P-114   2014

ch 16   Temperature & Heat

Temperature: a <u>macroscopic</u> property (can be sensed & measured with simple instruments) that is a consequence of the <u>microscopic</u> behavior of matter.
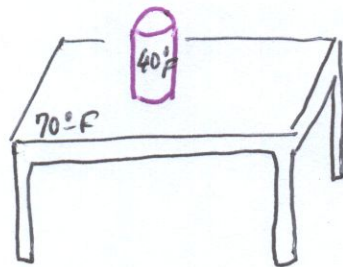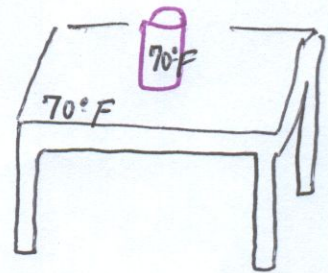

From a fridge
40 °F

Table @ room
temperature 70° F
@ 12:00PM

@ 4:00PM

1) Soda can was placed in thermal contact with table at higher temperature

2) Soda can absorbed heat and got its temperature raised.

Conclusion: two objects in thermal contact can <u>exchange heat</u> to arrive at a thermodynamic equilibrium or same temperature,

<u>Temperature</u> can be <u>measured</u> using <u>thermometers</u> (gas at constant volume, mercury Hg, electronic, etc...)

↳ Different scales or units {

Fahrenheit °F   : $T_F = T_C \frac{9}{5} + 32$

Celsius   °C (S.I.): $T_C = \frac{5}{9}(T_F - 32)$

Kelvin   °K (S.I.): $T_C = T_K - 273.16°$

<u>Triple</u> point of water : 273.16 °K

↳ gas, liquid, solid coexist @ this temp.

<u>Reason for adoption</u>

Fahrenheit   Body temp ~ 100 °F

Celsius   Water boiling temp ~ 100 °C
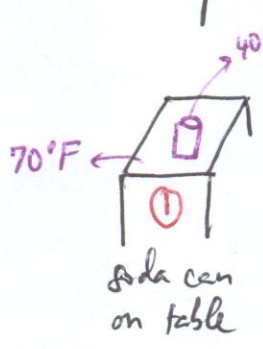
Kelvin   No negative temp in °K   (0 °K = lowest temp. in the universe)

Physics is most importantly <u>quantitative</u> ↔ predictions ↔ equations :

<u>Heat transfer</u> (or exchange) rate H : how fast heat is transferred.
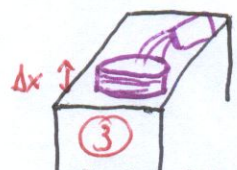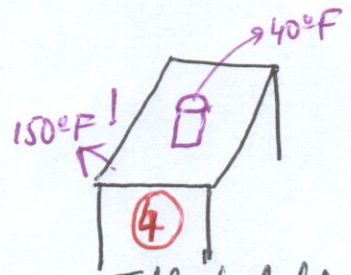
(intuitive)

↓

Compare different intuitive situations :

① soda can on table

70°F → ☐ → 40°F

② soda poured out on plate

Faster heat transfer: larger thermal contact area b/w soda & table

③ thick plate same surface area

Δx ↕

Slower than ② same contact area but heat needs to travel through Δx

④ Table heated to 150° F before soda is placed on

150°F! → ☐ → 40°F

Faster than ① due to larger temperature difference

$$H = \ominus \ k \cdot \frac{A}{\Delta x} \, \Delta T \longrightarrow \text{Heat loss rate (heat transfer out of)}$$

k : thermal constant, depends on materials (better thermal conductors have high k)

A : contact surface area b/w the two objects (area → m²)

Δx : thickness of contact medium (length → m)

ΔT : temperature difference b/w two objects (temp → °K)

H : heat transfer rate or work per unit time ($\frac{J}{s}$ → W)

Unit for thermal constant E: Dimensional Analysis:

Basic dimensions $\begin{cases} \text{Length } L \\ \text{mass } M \\ \text{time } \boxed{T} \\ \text{temperature } \boxed{T} \end{cases}$

Work or energy or heat $= [E] = [\frac{1}{2}][m][v]^2 = M \cdot \dfrac{L^2}{T^2}$

$\underbrace{\quad}_{1}$

(Numbers have no dimension)

$[k] = \dfrac{[H][\Delta x]}{[A][\Delta T]} = \dfrac{\frac{[E]}{T_{(time)}} \cdot \cancel{L}}{L^{\cancel{2}} \cdot T'_{(temp)}} = \dfrac{[E]}{T_{time}} \cdot \dfrac{1}{L \cdot T_{temp}}$

$\boxed{\text{units:}}$

$\begin{cases} \text{S.I.} \quad \dfrac{W}{m \cdot {}^{\circ}K} \\[20pt] \text{British:} \quad \dfrac{Btu \cdot in}{h \, ft^2 \, {}^{\circ}F} \end{cases}$

Thermal constant & R (R-factor) : $R = \dfrac{\Delta x}{k}$

→ Larger R is better insulator

→ $H = -k \dfrac{A}{\Delta x} \Delta T = - \dfrac{A}{R} \Delta T$

→ Units for R $\begin{cases} \text{SI:} \quad \dfrac{m^2 \, {}^{\circ}K}{W} \\[20pt] \text{British:} \quad \dfrac{ft^2 \, {}^{\circ}F}{\frac{Btu}{h}} \end{cases}$

Mechanisms of Heat Transfer: conduction, convection, radiation

pan with water
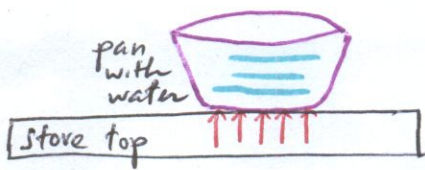
stove top

Pan: metal wall & bottom

1) From stove top to bottom of pan: free electrons in metal bottom receive heat → higher KE → move faster → hit other electrons more frequently and with more energy. Heat ~~by cor~~ transfer by <u>conduction</u> through bottom of pan to lowest H_2O molecules
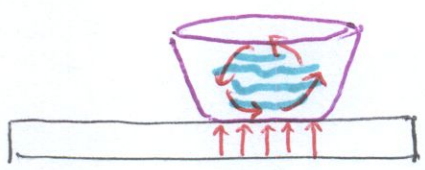
2) H_2O molecules transfer heat to each other by <u>convection</u>
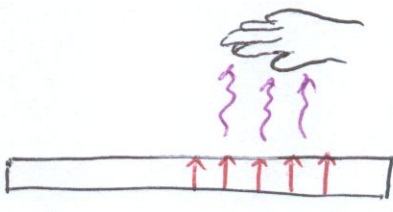
3) Heat transfer from stove top to hand by <u>radiation</u> (no materials are needed, happens in outer space).
Other example: heat from sun

Convection:

AC outlets are installed high : cold air molecules will sink
Baseboard heaters are installed low : hot air molecules will rise

Why? → hot air molecules → more K.E → more active : larger
average separation b/w molecules → lower density →
pushed up by boyancy (this only happens ~~with~~
in fluid ⟷ with materials)

→ cold air molecules → less KE → less active : less
average separation b/w molecules → higher density →
sinked by gravity

These two processes form the convection cycle

Radiation:

Stefan - Boltzman Law:

Heat loss rate by radiation of an object of area
A & temperature T is given by:
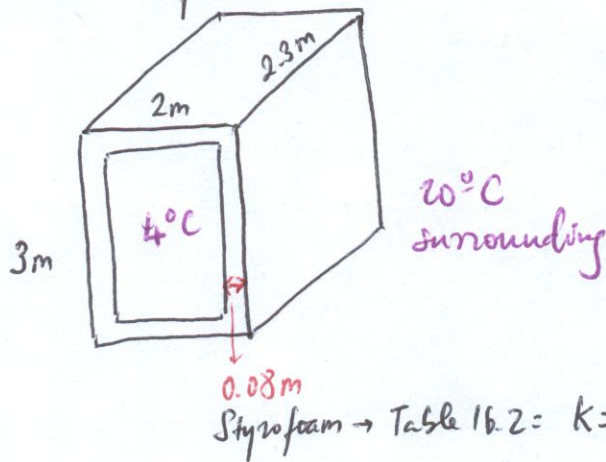
$$P = e \cdot \sigma \cdot A \cdot T^4$$

emissivity
constant

emissivity
constant
(no dimension)

Stefan - Boltzman
constant

$$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 \,{}^0K^4}$$

16.57

Step 1: diagram with information

2.3m
2m
4°C
3m
0.08m

20°C surrounding

Styrofoam → Table 16.2: $k = 0.029 \frac{W}{m\,°K}$ Thermal conductivity

if there was no refrigeration: heat is transferred into cooler @ rate $H_{surrounding} = -k\frac{A}{\Delta x}\Delta T$

Step 2: relevant equation(s)

$$H_{cooler} = +k\frac{A}{\Delta x}\Delta T$$

$\Delta x = 0.08m$

Temp. difference: $\Delta T = \begin{cases} 20°C - 4°C = 16°C = 16°K \\ \\ (20° + 273.16) - (4° + 273.16) = 16°K \end{cases}$

$A = \underset{\text{left & right}}{2 \times 6.9\,m^2} + \underset{\text{front & back}}{2 \times 6\,m^2} + \underset{\text{top & bottom}}{2 \times 4.6\,m^2} = 35\,m^2$
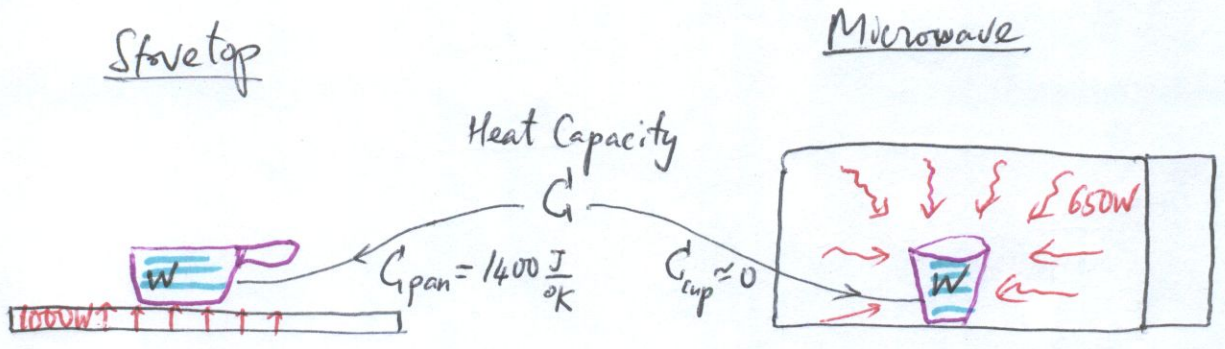
Step 3: solve for unknowns: heat removal rate by refrigeration
unit:

$$H_{removal} = -k\frac{A}{\Delta x}\Delta T = -0.029 \cdot \frac{35}{0.08} \cdot 16 = -203\,W$$

16.49| What is m_w so it would be quicker to heat using stove top instead of microwave?

Step 1: Diagram with information

Stovetop   Microwave

Heat Capacity

C

$C_{pan} = 1400 \frac{J}{^\circ K}$ $C_{cup} \approx 0$ $\{ 650W$

W   1000W   W

→ Heat capacity $C \left(\frac{J}{^\circ K}\right)$ $C \equiv m \cdot c$ $\begin{cases} m: & \text{object's mass} \\ \\ c: & \text{specific heat} \\ & \text{(specific to material)} \end{cases}$

      └→ in SI, C is how much heat in Joules (J) is needed to raise the temperature of an object of mass m one degree kelvin ($^\circ K$)

          └→ $\begin{cases} C_{H_2O} = 4184 \frac{J}{kg\,^\circ K} \\ \\ C_{Cu} = 386 \frac{J}{kg\,^\circ K} & \text{(for comparison)} \end{cases}$

→ Specific heat $c \left(\frac{J}{kg\,^\circ K}\right)$ in SI (International System of units)

          c is how much energy in Joules (J) is needed to raise the temperature of one kilogram (kg) of a specific material one degree kelvin ($^\circ K$)

→ Total heat change $\boxed{\Delta Q = C \Delta T = mc \Delta T}$

          change heat
          or
          increment

→ $\begin{cases} \text{Stove top : heat supplied by stove top goes into heating pan } \left(C_{pan} = 1400 \frac{J}{^\circ K}\right) \\ \quad\quad\quad\quad \text{and heating water } \left(C_w = m_w \, 4184 \frac{J}{kg\,^\circ K}\right) \\ \text{Microwave : heat supplied by microwave goes into heating water } \left(C_w = m_w \, 4184 \frac{J}{kg\,^\circ K}\right) \\ \quad C_{cup} = 0 \end{cases}$

Step 2:   Relevant equations :   quicker to heat ⟷ comparing times

$$\Delta t_S \quad \& \quad \Delta t_M$$
(stovetop)   (Microwave)

Stovetop

Heat supply @ $1000 W = 1000 \frac{J}{s} = \frac{\Delta Q_s}{\Delta t_s}$

$$\Delta t_s = \frac{\Delta Q_s}{1000}$$

$$= \frac{C_{pan} \cdot \Delta T + m_W c_W \cdot \Delta T}{1000}$$

Microwave

Heat supply @ $625 W = \frac{625 J}{s}$

$$\frac{\Delta Q_m}{\Delta t_m} = 625 \frac{J}{s}$$

$$\Delta t_m = \frac{\Delta Q_m}{625}$$

$$= \frac{m_W c_W \cdot \Delta T}{625}$$

What m_W using stovetop would be quicker than using microwave?

$$\Delta t_s \quad < \quad \Delta t_m$$

$$\frac{(C_{pan} + m_W c_W)\Delta T}{1000} \quad < \quad \frac{m_W c_W \Delta T}{625}$$

Step 3:   Solve for m_W : $C_{pan} + m_W c_W < \frac{1000}{625} m_W c_W$

$$C_{pan} < \left(\frac{1000}{625} - 1\right) m_W c_W$$

$$\frac{C_{pan}}{\left(\frac{1000}{625} - 1\right) c_W} < m_W$$

$$\frac{1400 \frac{J}{°K}}{\left(\frac{375}{625}\right) 4184 \frac{J}{kg °K}} = \boxed{\textcolor{red}{0.558 kg < m_W}}$$

Heat balance equation : $(T_1 > T_2)$

→ When two objects of different temperatures T_1 & T_2 are brought into thermal contact with each other, heat will transfer from hotter object to colder object until both arrive at a same temperature T_f (thermodynamic equilibrium)

→ If these two objects are isolated: total heat change is zero :

$$\Delta Q_1 + \Delta Q_2 = 0$$

$T_1 > T_2 \rightarrow$ loss $\rightarrow$ gain

$$\underbrace{m_1 C_1 (T_f - T_1)}_{\Delta Q_1} + \underbrace{m_2 C_2 (T_f - T_2)}_{\Delta Q_2} = 0$$

→ Given $m_1, m_2, T_1, T_2 \rightarrow$ provide T_f

→ Given $m_1, T_1, T_2, T_f \rightarrow$ provide m_2

etc...

} Given a set of 4 variables (values), calculate the value of the fifth one.

Ch 17   Thermal Behavior of Matter :

↳(i) If we continue to supply heat to a
pan with boiling water, where does this
heat go ?

(ii) Boiling water is always @ 100°C or 273.16°K :
$$\Delta Q = m_w C_w \Delta T , \quad \text{since } \Delta T = 0 ; \Delta Q = 0$$
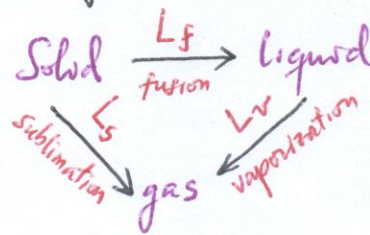
(iii) Heat supply goes into phase change liquid → gas
(after a while there is no more water in pan)

When heat is supplied
to matter
ΔQ

$\Big\{$

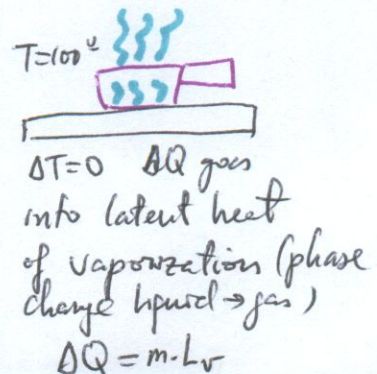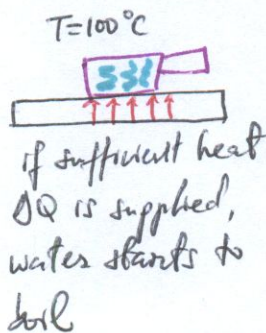1) It temperature will change $\Delta T = \dfrac{\Delta Q}{m \cdot C}$

2) If sufficient heat is supplied, matter will undergo a
phase change $\Delta Q = m \cdot L$ (L: latent heat)

$$\text{Solid} \xrightarrow[\text{fusion}]{L_f} \text{liquid}$$

sublimation L_s ↘   ↙ L_v vaporization

gas

3) If heat is supplied to a solid : it may undergo
expansion $\Big\{$
- linear : $\alpha \equiv \dfrac{\frac{\Delta L}{L}}{\Delta T}$ (L: length)
(T: temperature)
- volume $\beta \equiv \dfrac{\frac{\Delta V}{V}}{\Delta T}$ (V: volume)

Outcomes 1) & 2)

$T = 25°C$

$T\uparrow$

$\Delta Q = mc\Delta T$

$T = 100°C$

if sufficient heat
ΔQ is supplied,
water starts to
boil

$T = 100°$

$\Delta T = 0$ ΔQ goes
into latent heat
of vaporization (phase
change liquid → gas)
$\Delta Q = m \cdot L_v$

3 phases of matter $\begin{cases} \text{gas} \\ \\ \text{liquid} \\ \\ \text{solid} \end{cases}$

Density
low

↓

high

Molecules
almost free

interactions among
molecules

molecules form
a structure
(electrons can
move around,
as in metals)
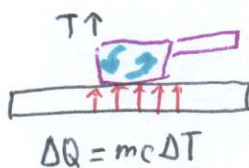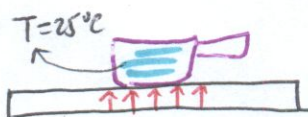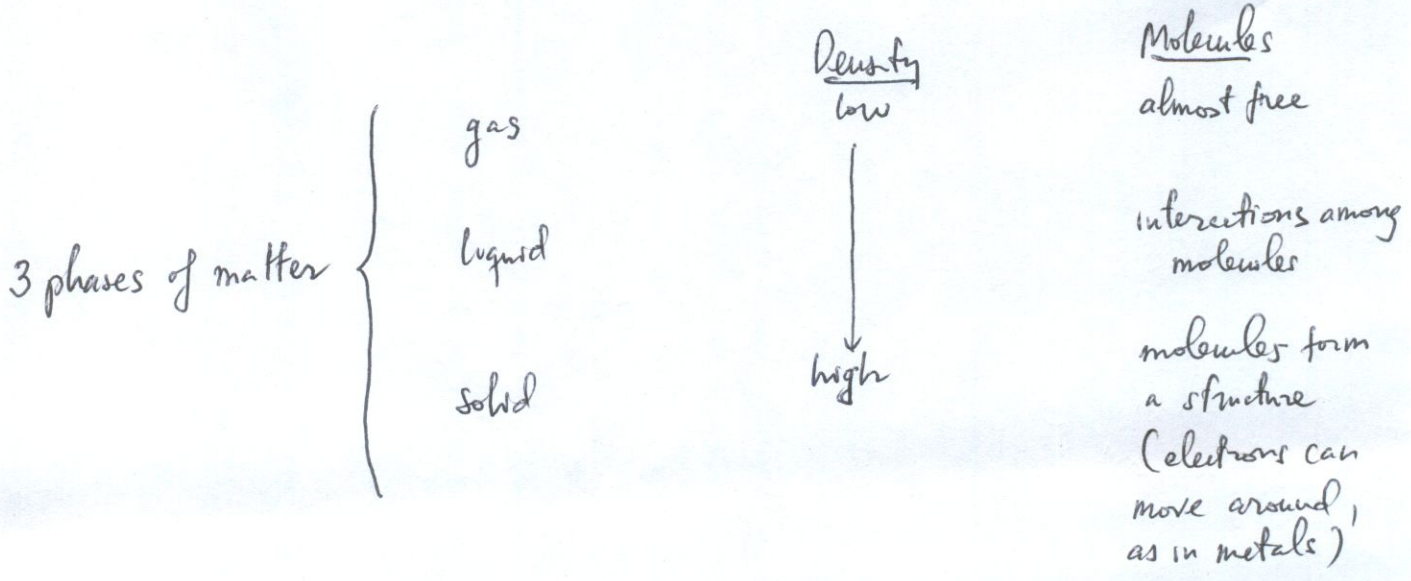
Easiest phase to describe quantitatively with an equation: [ideal gas]

Ideal Gas: we assume gas molecules are completely free (we ignore
interactions b/w them)

→ equation of state for an ideal gas is intuitive:

↳relates: macroscopic properties (T: temp. of gas, V: volume of
gas; P: pressure of gas) with the microscopic
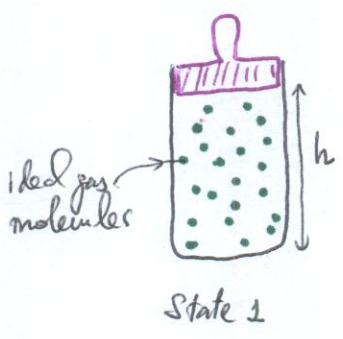number of gas molecules N

Observe gas in a cylinder controlled by a piston:

Gas is compressible
(molecules can come
closer together)

ideal gas molecules
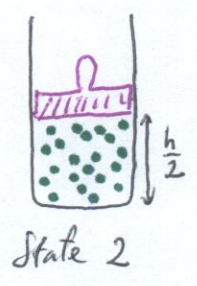
State 1

State 2

1) Piston fits cylinder perfectly ⟷ no leaks ⟷ # of gas molecules in
both states is N

2) Height is reduced by half → $V_1 = V$; $V_2 = \dfrac{V}{2}$

3) Gas molecules hit cylinder all the time transferring momentum ⟷ pressure
(momentum ~~pressure~~ transfer per unit area).

There is a double number of momentum transfer per unit area in state 2 compared to state 1 → $P_1 = P$; $P_2 = 2P$

4) We assume piston is going down very slowly = so molecules won't get hit by piston (they can slowly adjust to smaller volume in sma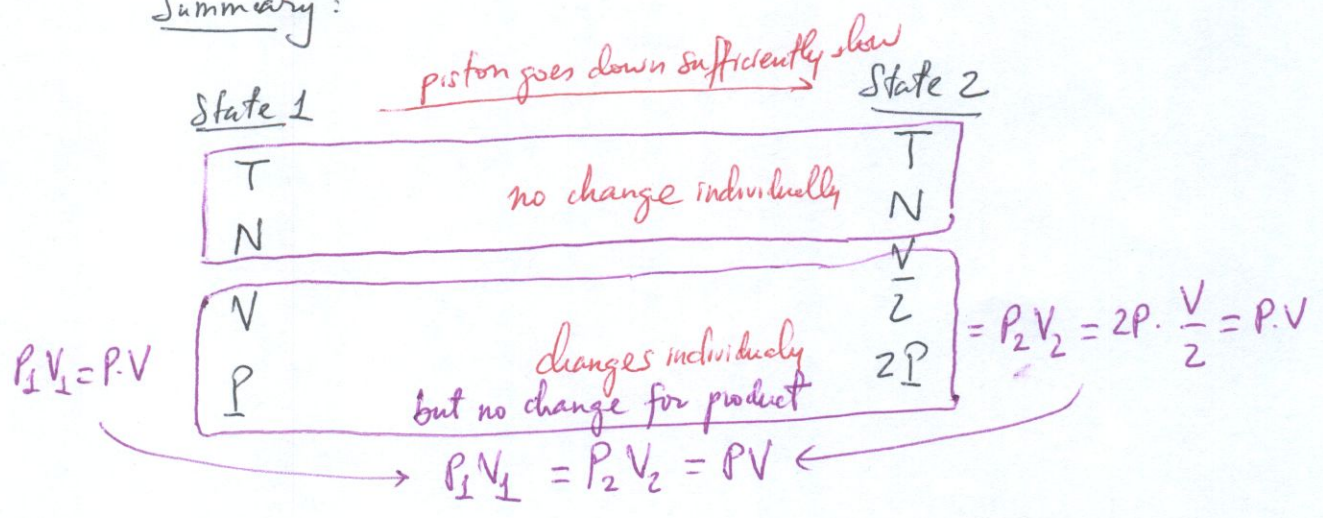ll increments). No hit, no increase in kinetic energy KE of each molecule → no temperature increase ↔ T is constant (isothermal process)

Summary :

State 1 ——— piston goes down sufficiently slow ——→ State 2

$P_1 V_1 = P \cdot V$

no change individually

changes individually but no change for product

$= P_2 V_2 = 2P \cdot \dfrac{V}{2} = P \cdot V$

$P_1 V_1 = P_2 V_2 = PV$

Equation of state for an ideal gas :

$$\boxed{P \cdot V = k \, N \cdot T}$$

dimension & scale

k : Boltzman's constant $= 1.38 \cdot 10^{-23} \; \dfrac{J}{{}^\circ K}$

why?

↓

SI units
$\begin{cases} P \cdot V = \dfrac{F}{A} \cdot A \cdot h = \text{Work} \\ \qquad\qquad \to \text{unit : } J \\ N = \text{\# molecules, no dimension} \\ T : \text{temperature} \to \text{unit } {}^\circ K \end{cases}$

$J = \dfrac{J}{{}^\circ K} \; {}^\circ K$

$\underset{PV}{} = k \; N \; T$

Alternative form for ideal gas equation of state:

$$P \cdot V = k \cdot N \cdot T$$

$$N \equiv n \cdot N_A$$

$n =$ number of moles

$N_A :$ Avogadro number $:= 6.022 \cdot 10^{23}$

$$P \cdot V = k \, n \, N_A \, T = n \cdot (k N_A) \cdot T$$

$$\boxed{R = 1.38 \cdot 10^{-23} \cdot 6.022 \cdot 10^{23} = 8.314 \, \frac{J}{°K \, mol}}$$

ideal gas constant R

$$\boxed{P \cdot V = n R \cdot T}$$

More realistically there are some interactions b/w gas molecules

Real Gas equation of state:

$$\boxed{\left(P + \frac{n^2 a}{V^2}\right) \cdot \left(V - n \cdot b\right) = n R \cdot T \qquad ; \quad a \& b \text{ are constant}}$$

Phase diagrams : pressure vs. temp. graphs

Normal matter phase diagram
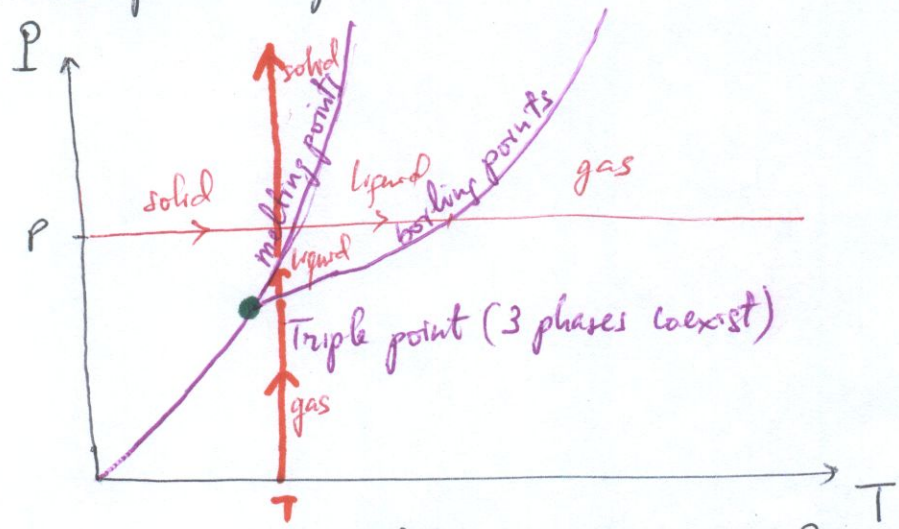
@ constant T as we increase pressure $\underline{P}$: gas → liquid → solid

Water phase diagram

@ constant T as we increase P: gas → liquid → solid → liquid

If we apply sufficient pressure on ice, it melts.
(not just melts by increased T but also by increased P)

16.67)   Example 16.4:

Step 1:   Total heating cost for one month?

$\triangle ABC$: $h\cos 30° = 14$

$h = \dfrac{14}{\cos 30°}$

Area $\triangle ABD = 28 \cdot 14 \cdot \tan 30° \cdot \dfrac{1}{2}$

$\dfrac{AC}{14} = \tan 30° \Rightarrow AC = 14 \cdot \tan 30°$

Total surface area: 4 sides + 2 roofs + 2 side roofs =
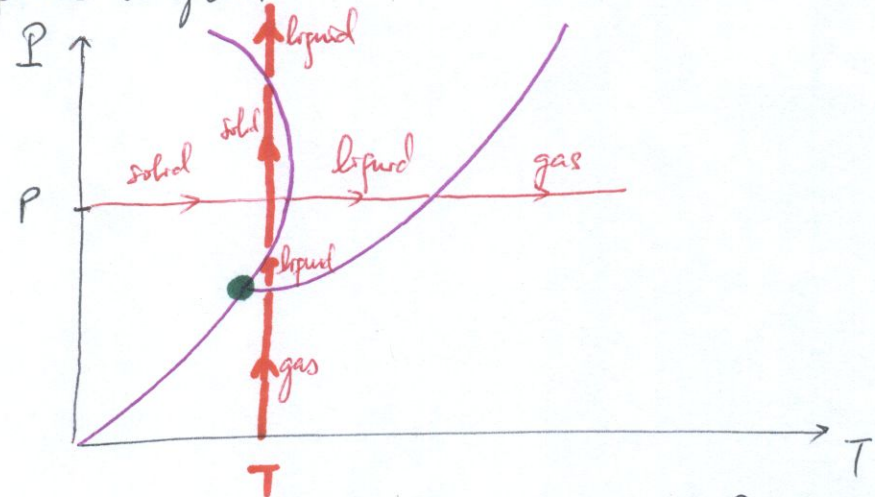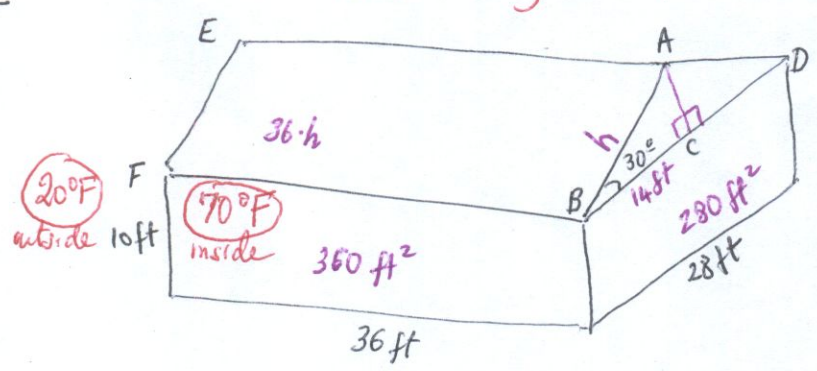
$$= 360 \cdot 2 + 280 \cdot 2 + 36 \cdot \frac{14}{\cos 30°} \cdot 2 + 28 \cdot 14 \cdot \tan 30° \cdot \frac{1}{2} \cdot 2$$

              $\underbrace{\hspace{4cm}}$ $\underbrace{1164}$ $\underbrace{113 \cdot 2}$

              → walls

$$= \boxed{1506 \text{ ft}^2} \quad + \quad \boxed{1164 \text{ ft}^2}$$

$R_{wall} = 0.17 \quad + \quad 11 \quad\quad + \quad 0.65 + 0.55 \quad = 12.37 \; \dfrac{\text{ft}^2 \cdot °F}{\dfrac{Btu}{h}}$

wall = plaster + fiberglass insulation + plywood + cedar shingles

                                    $\boxed{R-11}$

$\left(R \equiv \dfrac{\Delta x}{k} \right)$

                          $\boxed{R-30}$

$R_{roof} = 0.17 + \quad 30 \quad + \quad 0.65 + 0.55 \quad = 31.37 \; \dfrac{\text{ft}^2 \cdot °F}{\dfrac{Btu}{h}}$

Step 2   Relevant equation :   Heat loss rate (we need to supply what is lost to keep interior of house at 70°F)

$$H = -k\frac{A}{\Delta x}\Delta T = -\frac{A}{R}\Delta T$$

$$R \equiv \frac{\Delta x}{k}$$

$H_{Walls} = -\dfrac{A_{wall}}{R_{wall}}\Delta T = -\dfrac{1506}{12.37}(70-20)$

$\qquad\qquad = -6073 \; \dfrac{Btu}{h}$

$H_{Roofs} = -\dfrac{A_{Roofs}}{R_{Roofs}}\Delta T = -\dfrac{1164}{31.37} \cdot 50$

$\qquad\qquad = -1855 \; \dfrac{Btu}{h}$

→ Total heat loss rate $H_{Total} = -7928 \; \frac{Btu}{h}$

→ Every hour we need to supply 7928 Btu to keep house interior @ 70°F

→ Every month we need $7928 \times 24 \times 30 = 5.7 \cdot 10^6 \; Btu$

→ For each gallon of heating oil we can extract: $10^5 \; Btu$:

$5.7 \cdot 10^6 \; Btu \cdot \dfrac{1 \; Gal}{10^5 \; Btu} = 57 \; Gallons$

→ If oil price is \$2.20/Gal

↳ Per month: $57 \; Gal \cdot \dfrac{\$2.20}{Gal} = \$126$

(no windows)

Step 3: Now similar calculations taking into considerations 10 windows

10 windows $\begin{cases} \text{size or } A = 2.5 ft \times 5 ft = 12.5 \; ft^2 \\ \text{single-glazed } R = 0.9 \quad \to \quad H = -\dfrac{A}{R} \Delta T \\ \text{4 south facing: solar energy} \to \text{heat gain rate } H_{\substack{South \\ facing \\ window}} = \oplus 30 \; \frac{Btu}{h \cdot ft^2} \cdot A_{window} \end{cases}$

Total heat loss rate with windows:

$H_{\substack{with \\ windows}} = H_{\substack{Walls \\ R-12.37 \\ (loss)}} + H_{\substack{Windows \\ R-0.9 \\ (loss)}} + H_{\substack{South \\ facing \\ windows \\ (gain)}} + H_{\substack{Roofs \\ R-31.37}}$

$= -\dfrac{1506-125}{12.37} 50 - \dfrac{125}{0.9} 50 + 30 \cdot 12.5 \cdot \underset{\substack{\uparrow \\ 4 \; south \\ facing \\ windows}}{4} - \dfrac{1164}{31.37} 50$

$\underbrace{\qquad\qquad\qquad\qquad} \qquad\qquad\qquad\qquad \underbrace{\qquad\quad}$

$= \qquad\qquad -11026 \qquad\qquad\qquad\qquad\qquad -1855$

$= \qquad\qquad -12881 \; \frac{Btu}{h}$

$\rightarrow \qquad H_{\substack{Total \\ w/windows}} = -12\,881 \; \dfrac{Btu}{h}$

$\hookrightarrow$ Cost per month : $12\,881 \times 24 \times 30 \; Btu \cdot \dfrac{1\,Gal}{10^5\,Btu} \dfrac{\$2.20}{Gal} = \$204$

$\hookrightarrow$ Solar gain $= 30 \dfrac{Btu}{h \cdot ft^2} \times \underbrace{12.5 \times 4}_{50} \times \textcircled{24} \times 30 \times \dfrac{\$2.70}{10^5} = \$23.96$ per month

(for curiosity)

24 hrs of
sun light per
day

$\downarrow$
add to $204
if you have a tree
blocking south facing
windows!

(since gain of $30 \dfrac{Btu}{h \cdot ft^2}$ was "average rate")

Ch 18   Heat, Work, 1st Law of Thermodynamics (TD)

Conservation of energy = relating
heat, work, and internal energy of
a system

$$\Delta U = Q - W$$

$$\begin{cases} Q = \text{heat absorbed by system} \\ W = \text{work done by system} \\ \Delta U = \text{change of internal energy} \\ \qquad \text{of system} \end{cases}$$

(If you define W as work done on system then $\Delta U = Q + W$)

State variables :

1) Given a state, state variables are independent of
how the system arrived at that state. State
variables don't depend on a process
   Examples: $\Delta U, T, P, V$

2) Q & W are not state variables. They depend on
    a particular process $\begin{cases} \text{isobaric (constant pressure } P) \\ \text{isovolumic (constant volume } V) \\ \text{isothermal (constant temperature } T) \\ \text{adiabatic } (Q = 0) \end{cases}$

   Note : 1st Law of TD = while Q & W are not state variables
   their difference Q - W is a state variable.

PV diagram: each point on this diagram represents a state
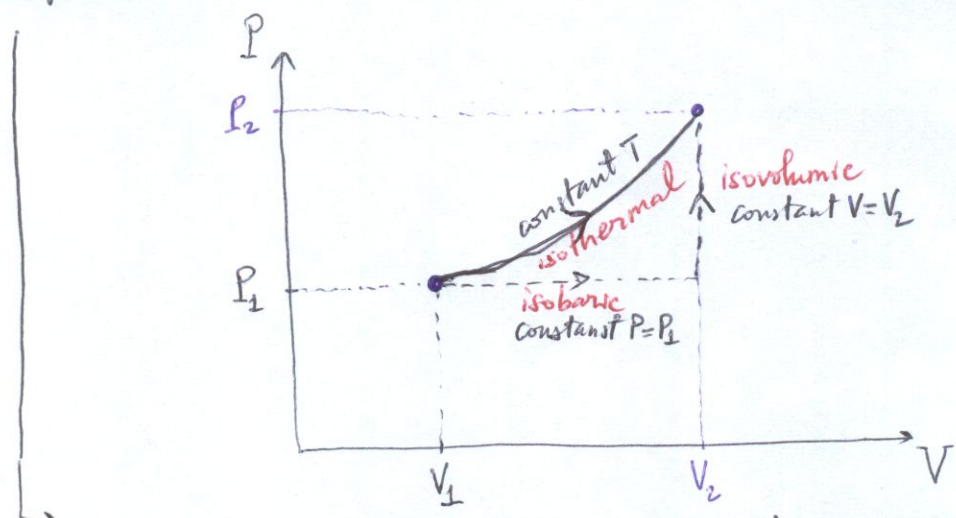
Allows us to calculate work done by system $W = \int P \cdot dV$

↑
Area under $P(V)$
(Area under the pressure curve in a PV diagram)

(Mechanics: $W = F \cdot \Delta x = P \cdot \underbrace{A \cdot \Delta x}_{V}$)

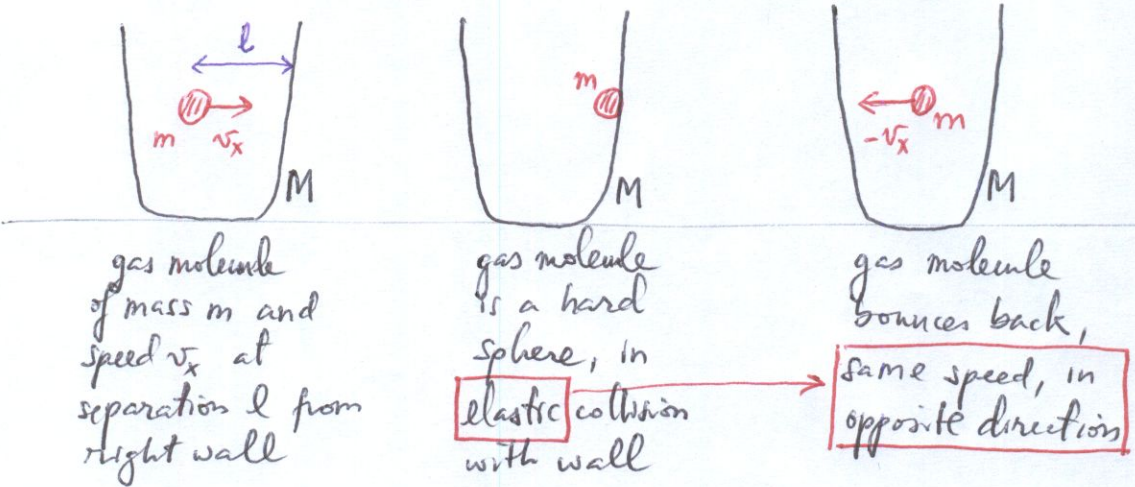Microscopic to Macroscopic Connection in an Ideal Gas:

Microscopic: pressure exerted by a gas molecule on its container

$\quad\quad\hookrightarrow$ Macroscopic pressure : adding up these pressures for N molecules $\quad (N_A = 6.023 \cdot 10^{23})$

$$P = \sum_{i=1}^{N} P_i$$

Pressure by a single molecule comes from the transfer of linear momentum when it collides with the containing wall. $\left(\vec{F}_{net} = \dfrac{d\vec{p}}{dt}\right)$

gas molecule of mass m and speed v_x at separation ℓ from right wall

gas molecule is a hard sphere, in elastic collision with wall

gas molecule bounces back, same speed, in opposite direction

What is the linear momentum transfer (gas to wall) at the elastic collision. molecule

Any collision $\vec{F}_{net} = 0 \rightarrow \dfrac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p}$ is conserved $\vec{P_i} = \vec{P_f}$

Before collision $\quad$ After collision

Conservation of linear momentum:

$$m v_x \quad = \quad -m v_x + M V_x$$

Momentum acquired by container after collision

or $\boxed{2m v_x = M V_x}$ $\quad V_x = \dfrac{2m v_x}{M} \simeq 0$ (container barely moves!)

→ Pressure by gas molecule i: $\underset{\underset{\text{Capital P}}{\downarrow}}{P_i} = \frac{F_i}{A} = \frac{\frac{\Delta p_i}{\Delta t}}{A} = \frac{\frac{2mv_{x_i}}{2\ell/v_{x_i}}}{A} = \frac{mv_{x_i}^2}{A \cdot \ell}$

lower case p linear momentum

Capital P pressure

volume

$\Delta t = \frac{\ell}{v_{x_i}}$ (one way)

$\Delta t = \frac{2\ell}{v_{x_i}}$ (round trip)

→ Macroscopic pressure $\quad P = \sum\limits_{i=1}^{N} P_i = \frac{m}{vol} \sum\limits_{i=1}^{N} v_{x_i}^2$

$$= \frac{mN}{vol} \underbrace{\left(\frac{\sum\limits_{i=1}^{N} v_{x_i}^2}{N} \right)}_{\overline{v_x^2}}$$

$\overline{v_x^2}$ (average of v_x^2 over molecules)

$$P = \frac{mN}{vol} \overline{v_x^2}$$

↑
macroscopic   *microscopic*

$\begin{cases} m: \text{mass of a molecule} \\ \overline{v_x^2}: \text{average } \boxed{x\text{-component}} \text{ of the velocity, squared, of a molecule.} \end{cases}$

We included momentum transfer in one direction (x-direction). In general there can be momentum transfer in 3 directions:

$$\boxed{\overline{v_x^2}} = \frac{1}{3} \boxed{\overline{v^2}}$$

x-component only   total velocity in 3D

$$P = \frac{1}{3} \frac{mN}{vol} \overline{v^2} = \underset{vol \equiv V}{\frac{mN}{3V} \overline{v^2}} \quad \text{or} \quad \boxed{PV = mN \frac{\overline{v^2}}{3}}$$

$$\boxed{PV = kNT}$$

Equation of state of an ideal gas:

↳ $kT = m \frac{\overline{v^2}}{3} \Rightarrow \boxed{\frac{3}{2} kT = \frac{1}{2} m \overline{v^2}}$

macroscopic   *microscopic*

Temp   *m, v of a molecule*

$$\text{macroscopic} \qquad \text{microscopic}$$

$$\frac{3}{2}kT \quad = \quad \frac{1}{2}m\overline{v^2}$$

$$\downarrow$$

average KE of one
gas molecule in 3D
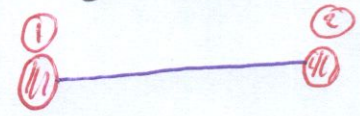($\vec{v}$: total velocity in 3D)

Profound consequences:

1) KE of gas molecules determines the overall temperature
   of the gas

2) If gas is confined in 1D : $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$

$$\downarrow$$

$$\frac{1}{2}kT = \frac{1}{2}m\overline{v_x^2}$$

→ KE of a gas contributes $\frac{1}{2}kT$ per degree of freedom

3) Diatomic molecules

without invisible rod   with invisible rod
(2 individual ~~molecules~~   (diatomic molecule)
          atoms)

3 d.o.f's atom 1   3
+ 3 d.o.f's atom 2   + 3
———————————   − 1 (invisible rod
6 d.o.f's   —————— or interaction
                               5 d.o.f's. to form a
                                         diatomic
                                         molecule)

$$\quad \rightsquigarrow \frac{6}{2}kT \qquad\qquad \rightsquigarrow \frac{5}{2}kT$$

$$\rightsquigarrow c_V \begin{cases} \text{monoatomic gas}: \frac{3}{2}R \\[2mm] \text{diatomic gas}: \frac{5}{2}R \end{cases}$$

specific heat
at constant
volume

$$\rightsquigarrow \gamma \begin{cases} \text{monoatomic} = \gamma = 1.67 \\[2mm] \text{diatomic} : \gamma = 1.4 \end{cases}$$
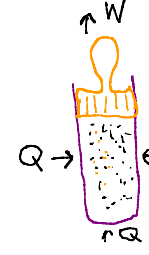
Isothermal processes (constant T)

Work done by an ideal gas undergoing an isothermal process ① → ②

$$W = \int_1^2 P\, dV \underset{T\,constant}{=} nRT \int_i^2 \frac{dV}{V} = nRT\,[\ln V]_1^2 = nRT(\ln V_2 - \ln V_1) = nRT \ln \frac{V_2}{V_1}$$
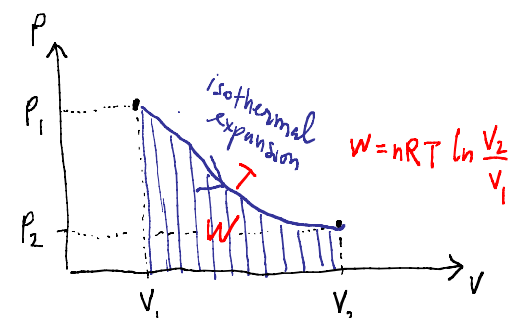
ideal gas
eq. of state
$PV = nRT$
$P = \dfrac{nRT}{V}$

$$W = nRT \ln \frac{V_2}{V_1} \quad (\text{work done by a gas})$$

$\begin{cases}(i)\ V_2 > V_1 \ (\text{isothermal expansion}) \to \ln\frac{V_2}{V_1} > 0 \\ \qquad \Rightarrow W > 0 \ (\text{gas did work}) \\ (ii)\ V_2 < V_1 \ (\text{isothermal compression}) \to \ln\frac{V_2}{V_1} < 0 \\ \qquad \to W < 0 \ (\text{gas received work})\end{cases}$

Practical situation of (i):

Heat supplied Q → gas expands pushing piston up doing work

$Q \to \quad \leftarrow Q$ If T is constant → $W = nRT \ln \frac{V_2}{V_1}$

PV diagram

isothermal expansion

$W = nRT \ln \frac{V_2}{V_1}$

1) what is the change of internal energy for this gas from ① to ②?

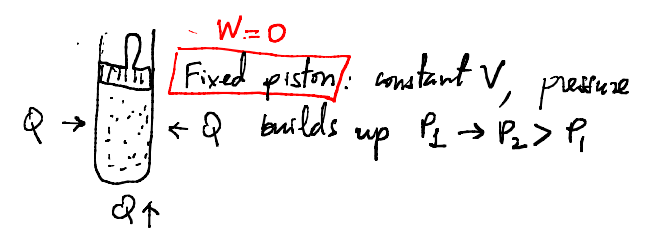$$\Delta U = U_2 - U_1 = \frac{3}{2}kT - \frac{3}{2}kT = 0$$

(i) ideal gas
no interaction: $U = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$
(ii) isothermal process $T_2 = T_1 = T$

2) What is the heat absorbed by gas in this isothermal process?
Q? 1st Law of TD: $\Delta U = Q - W$
$0 = Q - W \Rightarrow Q = nRT \ln \frac{V_2}{V_1}$

In an isothermal process an ideal gas absorbs heat Q and returns an equal amount of work $W = nRT \ln \frac{V_2}{V_1}$

Isovolumic process (constant volume V)

$$W = \int_1^2 P\, dV = 0$$

$W = 0$
Fixed piston: constant V, pressure
$Q \to \quad \leftarrow Q$ builds up $P_1 \to P_2 > P_1$
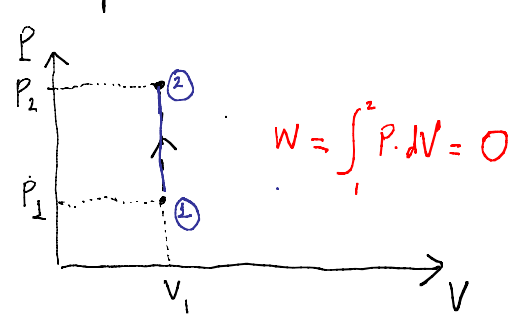
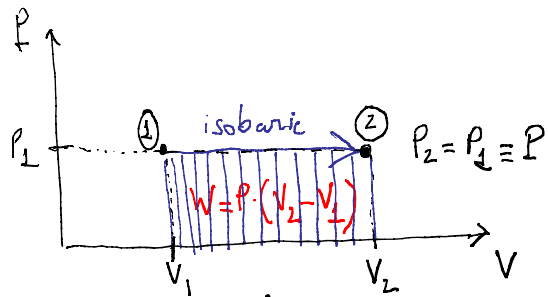1) 1st Law of TD : $\Delta u = Q - W = Q$

$\underset{\substack{\uparrow \\ W=0 \\ \text{isovolumic process}}}{}$

2) Specific heat at constant volume $C_v = \dfrac{1}{n} \dfrac{Q}{\Delta T} = \dfrac{1}{n} \dfrac{\Delta u}{\Delta T} \Rightarrow \boxed{\Delta u = n C_v \Delta T}$

Isobaric Process (constant pressure P)

$P_2 = P_1 \equiv P$

$$W = \int_1^2 P \cdot dV = P \int_1^2 dV = P [V]_1^2 = P \cdot (V_2 - V_1)$$

1) Specific heat at constant pressure $C_p = \dfrac{1}{n} \dfrac{Q}{\Delta T} \Rightarrow \boxed{Q = n C_p \Delta T}$

2) Ideal gas equation: $P \cdot \Delta V = n R \Delta T$

$ W = n R \Delta T$

 *isobaric process!*

3) 1st Law TD : $\Delta u = Q - W$

$ \cancel{n} C_v \cancel{\Delta T} = \cancel{n} C_p \cancel{\Delta T} - \cancel{n} R \cancel{\Delta T} \Rightarrow C_v = C_p - R$ or $\boxed{C_p = C_v + R}$

$\left.\begin{array}{l} C_p = C_v + R \\ R = 8.314 \dfrac{J}{mol\,{}^\circ K} \end{array}\right\} C_p > C_v$: a gas at constant pressure can absorb more heat than a gas at constant volume

Why?

$Q = \Delta u + W \rightarrow \begin{cases} \text{const } P: \; Q = \Delta u + W \\ \text{const } V: \; Q = \Delta u \end{cases} \begin{array}{c} \\ C_p > C_v \end{array}$

$(\Delta u = Q - W)$

(i) at constant volume V gas can't do any work to release heat absorbed, so it can't absorb lot of heat

(ii) at constant pressure P, it can change volume or can do work to release heat absorbed, so it can absorb more heat

Adiabatic Process (Q = 0) $\Leftrightarrow$ $\boxed{\Delta u = -W}$
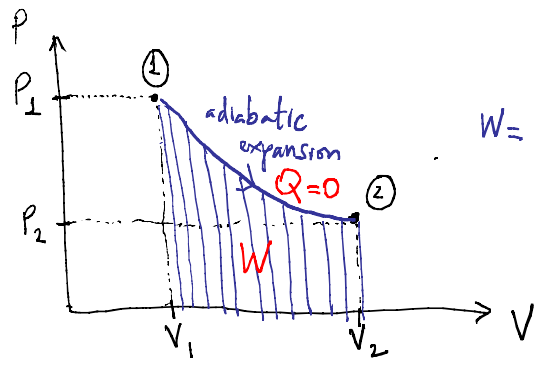
$W = \dfrac{P_1 V_1 - P_2 V_2}{\gamma - 1}$, $\gamma = $ gas coefficient $\equiv \dfrac{C_p}{C_v} = \dfrac{C_v + R}{C_v} > 1$

$\downarrow$

depends on type of gas molecules (monoatomic, diatomic, ...)

1) In an adiabatic process: $P \cdot V^\gamma = $ constant

Adiabatic expansion ① → ② $\boxed{P_1 V_1^\gamma = P_2 V_2^\gamma}$

Ideal gas equation $PV = nRT$ or $P = \frac{nRT}{V}$ → $\boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}}$

2) Values for $\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$

$\begin{cases} n \equiv \frac{N}{N_A} \quad (N_A = 6.022 \cdot 10^{23}) \\ R \equiv k N_A \end{cases}$

Equations for C_v: for N gas molecules

(i) Monoatomic gas: each atom or molecule has 3 d.o.f

$$C_v = \frac{1}{n} \frac{\Delta u}{\Delta T} = \frac{1}{n} \cdot \frac{\frac{3}{2} k \Delta T \cdot N}{\Delta T} = \frac{1}{\frac{N}{N_A}} \cdot \frac{3}{2} k \cdot N = \frac{3}{2} k \cdot N_A = \frac{3}{2} R$$

(ii) Diatomic gas: each molecule has 5 d.o.f.

$$C_v = \frac{1}{n} \frac{\Delta u}{\Delta T} = \frac{1}{n} \frac{\frac{5}{2} k \Delta T \cdot N}{\Delta T} = \frac{1}{\frac{N}{N_A}} \frac{5}{2} k \cdot N = \frac{5}{2} k N_A = \frac{5}{2} R$$

$\gamma = \frac{C_v + R}{C_v}$
$\begin{cases} \text{monoatomic:} \quad \gamma = \frac{\frac{3}{2} R + R}{\frac{3}{2} R} = \frac{5}{3} = 1.67 \\ \text{diatomic:} \quad \gamma = \frac{\frac{5}{2} + 1}{\frac{5}{2}} = \frac{7}{5} = 1.4 \end{cases}$

18.33

Sizes of air bubble e $\begin{cases} P_H = 125 \, mm \, Hg \\ P_L = 80 \, mm \, Hg \end{cases}$ excesses over atmospheric pressure $\begin{cases} P_H = 125 + 760 \, mm \, Hg \\ P_L = 80 + 760 \, mm \, Hg \end{cases}$

1 Atm = 760 mm Hg
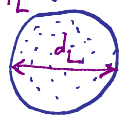
Step 1: Diagram with information

Bubble in blood:

↑ spherical shape

P_L (Low blood pressure)

P_H (High blood pressure)

$V = \frac{4}{3} \pi R^3$

$d_L = 1.52 \, mm$

d_H ?

Step 2: Relevant equation: ideal gas equation $P \cdot V = nRT$ ← constant!

$T = 37°C$ constant body temperature

$$P_L \cdot V_L = P_H \cdot V_H$$

Step 3: $P_L \cdot \frac{4}{3} \pi \left(\frac{d_L}{2}\right)^3 = P_H \cdot \frac{4}{3} \pi \cdot \left(\frac{d_H}{2}\right)^3 \Rightarrow P_L \cdot d_L^3 = P_H \cdot d_H^3 \Rightarrow d_H = \left(\frac{P_L}{P_H}\right)^{1/3} \cdot d_L = \left(\frac{840}{885}\right)^{1/3} \cdot 1.52 \, mm$

a) $d_L = 1.49 \, mm$

b) From $P_L \to P_H$ as bubble is squeezed, gas is compressed as it receives work ↑ blood or ultimately from heart. W? for $P_L \xrightarrow{\text{isothermal!}} P_H$

$$W = nRT \ln \frac{V_H}{V_L} \xrightarrow{PV = nRT \text{ or } V = \frac{nRT}{P}} W = nRT \ln \frac{P_L}{P_H}$$

$$W = \underbrace{P_L V_L}_{nRT} \ln \frac{P_L}{P_H} = (80+760) \cdot 133.3 \cdot \frac{4\pi}{3} \left(\frac{1.52 \cdot 10^{-3}}{2}\right)^3 \ln\left(\frac{80+760}{125+760}\right) = -12312 \cdot 10^{-9} J$$

SI units

Pressure : Pa (Pascal)

$1\,mm\,Hg = 133.3\,Pa$

Volume : m^3

$1\,mm = 10^{-3} m$

$W = \ominus 12\,\mu J$

$\hookrightarrow$ work received by gas

$\mu = 10^{-6}$ (micro)

18.58] Mixture Ar & O_2

monoatomic ($\gamma = 1.67$)   diatomic ($\gamma = 1.4$)

$\left\{ \begin{array}{l} \text{in adiabatic} \\ \text{expansion} \end{array} \right.$ $\left. \begin{array}{l} V_1 \rightarrow V_2 = 2V_1 \\ P_1 \rightarrow P_2 = \frac{P_1}{3} \end{array} \right\}$ Fraction of Argon f in mixture

$Ar\,(f)$ & $O_2\,(1-f)$

1) Calculate f from $\gamma_{mixture}$ which depends on mixture composition

2) Obtain $\gamma_{mixture}$ from adiabatic equation : $P_1 V_1^\gamma = P_2 V_2^\gamma$ $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma$

$\Rightarrow \ln \frac{P_1}{P_2} = \gamma \ln \frac{V_2}{V_1} \rightarrow \boxed{\gamma = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln 3}{\ln 2} = 1.58}$ for mixture $\left\{ \begin{array}{l} Ar : f \\ O_2 : 1-f \end{array} \right.$

$\hookrightarrow$ Derive equation for γ for mixture :

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$$

Derive equation for $C_V = \underbrace{\frac{3}{2}R \cdot f}_{\text{monoatomic } Ar} + \underbrace{\frac{5}{2}R \cdot (1-f)}_{\text{diatomic } O_2} = R\left(\frac{5}{2} - f\right)$

$$\gamma = \frac{R\left(\frac{5}{2}-f\right) + R}{R\left(\frac{5}{2}-f\right)} = \frac{\frac{5}{2}-f+1}{\frac{5}{2}-f} = \frac{\frac{7}{2}-f}{\frac{5}{2}-f}$$

3) Solve for f : $1.58 = \frac{3.5 - f}{2.5 - f} \rightarrow 1.58(2.5-f) = 3.5-f \rightarrow 2.5 \cdot 1.58 = 3.5 + f(1.58-1)$

$f = \frac{2.5 \cdot 1.58 - 3.5}{0.58} = 0.78$ $\rightarrow$ Mixture composition $\left\{ \begin{array}{l} 78\% \;\; Ar \\ 22\% \;\; O_2 \end{array} \right\}$ given their behavior during an adiabatic process