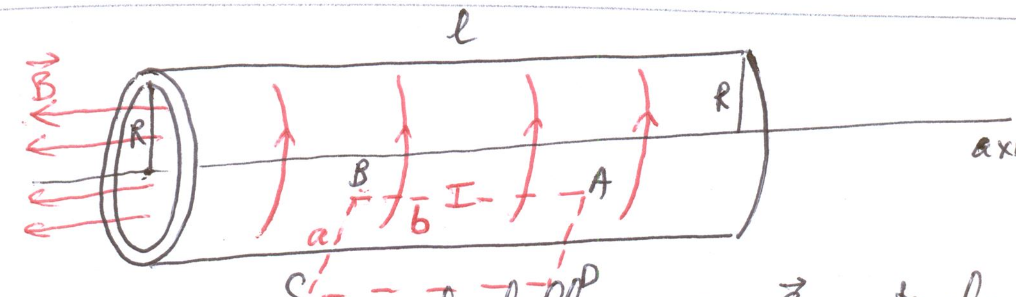
$$\oint \vec{B}\cdot d\vec{l} = B\,2\pi r$$

$$\mu_0 I_{enclosed} = \mu_0 (I - I) = 0$$

$$B\cdot 2\pi r = 0 \Rightarrow \boxed{B = 0}$$

↓ effect of coaxial cable is to shield the environment from the fields created by current inside.

26.74



→ Current I is wrapping around shell → RHR: $\vec{B}$ points along axis to the left: → remember $\vec{B}$ due to a ring of current points along axis of ring away from it

c) $B(r)$? $r < R$: Amperian loop is a rectangle of sides a & b

$$\oint_{ABCD} \vec{B}\cdot d\vec{l} = \int_{AB} \vec{B}\cdot d\vec{l} + \underbrace{\cancel{\int_{BC}}}_{0} + \underbrace{\cancel{\int_{CD}}}_{0} + \underbrace{\cancel{\int_{DA}}}_{0}$$

$= Bb +$

$$\mu_0 I_{enclosed} = \mu_0 I \cdot \frac{b}{l}$$

↳ side of length b (one side) is inside shell & parallel to its axis and $\vec{B}$

the other side of length b is outside shell where $\vec{B} = 0$

the other two sides of length a are $\perp$ to $\vec{B}$

$$\Rightarrow \quad B \cdot \cancel{b} = \mu_0 I \frac{\cancel{b}}{l} \rightarrow \boxed{B = \frac{\mu_0 I}{l}}$$

b) $B(r) \quad r > R \quad \rightarrow \quad B = 0$