Ch23 Electrostatic Energy & Capacitors

Ch22:

↳ Electric potential: $\Delta V_{12} = \dfrac{\Delta U_{12}}{q'}$ ← change electric potential energy (J)

q' ← probe or test charge (C)

$(V = \frac{J}{C})$

↳ Electric potential energy $\boxed{\Delta U_{12}} = -W_{12} = -\int_1^2 \vec{F} \cdot d\vec{r}$ ← electric force ← split Q=0 into + & − charges

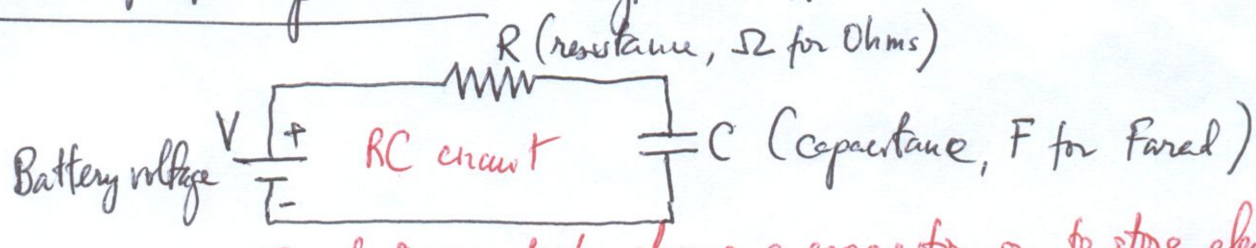
"charging a battery"

↳ We can store electrostatic potential energy by separating charges of opposite signs (Similarly to storing elastic potential energy by stretching a spring!)

↓

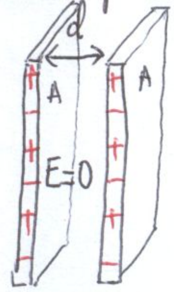
Capacitors

Parallel-plate Capacitors : symbol: ||



Simplest circuit to charge a capacitor or to store electrostatic potential energy

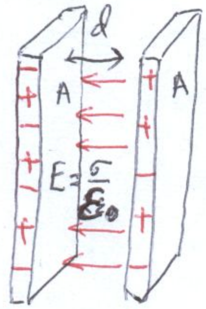
Parallel-plate capacitor



$Q=0$ $Q=0$

To charge: move one charge ⊖ from right plate to left plate

Work against $E=0$ (easiest charge to move!)



$Q=-e$ $Q=+e$

$\sigma = \frac{e}{A}$

Moving 2nd charge from right plate to left plate:

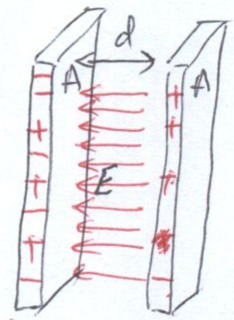
work against $E=\frac{\sigma}{\varepsilon_0}$ (R to L)

↓

Force on a negative charge

$F=q'\cdot E$

(L to R)!



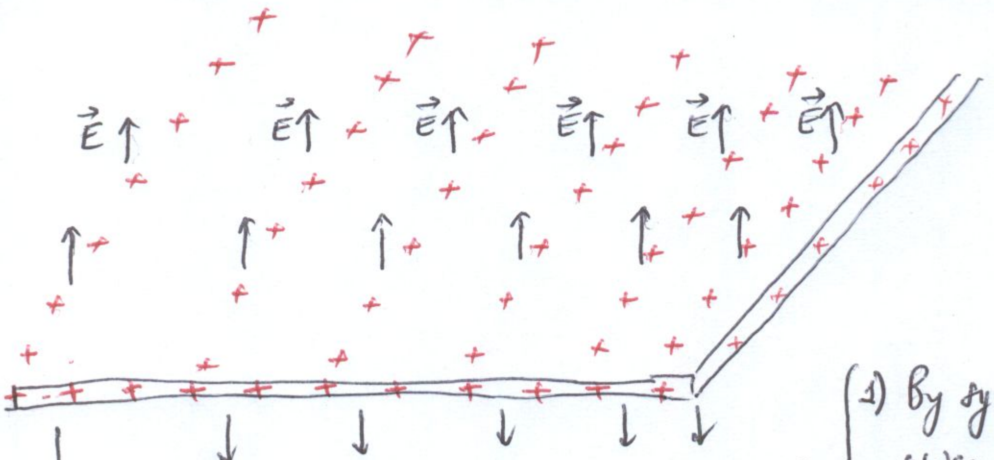
$Q=-2e$ $Q=+2e$

Conclusion: 1) It takes energy to store energy in a capacitor or battery

2) It takes more energy to move later charges from one plate to another (to store energy) since we have to work against the electric field created by the new charge in the plates!

Electric field due to an infinite plate or plane of charge: $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

↳ Gauss Law $\phi = \frac{q_{enclosed}}{\varepsilon_0}$



Positively charged ∞ plate → electric field $\vec{E}$

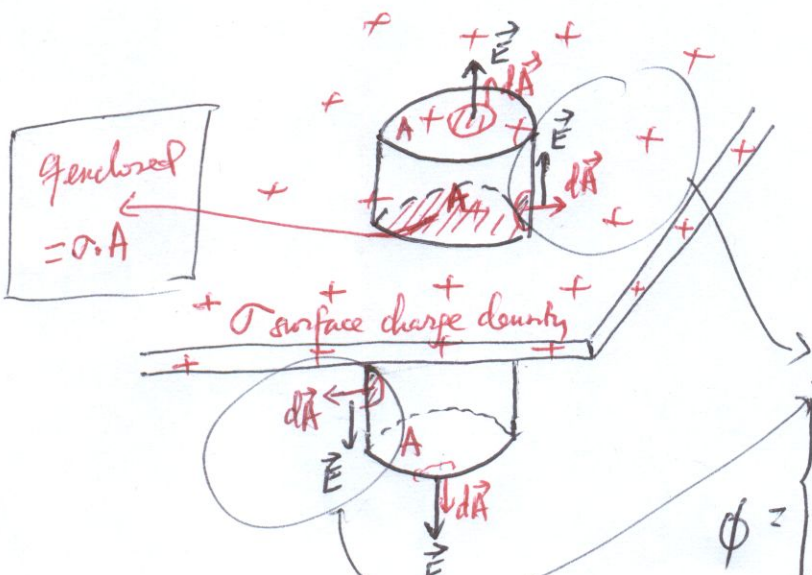
1) By symmetry $\vec{E}$ point away & perpendicular to the plane of +charge
2) $\vec{E}$ will have same strength at same separation from plate

Apply Gauss Law to find an equation for $\vec{E}$:

↳ Gaussian surface ↔ $\phi = \oint \vec{E} \cdot d\vec{A} = EA$

Convenient: Gaussian surface where
1) $\vec{E} \parallel d\vec{A}$
2) $\vec{E}$ constant

↳ Cylindrical with top & bottom cross-sectional areas parallel to the plate.



$\phi =$
- No contribution to ϕ since $\vec{E} \cdot d\vec{A} = 0$ there
- Only top & bottom cross-sectional areas of cylindrical Gaussian surface contribute $\phi_{G\text{-surface}} = 2EA$

Gauss Law

1) $\phi_{G\text{-surface}} = 2EA$ (E: electric field; A: cross-sectional surface area of Gaussian cylindrical surface)

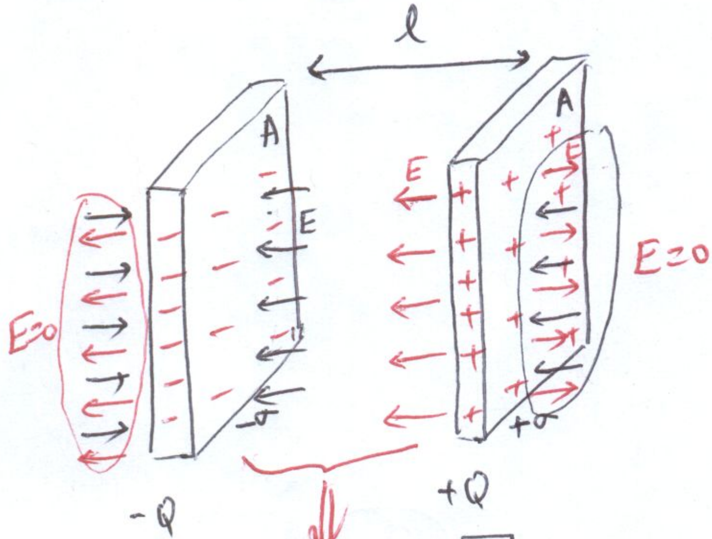
2) $q_{enclosed} = \sigma \cdot A$ (σ: surface charge density of the plate of charge)

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

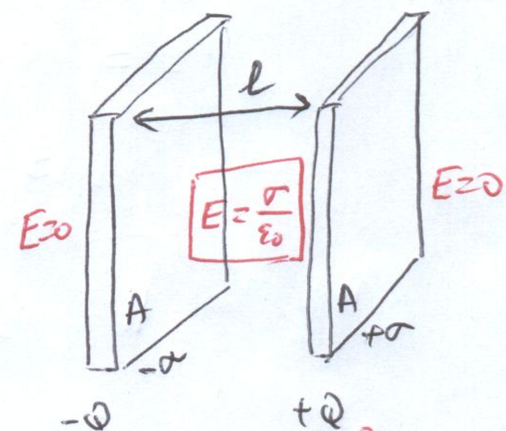
$\downarrow$

Electric field due to an infinite plate of charge is the surface charge density divided by $2\varepsilon_0$

How much energy U can we store in a parallel plate capacitor?



or



$$\Downarrow$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}}$$

Results from ∞ plate of charge: $l \ll \sqrt{A}$

(when you are close enough to a finite plate, it looks like infinite plate!)

infinitesimal probe charge

electric potential b/w plates

$$dU = -dW = -dq \cdot V$$

$$V = -\int \vec{E} \cdot d\vec{l} = -El$$

$$\Rightarrow dU = dq\, E l = dq \frac{\sigma}{\varepsilon_0} l = dq \frac{q}{A} \frac{l}{\varepsilon_0}$$

$$U = \int dU = \frac{l}{A\varepsilon_0} \underbrace{\int_0^Q q\, dq}_{\left[\frac{q^2}{2}\right]_0^Q} = \frac{l}{A\varepsilon_0} \frac{Q^2}{2}$$

$$U = \frac{\ell}{A\varepsilon_0}\frac{Q^2}{2} = \frac{\ell}{(A\varepsilon_0)^2} A\varepsilon_0 \frac{Q^2}{2} = \frac{1}{2}\boxed{\ell A}\varepsilon_0 E^2$$

Vol b/w plates! → where electrostatic potential energy is store!

Note: $\frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = E$!

$$\left[u \equiv \frac{U}{vol} = \text{electrostatic potential energy } \underbrace{\text{density}}_{\text{unit: } \frac{J}{m^3}} = \frac{\frac{1}{2}\cancel{\ell A}\varepsilon_0 E^2}{\cancel{\ell A}} = \frac{1}{2}\varepsilon_0 E^2 \right]$$

Total energy stored in a parallel plate capacitor

(i) $U = \frac{1}{2} vol.\, \varepsilon_0 E^2 \rightarrow u = \frac{U}{vol} = \frac{1}{2}\varepsilon_0 E^2$ (~ kinetic energy: $\frac{1}{2}mv^2$)

(ii) $U = \frac{1}{2}\varepsilon_0 E^2 (A\ell)\frac{\ell}{\ell} = \frac{1}{2}\underbrace{\frac{A\varepsilon_0}{\ell}}_{C}\underbrace{E^2\cdot\ell^2}_{V^2} = \frac{1}{2}CV^2$

Capacitance $C \equiv \frac{Q}{V} = \frac{Q}{E\cdot\ell} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\cdot\ell} = \frac{Q}{\sigma}\frac{\varepsilon_0}{\ell} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A}}\frac{\varepsilon_0}{\ell}$

$$= \frac{A\varepsilon_0}{\ell}$$

Kinetic	Electrostatic
$\frac{1}{2}mv^2$	$\frac{1}{2}CV^2$
m ↓ inertia to change in speed v	↓ inertia to change in electric potential V

Energy stored:

Kinetic (motion)

$\frac{1}{2} m v^2$

mass m: inertia to changes in velocity v

Electric (capacitor)

$\frac{1}{2} C V^2$

Capacitance C: inertia to changes in electric potential V

Magnetic (solenoid)

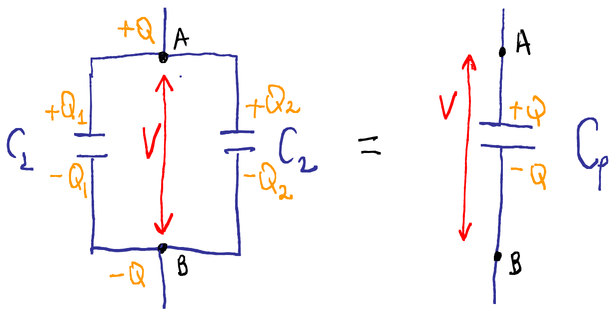
$\frac{1}{2} L I^2$

inductance L: inertia to changes in electric current I

Connecting two capacitors C_1 & C_2

Parallel Connection

Same potential difference V across the two



Facts: (i) $Q = Q_1 + Q_2$

(ii) The equivalent capacitor b/w A & B is delivering the same potential difference V, and same charge Q

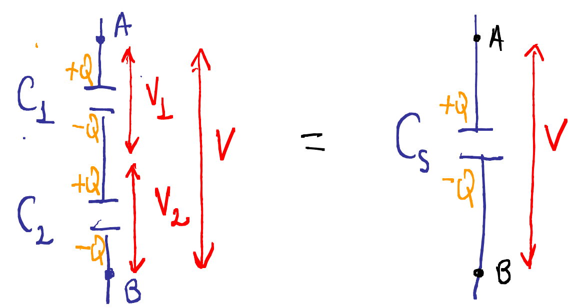
(iii) $C_1 \equiv \frac{Q_1}{V}$; $C_2 \equiv \frac{Q_2}{V}$

Consequence: $\boxed{C_p \equiv \frac{Q}{V}} \overset{(i)}{=} \frac{Q_1 + Q_2}{V} = \boxed{C_1 + C_2}$

Parallel connection increases the capacitance

Series Connection

Same charge Q in both capacitors



Facts: (i) $V_1 \neq V_2$; $V = V_1 + V_2$

(ii) The equivalent capacitor b/w A & B should deliver same potential difference V, and same charge Q

(iii) $C_1 \equiv \frac{Q}{V_1}$; $C_2 = \frac{Q}{V_2}$

Consequence: $C_s = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{1}{\frac{V_1}{Q} + \frac{V_2}{Q}}$

$= \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$

$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$ or $C_s = \frac{1}{\frac{C_1 + C_2}{C_1 C_2}} = \frac{C_1 C_2}{C_1 + C_2}$

If $C_1 = C_2 = C \Rightarrow C_s = \frac{C \cdot \cancel{C}}{2\cancel{C}} = \frac{C}{2}$

Series connection reduces the capacitance

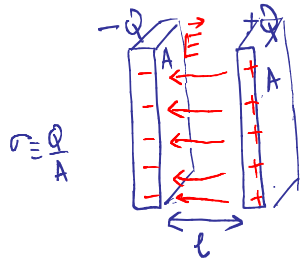
Ways to increase the capacitance:

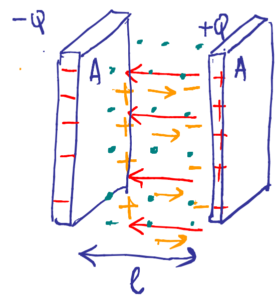
1) Connecting multiple capacitors in parallel
2) Inserting a dielectric material b/w the plates:

Background: $C = \frac{Q}{V} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\cdot \ell} = \frac{\cancel{\sigma}\cdot A}{\frac{\cancel{\sigma}}{\varepsilon_0}\cdot \ell} = \frac{A\varepsilon_0}{\ell}$

parallel plate

$V = E\cdot \ell = \frac{\sigma}{\varepsilon_0}\cdot \ell$





(i) no insert or vacuum b/w plates

$E = E_0 = \frac{\sigma}{\varepsilon_0}$ $\begin{cases} \sigma: \text{surface charge density} \\ \varepsilon_0: \text{dielectric constant of vacuum} \end{cases}$

(ii) $C = C_0 = \frac{Q}{V_0}$ $\begin{cases} Q \text{ charge on each plate} \\ V_0 = E_0\cdot \ell \;:\; \text{potential across plates with no insert} \end{cases}$

(i) dielectric insert b/w plates of dielect constant $\varepsilon = K\cdot\varepsilon_0$. K (kappa) is the dielectric coefficient (no dimension, >1)

(ii) Electrons in the dielectric are attracted to the + charges in the right plate leaving + charges attracted to the left plate. So dielectric insert has it own electric field pointing in the opposite direction

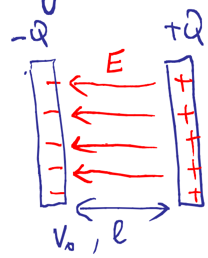
$\Rightarrow$ Electric field b/w plates is reduced: $E = \frac{E_0}{K}$

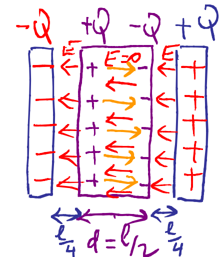
(iii) $C = \frac{Q}{V} = \frac{Q}{E\cdot\ell} = \frac{Q}{\frac{E_0}{K}\cdot\ell} = K\underbrace{\frac{Q}{E_0\cdot\ell}}_{C_0} = K\cdot C_0$

Capacitance is increased by a factor of K when a dielectric is inserted b/w plates

3) Inserting a conducting slab b/w plates: $\hookrightarrow$ width $d < \ell$

$\hookrightarrow$ many free electrons





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(i) Due to free electrons arrangement inside conducting slab, the electric field inside slab is cancelled $E=0$

(ii) If conducting slab width is $\frac{\ell}{2}$ the effective separation b/w plates is reduced to $\frac{\ell}{2}$!

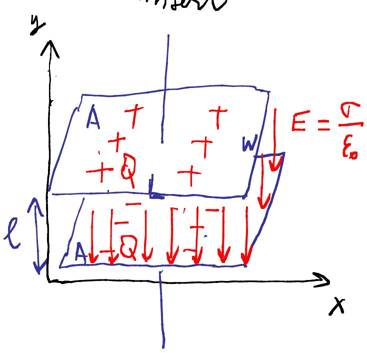
$$C_0 = \frac{Q}{V_0} = \frac{Q}{E \cdot \ell}$$

$$C = \frac{Q}{V} = \frac{Q}{E\cdot\frac{\ell}{2}} = 2\underbrace{\frac{Q}{E\cdot\ell}}_{C_0} = 2C_0$$

Capacitance is increased when a conducting slab is inserted b/w plates

23.69

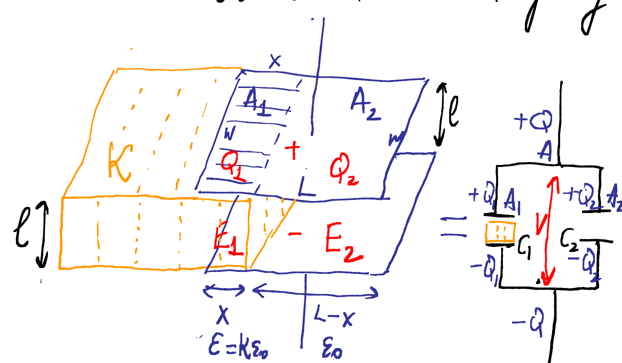
No dielectric insert



$$C_0 = \frac{Q}{V_0}$$

$$U_0 = \frac{1}{2} C_0 V_0^2$$

With dielectric inserted a distance x into spacing



equivalent to having two capacitors in parallel

$$\boxed{C_p = C_1 + C_2}$$

$Q = Q_1 + Q_2$

$A = A_1 + A_2$ $\begin{cases} A_1 = x \cdot w \\ A_2 = (L-x)w \end{cases}$

With dielectric	Without dielectric
Q_1	Q_2
A_1	A_2
$\varepsilon = K\varepsilon_0$	ε_0
E_1	E_2

We want to find $C_p = C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V} \rightarrow \begin{cases} Q_1 \\ Q_2 \end{cases}$?

Observations: (Parallel plate capacitors)

(i) $E_1 = \frac{\sigma_1}{\varepsilon} = \frac{\frac{Q_1}{A_1}}{K\varepsilon_0} = \frac{Q_1}{x \cdot w\, K\varepsilon_0}$; (ii) $E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{\frac{Q_2}{A_2}}{\varepsilon_0} = \frac{Q_2}{(L-x)w\varepsilon_0}$

(iii) Field continuity b/w two regions $(\varepsilon, \varepsilon_0)$: $E_1(x) = E_2(x)$ (at one location x, there can't be two different electric fields!) $\frac{Q_1}{x\cancel{w}K\cancel{\varepsilon_0}} = \frac{Q_2}{(L-x)\cancel{w}\cancel{\varepsilon_0}} \Rightarrow \boxed{Q_1 = Q_2 \frac{xK}{L-x}}$

Q_1 is a fraction of Q_2 depending on how far in dielectric is inserted

(iv) Total capacitance with dielectric inserted a distance x into spacing is $C = \frac{Q_1 + Q_2}{V} = \frac{Q_2 \frac{xK}{L-x} + Q_2}{V} = \frac{Q_2}{V}\left(\frac{Kx}{L-x} + 1\right)$

(v) $\frac{Q_2}{V} = \frac{Q_2}{E_2 \cdot \ell} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{(L-x)w\varepsilon_0} \cdot \ell} = \frac{(L-x)w\varepsilon_0}{\ell}$

(vi) $\underset{(iv)\&(v)}{C =} \frac{(L-x)w\varepsilon_0}{\ell} \cdot \left(\frac{Kx}{L-x} + 1\right) = \frac{w\varepsilon_0}{\ell}\left(Kx + L - x\right)$

$$C(x) = \frac{w\varepsilon_0}{\ell}\left[x(K-1) + L\right]$$

Total capacitance increases linearly with x (as the dielectric is inserted into spacing)

Three questions in 23.69:

a) What is $C\left(x = \frac{L}{2}\right)$? $\rightarrow C\left(\frac{L}{2}\right) = \frac{w\varepsilon_0}{\ell}\left[\frac{L}{2}(K-1) + L\right] = \frac{w\varepsilon_0 L}{2\ell}\left[K - 1 + 2\right]$

$= \frac{w\varepsilon_0 L}{2\ell}(K+1)$

b) What is $U(x)$?

$U(x) = \frac{1}{2}C(x)V^2(x) = \frac{1}{2}\frac{w\varepsilon_0}{\ell}\left[x(K-1) + L\right] \cdot V^2(x)$

$V = E \cdot \ell \rightarrow V(x) = E(x) \cdot \ell = E_1(x) \cdot \ell = \frac{\boxed{Q_1}}{xwK\varepsilon_0} \cdot \ell$

$E_1(x) = E_2(x) \equiv E(x)$

$\begin{cases} Q = Q_1 + Q_2 = \\ Q_1 = Q_2 \frac{Kx}{L-x} \Rightarrow Q_2 = Q_1 \frac{L-x}{Kx} \end{cases}$ $\rightarrow Q = Q_1 + Q_1 \frac{L-x}{Kx} = Q_1\left(1 + \frac{L-x}{Kx}\right) \Rightarrow Q_1 = \frac{Q}{\left(1 + \frac{L-x}{Kx}\right)}$

$Q_1 = Q \frac{Kx}{Kx + L - x}$

$\boxed{Q_1 = Q \frac{Kx}{x(K-1) + L}}$

$\boxed{V(x) = \frac{Q_1}{xwK\varepsilon_0}\ell} = \frac{Q\cancel{Kx}}{[x(K-1)+L]} \frac{\ell}{\cancel{x}w\cancel{K}\varepsilon_0} = \frac{Q\ell}{w\varepsilon_0[x(K-1)+L]}$

$U(x) = \frac{1}{2}\frac{w\varepsilon_0}{\ell}\left[x(K-1)+L\right]\frac{Q^2\ell^2}{w^2\varepsilon_0^2[x(K-1)+L]^2} = \boxed{\frac{1}{2}\frac{Q^2 \cdot \ell}{[x(K-1)+L] \cdot w\varepsilon_0}}$

$U\left(x = \frac{L}{2}\right) = \frac{1}{2}\frac{Q^2\ell}{\left[\frac{L}{2}(K-1) + L\right]w\varepsilon_0} = \frac{1}{2}\frac{Q^2\ell}{\frac{L}{2}\underbrace{\left[K - 1 + 2\right]}_{K+1}w\varepsilon_0} = \frac{1}{2}\frac{Q^2\ell\,2}{L(K+1)w\varepsilon_0}$

$U\left(x = \frac{L}{2}\right) = U_0 \frac{2}{K+1}$

Without dielectric insert: $U_0 = \frac{1}{2}C_0V_0^2 \overset{1)}{=} \frac{1}{2}C_0\frac{Q^2}{C_0^2} = \frac{Q^2}{2C_0} \overset{2)}{=} \frac{Q^2}{2\frac{\varepsilon_0 L w}{\ell}} = \frac{Q^2\ell}{2Lw\varepsilon_0}$

1) $C_0 = \frac{Q}{V_0} \rightarrow V_0 = \frac{Q}{C_0}$

2) Parallel plate capacitor: $C_0 = \frac{\varepsilon_0 A}{\ell} = \frac{\varepsilon_0 L w}{\ell}$

$$U\left(x=\frac{L}{2}\right) = U_0 \frac{2}{\kappa+1} < U_0$$

c) Force on slab?

$$\boxed{F = -\frac{dU}{dx}} \quad \text{or} \quad E = -\frac{dV}{dx} \Rightarrow qE = -\frac{d(qV)}{dx}$$

$\cdot q \qquad \cdot q$

$$F = -\frac{d(U)}{dx}$$

We needed $U(x)$

$$U(x) = \frac{1}{2}\frac{Q^2 \ell}{[x(\kappa-1)+L]w\varepsilon_0} = U_0 \frac{L}{(\kappa-1)x+L}$$

$$F(x) = -\frac{dU}{dx} = -U_0 L \frac{d}{dx}\left[(\kappa-1)x+L\right]^{-1} = U_0 L\left[(\kappa-1)x+L\right]^{-2}(\kappa-1)$$

$$= \frac{U_0 L(\kappa-1)}{[(\kappa-1)x+L]^2}$$

$$F\left(x=\frac{L}{2}\right) = \frac{U_0 L(\kappa-1)}{\left[(\kappa-1)\frac{L}{2}+L\right]^2} = \frac{U_0 \cancel{L}(\kappa-1)}{\frac{L^{\cancel{2}}}{2^2}\left[(\kappa-1)+2\right]^2} = \frac{4U_0(\kappa-1)}{L(\kappa+1)^2} \underset{U_0 = \frac{1}{2}C_0 V_0^2}{=}$$

$$= \frac{2C_0 V_0^2(\kappa-1)}{L(\kappa+1)^2}$$

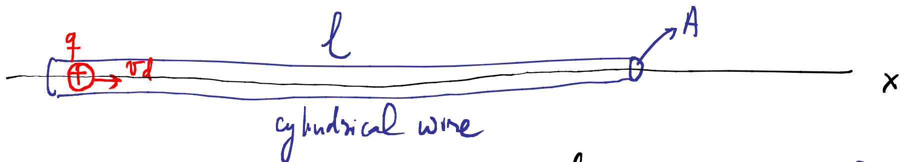
Ch24 Electric Current I

$$I = \begin{cases} \dfrac{\Delta q}{\Delta t} & \text{average current} \\ \dfrac{dq}{dt} & \text{instantaneous current} \end{cases} \quad \text{Unit } \frac{C}{s} = A \text{ (Amp)}$$

Current: { charges in motion
a macroscopic quantity as a consequence of microscopic behavior (motion of charges)

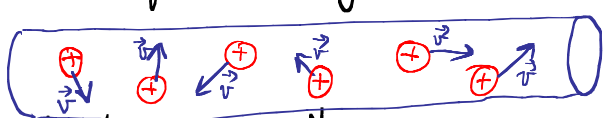
Temperature: macroscopic quantity as a consequence of motion of molecules

In an electrical wire (copper or aluminum)



v_d: drift velocity: average velocity along wire axis (normally small as it is an average over a random distribution of actual velocities)
small

v: actual velocity of charge: can point in any direction
very large



n: number of charge per unit volume $\left(n = \frac{N}{Vol}\right)$

A: cross-sectional area of wire

Average current $I = \frac{\Delta q}{\Delta t} = \frac{n \cdot q \cdot A \ell}{\frac{\ell}{v_d}} = n q A v_d$

macroscopic (I) — microscopic (n, q, v_d)

Let's calculate v_d for charges in a copper wire { $A = 1mm^2$, $I = 5A$, Each atom of Cu contributes $1.3e$ of charge

$$v_d = \frac{I}{nqA} = \begin{cases} I = 5A \\ q = 1.3 \cdot 1.6 \cdot 10^{-19} C \\ A = 1mm^2 = 10^{-6} m^2 \\ n : \end{cases}$$

Cu atoms per unit volume

$\Downarrow$

$\rho_{Cu} = 8920 \frac{kg}{m^3}$

One atom Cu weighs 63.55 a.u (periodic table)

$\downarrow$

$m_{Cu} = 63.55 \cdot 1.66 \cdot 10^{-27} kg$

$$\frac{\rho_{Cu}}{m_{Cu}} = \frac{8920 \cancel{kg}}{63.55 \cdot 1.66 \cdot 10^{-27} \cancel{kg}} = 8.5 \cdot 10^{28} \frac{\text{atoms Cu}}{m^3}$$

$n = \#\ \text{charge}/\text{vol} = 8.5 \cdot 10^{28}\ \frac{\text{charge}}{m^3}$

$$v_d = \frac{I}{nqA} = \frac{5}{8.5 \cdot 10^{28} \cdot 1.3 \cdot 1.6 \cdot 10^{-19} \cdot 10^{-6}} = 0.283\ \frac{mm}{s} \quad \text{(very slow!)}$$

Actual velocities for charges in Copper?

$$KE_{av} = \frac{1}{2} kT \times \text{d.o.f.}$$

$$\frac{1}{\cancel{2}} m v^2 = \frac{3}{\cancel{2}} kT \rightarrow v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 298.16}{9.11 \cdot 10^{-31}}} = 10^3\ \frac{m}{s}$$

(very large)

mass of one charge

Ohm's Law:

macroscopic: $I = \dfrac{V}{R}$ — Current is voltage divided by the resistance

SI units $A = \dfrac{V}{\Omega}$

$\left(\text{Amp} = \dfrac{\text{Volt}}{\text{Ohm}} \right)$

microscopic: $\vec{J} = \sigma \vec{E}$ — Current density vector $\vec{J}$ is the conductivity σ times the electric field $\vec{E}$

Current density $J \equiv \dfrac{I}{\text{Area}}$ $\left(\dfrac{\text{Amp}}{m^2} \right)$

Area ← cross-sectional area of wire

Conductivity σ : depend on materials (higher in metals with lots free e^-)

Resistance R : depend on materials (lower in metals), length of wire, and cross-sectional area of wire:

$\boxed{\sigma = \dfrac{1}{\rho}}$ conductivity is the inverse of resistivity

$R = \rho \cdot \dfrac{l}{A}$ $\begin{cases} \rho : \text{resistivity of material} = \dfrac{1}{\sigma} \\ l : \text{length of wire} \\ A : \text{cross-sectional area of wire} \end{cases}$

Power: $P = I \cdot V$ current times voltage $\begin{cases} = \dfrac{V^2}{R} \\ = I^2 R \end{cases}$

(i) Energy supplied per unit time
(ii) Heat loss or dissipated per unit time
} to maintain a current I at a voltage V

→ When charges are moved through a wire they collide with internal structure of the wire: this requires energy which is lost in the form of heat.

24.43 Given $\begin{cases} \text{Copper wire 12-gauge (diameter 2.1 mm)} \to r = 1.05 \cdot 10^{-3}\, m \\ I_{max} = 20\,A \end{cases}$ a) J? b) E?

a) $J = \dfrac{I}{\text{Area}} = \dfrac{20\,A}{\pi \cdot (1.05 \cdot 10^{-3})^2\, m^2} = (\quad) \dfrac{A}{m^2}$

circular cross-sectional area for cylindrical wire: $\text{Area} = \pi \cdot \left(\dfrac{d}{2}\right)^2$

b) Ohm's Law (microscopic version): $J = \sigma E \Rightarrow E = \dfrac{J}{\sigma} = \rho J$

Table 24.1 : Resistivity for Copper : $\rho_{Cu} = 1.68 \cdot 10^{-8}\ \Omega \cdot m$

$E = 1.68 \cdot 10^{-8} \cdot J = (\quad) \frac{N}{C}$

24.60

Given $\left\{ \begin{array}{l} \text{Aluminum wire 12-gauge (diameter 2.1 mm)} \\ \quad \hookrightarrow A = \pi (1.05 \cdot 10^{-3})^2 \\ I = 20\,A \\ \text{Each Al atom contributes } q = 3.5e = 3.5 \cdot 1.6 \cdot 10^{-19} C \end{array} \right\}$ v_d ?

$I = nqAv_d \rightarrow v_d = \dfrac{I}{nqA}$ $\left\{ \begin{array}{l} n ?\ \# \text{ Al atoms per unit volume} \\ q: \text{charge contribution per Al atom} \\ A: \text{cross-sectional area} \\ I: \text{current in the Al wire} \end{array} \right.$

We need to calculate n or # Al atoms per unit volume.

$\hookrightarrow = \dfrac{\text{density of Al}}{\text{mass of one Al atom}}$

$$n = \frac{\rho_{Al}}{m_{Al}} = \frac{2702\ \frac{kg}{m^3}}{26.98 \cdot 1.66 \cdot 10^{-27}\ kg} = 6 \cdot 10^{28}\ \frac{\text{Al atoms}}{m^3}$$

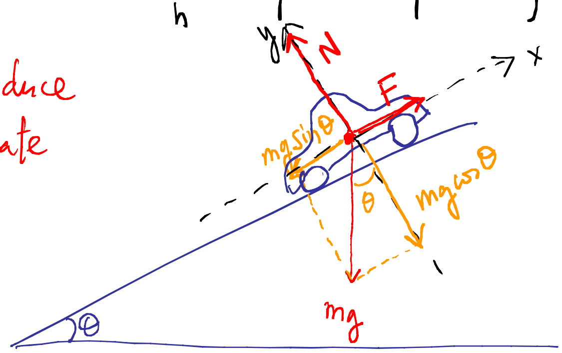
periodic table : $m_{Al} = 26.98\ \cancel{a.u.} \times 1.66 \cdot 10^{-27} \frac{kg}{\cancel{a.u.}}$

$$v_d = \frac{20}{6 \cdot 10^{28} \cdot 3.5 \cdot 1.6 \cdot 10^{-19} \cdot \pi \cdot (1.05 \cdot 10^{-3})^2} = 0.171 \cdot 10^{-3}\ \frac{m}{s} \text{ or } 0.171\ \frac{mm}{s}$$

24.65

Given $\left\{ \begin{array}{l} \text{hybrid car} \left\{ \begin{array}{l} m = 1200\,kg \\ 360\,V \text{ battery} \\ 180\,A \text{ max} \end{array} \right. \\ v = 60\ \frac{km}{h} \text{ up a slope } \theta \end{array} \right\}$ What is max. value for θ ? (ignoring friction)

Motor needs to produce a force F to compensate gravity $mg\sin\theta$



Observations:

(i) if $F - mg\sin\theta = m \cdot a = 0$
$\Rightarrow a = 0 \Rightarrow$ car goes up at constant speed as said in problem

(ii) if $F - mg\sin\theta = ma \neq 0$
$\Rightarrow a \neq 0 \Rightarrow$ car goes up at some acceleration

Car goes up slope θ @ constant speed $v = 60 \frac{\cancel{km}}{\cancel{h}} \cdot \frac{10^3 m}{1 \cancel{km}} \cdot \frac{1 \cancel{h}}{3600 s}$

$$v = \frac{60}{3.6} \frac{m}{s} = 16.667 \frac{m}{s}$$

$\rightarrow F = mg\sin\theta$

Power P :

Required : $\frac{Work}{\Delta t} = \frac{F \cdot \Delta x}{\Delta t} = F \cdot v = mg\sin\theta \cdot v$

Supplied by batteries : electrical power $= I \cdot V \left(= \frac{U}{\Delta t} = \frac{qV}{\Delta t} = I \cdot V \right)$

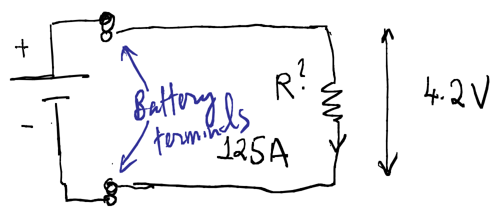
If electrical power supplied equal power required $\rightarrow$ car can do

$$mg\sin\theta \cdot v = I \cdot V \Rightarrow \sin\theta \leq \frac{IV}{mgv} \text{ or } \theta \leq \sin^{-1}\left(\frac{IV}{mgv}\right)$$

$$\theta \leq \sin^{-1}\left(\frac{180 \cdot 360}{1200 \cdot 9.81 \cdot 16.667}\right) = 0.3366 \cancel{rad} \cdot \frac{180^\circ}{\pi \cancel{rad}} = 19.29^\circ$$

24.50

Car battery :



Ohm's Law : $I = \frac{V}{R}$ or $R = \frac{V}{I} = \frac{4.2V}{125A} = 0.0336\ \Omega$

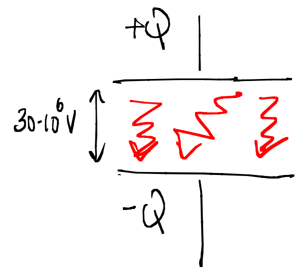
or $33.6\ m\Omega$

Normal value for R b/w battery terminals & starter is $1 m\Omega$

$\rightarrow$ Since this is now 33 times larger $\rightarrow$ there is corrosion at battery terminal

23.65

Lighting every 5s each : $Q = 30 C$, $V = 30 \cdot 10^6\ V$



$\downarrow$

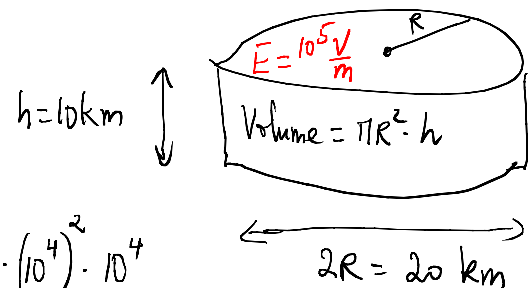
How much energy is discharged per lightning or flash?

$$U_F = Q \cdot V = 30 \cdot 30 \cdot 10^6 = 9 \cdot 10^8 J \text{ or } 0.9\ GJ \text{ (every 5s)}$$

How long storm will last if its energy U is not refilled?

Example 23.4

We need to calculate total energy in the thundercloud U:



$$U = \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\substack{u \\ \text{energy density}}} \cdot Vol = \frac{1}{2}\, 8.85 \cdot 10^{-12} \cdot \left(10^5\right)^2 \cdot \pi \cdot \left(10^4\right)^2 \cdot 10^4$$

$$= \underbrace{\frac{\pi}{2} 8.85}_{13.9} \cdot 10^{10}\ J = 140 \cdot 10^9\ J = 140\ GJ$$

Total energy of 140 GJ, every flash carries 0.9 GJ (every 5 s)

How long will storm last if its energy is fixed at 140 GJ?

$$\frac{140\ \cancel{GJ}}{\frac{0.9\ \cancel{GJ}}{5\ s}} = \frac{140 \cdot 5}{0.9}\ s = 777\ s \cdot \frac{1\ min}{60\ s} = 12.96\ min \sim 13\ min$$

(number of flashes: $\frac{140\ GJ}{0.9\ GJ} = 156$ flashes)