Ch 27 Electromagnetic Induction:

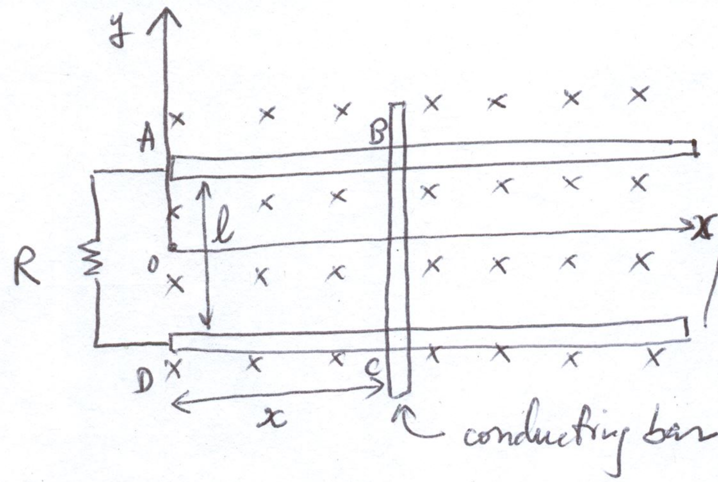
Faraday's Law: $\longrightarrow$ $\boxed{-\dfrac{d\Phi_B}{dt} = \mathcal{E}}$

$\mathcal{E}$: induced voltage (e.m.f) $\rightarrow$ from an induced $\vec{E}$

$-$: Conservation of energy or Lenz's law

Magnetic flux Φ_B (through a 3D surface) $= \int_{\text{surface integral}} \vec{B} \cdot d\vec{A}$

27.44



→ conducting rails

$\vec{B}$: uniform, into page

↳ $\phi_B = B \cdot A = Bxl$

Magnetic flux through ABCD depends on position x of conducting bar
→ ϕ_B varies as bar is moving $\vec{v} = v\hat{i}$ → induced voltage
→ current through R

a) Direction of this current? Determined by the "−" in Faraday's Law:

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(Bxl) = -Bl\underbrace{\frac{dx}{dt}}_{v} = -Blv$$

This current tends to create a magnetic field to neutralize the increase of ϕ_B as conducting bar moves right → current will be such to create a counter flux with a magnetic field pointing against the original ⇒ out of page : current in ABCD will be CCW or down at R

b) Agent pulling bar is doing $\underbrace{\text{work at what rate}}_{P}$?

$$P = \frac{\text{Work}}{\text{time}} = \frac{F_B \cdot X}{\text{time}} = F_B \cdot v$$

v ← speed of bar

F_B ↑ magnetic force on moving bar with a current (the induced current : $I = \frac{\mathcal{E}}{R}$)

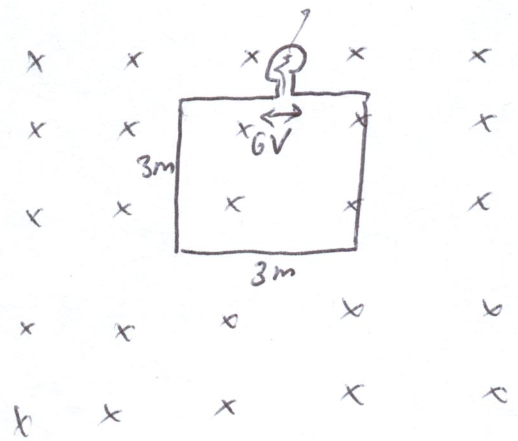
$$F_B = I \cdot l \cdot B$$

$$P = IlB \cdot v = \frac{\mathcal{E}}{R} lBv = - \frac{(Blv)^2}{R}$$

⇓ work is on bar

Alternative: $P = \underset{\downarrow \atop \frac{\varepsilon}{R}}{I} \cdot \underset{\downarrow \atop \varepsilon}{V} = \frac{\varepsilon^2}{R} = \frac{(Blv)^2}{R}$

27.37



$B = 2T$, uniform into page, then reduces steadily to 0 over Δt

a) $\Delta t = 3s$ to reach full brightness b) Induced current CW

Induced current that gives light comes from the change of the magnetic field over time → which implies a change in ϕ_B : $\Delta B \cdot A$

→ Faraday's Law: $\varepsilon = -\frac{\phi_B}{\Delta t} = -\frac{\Delta B \cdot A}{\Delta t} = -\frac{(0-2)\cdot 9}{\Delta t} = \frac{18}{\Delta t}$

→ Full brightness when $\varepsilon \to 6V$ → $6 = \frac{18}{\Delta t}$ → $\boxed{\Delta t = 3s}$

b) Induced current always to ~~opposed~~ oppose the change in magnetic flux : $\varepsilon = - \frac{d\phi_B}{dt}$.

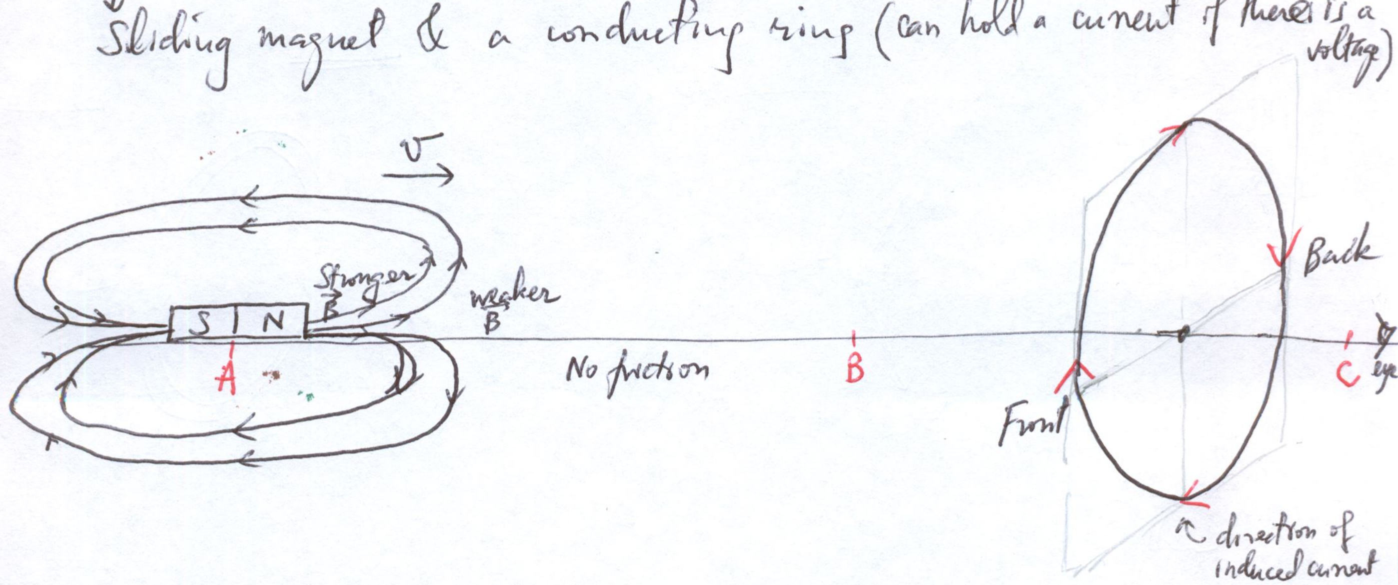
→ Change in ϕ_B: reduction of flux into page (as B decreases over Δt)

→ Induced current in loop will create a magnetic field also into the page to neutralize the flux reduction → current I in CW (RHR - CW current → $\vec{B}$ into page)

Visual Experiment: EM induction & Conservation of Energy

↓

Sliding magnet & a conducting ring (can hold a current if there is a voltage)



1) As magnet ~~...~~ slides from A to B at constant speed v: magnetic flux by magnet field through the cross-sectional area of the ring increases: $\rightarrow \frac{d\Phi_B}{dt} \rightarrow$ Faraday's Law of induction: $\mathcal{E} = - \frac{d\Phi_B}{dt}$

induced voltage opposes the change in Φ_B

Induced current goes up in the front side of ring or CW from right of ring to create a counter magnetic field to neutralize the increase in magnetic flux due to the approaching magnet.

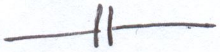
2) After the magnet is given a push, it travels from A to B at speed v. As it approaches the ring the speed will decrease as some of its KE is transferred to the induced current in the ring.

3) When magnet gets to C (it has passed the ring center): magnetic flux through ring now decreases → induced current decreases to 0 and reverses direction to create a counter magnetic field now pointing to the right, to neutralize the reduction of Φ_B. This reverse current will be gone when $\frac{d\Phi_B}{dt}$ goes back to 0, energy is returned to magnet which will pick up speed.

Inductance & Magnetic Energy

Electric energy storage

Capacitors



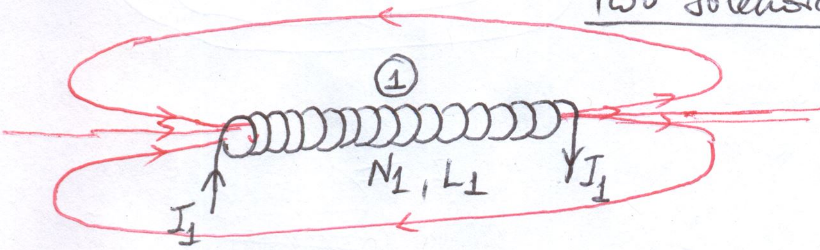
Capacitance: $C = \frac{Q}{V}$

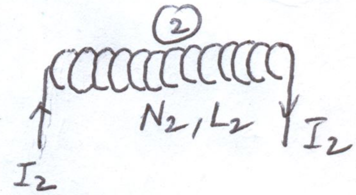
Magnetic energy storage

Inductors

$\begin{cases} \text{Mutual inductance: } M = \frac{\Phi_2}{I_1} \\ \text{Self inductance } \quad L = \frac{\Phi}{I} \end{cases}$

Two solenoids





$B_1 = \mu_0 n_1 I_1$ (constant along axis)

n_1 = number of turns per unit length $= \frac{N_1}{L_1}$

$B_2 = \mu_0 n_2 I_2$

$n_2 \equiv \frac{N_2}{L_2}$

1) B_1 goes through cross-sectional area of solenoid # 2 → magnetic flux $\Phi_2 = B_1 \cdot A_2 \cdot N_2 = \mu_0 n_1 I_1 A_2 N_2$.

If I_1 varies over time → solenoid # 2 will carry an induced voltage $\mathcal{E}_2$. By Faraday's Law $\quad \mathcal{E}_2 = -\frac{d\Phi_2}{dt}$

$$\text{or } -\mathcal{E}_2 = \frac{d\Phi_2}{dt} = \underbrace{\mu_0 n_1 N_2 A_2}_{\equiv M} \cdot \frac{dI_1}{dt}$$

(relates the induced voltage in solenoid 2 with a changing current in solenoid 1) ↔ <u>mutual</u> inductance.

→ $\Phi_2 = M I_1$

$$\boxed{M = \frac{\Phi_2}{I_1}}$$

2) Vice versa: B_2 by solenoid 2 goes through cross-sectional area A_1 of solenoid 1 → creating a magnetic flux through solenoid 1:

$$\phi_1 = B_2 \cdot A_1 \cdot N_1 = \mu_0 n_2 I_2 \cdot A_1 \cdot N_1$$

If I_2 varies over time → induces a voltage ε_1 in solenoid 1

$$-\varepsilon_1 = \frac{d\phi_1}{dt} = \underbrace{\mu_0 n_2 A_1 \cdot N_1}_{\equiv M} \frac{dI_2}{dt}$$

$\equiv M$ (relates voltage induced in one solenoid with the changing current in the other)

SI Unit: $M = \frac{V}{\frac{A}{s}} = \frac{V.s}{A} = H$ (Henry)

3) Also B_1 goes through the cross-sectional area A_1 of solenoid 1 (itself) → magnetic flux $\phi = B_1 \cdot A_1 \cdot N_1 = \mu_0 n_1 I_1 A_1 . N_1$

If I_1 varies over time, there is a self-induced voltage in solenoid 1 ⇒ $-\varepsilon = \frac{d\phi}{dt} = \underbrace{\mu_0 n_1 A_1 . N_1}_{\equiv L} \frac{dI_1}{dt}$

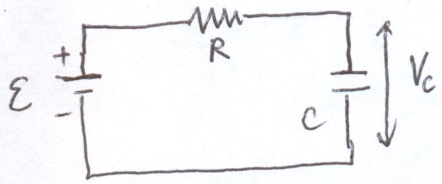
$\equiv L$ (self-inductance)

$\phi = LI$ or $\boxed{L = \frac{\phi}{I}}$

self-magnetic flux

SI Unit: $L \longrightarrow H$ (Henry)

RC



@ $t=0$ (switch is just closed)
capacitor initially uncharged
$\boxed{V_c=0}$ → C acts like a short-circuit

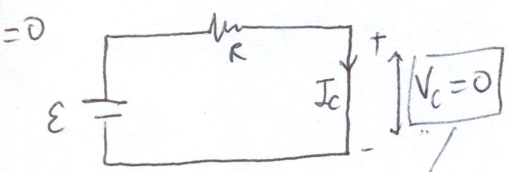
$t\to\infty$ (sufficiently long after switch is closed so C is fully charged)
$\boxed{I_c=0}$ → C acts like an open-circuit

$$I_c(t) = \underbrace{\boxed{\frac{\varepsilon}{R}}}_{} \cdot e^{-\frac{t}{\tau_{RC}}}$$

$I_c(0)$, max current when $t=0$

τ_{RC} : time constant = $\cancel{RC}$ RC

$t=0$



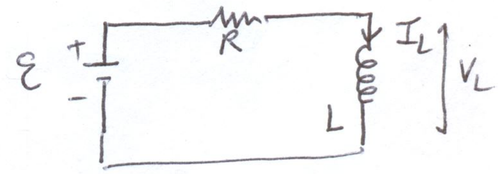
$I_c = \frac{\varepsilon}{R}$

→ As circuit is closed, V_c was 0, stays at 0, V_c does not change instantaneously

→ C = electric inertia to changes in Voltage V_c

→ $U = \frac{1}{2}CV^2$

RL



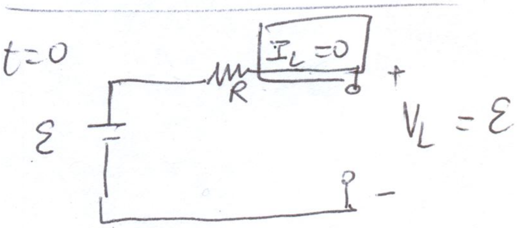
$t=0$ (switch is just closed)
$\boxed{I_L=0}$ → L acts like an open-circuit

$t\to\infty$, $I_L\to$ max, $\boxed{V_L=0}$
L acts a short-circuit

$$V_L(t) = \boxed{\varepsilon}\, e^{-\frac{t}{\tau_{LC}}}$$

$\tau_{LC} = \frac{L}{R}$: time constant

$t=0$



→ As circuit is closed, I_L was 0, stays at 0, I_L does not change instantaneously

L → magnetic inertia to changes in current I_L

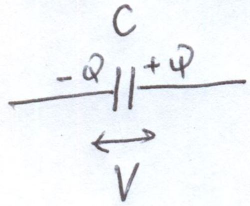
$U = \frac{1}{2}LI^2$

($KE = \frac{1}{2}mv^2$: m was inertia to changes in velocity v)

Grav, electric, magnetic forces are all inverse-square laws!

Magnetic Energy:

Electric energy



$$\boxed{U_C = \frac{1}{2} C V^2} \quad (J)$$

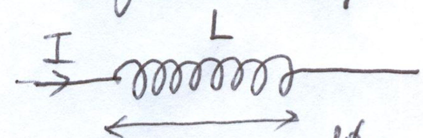
C (F)
V (V)

Volume b/w plates: $A \cdot d$
(A: plate area, d: plate gap separation)

Energy density: $u_C = \frac{U_C}{A \cdot d}$

$$C = \frac{A\varepsilon_0}{d} \rightarrow \boxed{u_C = \frac{1}{2}\varepsilon_0 E^2} \left(\frac{J}{m^3}\right)$$

Magnetic energy



$$\mathcal{E}_L = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

$\phi = L \cdot I$

$$U_L = \int_0^t \underset{\downarrow}{P_L}\, dt = \int_0^t I \cdot |\mathcal{E}_L|\, dt = L\int_0^t I \frac{dI}{\cancel{dt}} \cancel{dt}$$

energy stored per unit time or power

$$= \frac{1}{2} L \left[I^2\right]_{t=0}^{t} \underset{\downarrow}{=} \frac{1}{2} L I^2 \quad (J) \quad \begin{cases} L\ (H) \\ I\ (A) \end{cases}$$

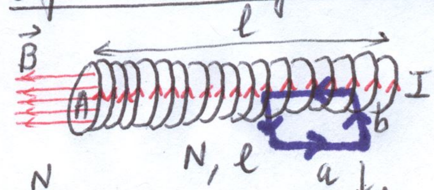
$I(0) = 0$ (L is inertia to changes in I)

Volume of solenoid $= A \cdot l$ (A: cross-sectional area, l: length)

Magnetic energy density:

$$u_L = \frac{U_L}{A \cdot l}$$

Self-inductance of a solenoid: $L = \frac{\phi}{I}$



I (in each loop, up in the front side)

$n = \frac{N}{l}$

$I_{enclosed} = I \cdot \underbrace{\frac{a}{l} N}_{} = Ian$

circular loops enclosed by Amperian loop.

Amperian loop: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$

$B \begin{cases} \text{uniform inside} \\ 0 \text{ outside} \end{cases}$

$B \cdot a = \mu_0 I \frac{a}{l} N$

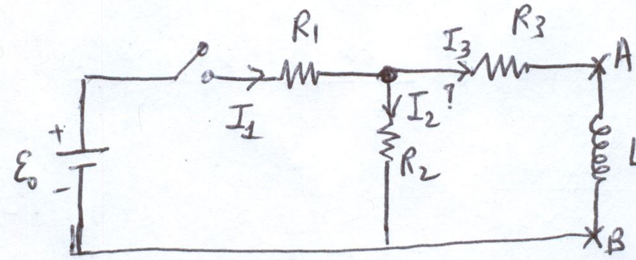
$$\boxed{B = \mu_0 n I}$$
$$\boxed{B = \mu_0 \frac{NI}{l}}$$

$$L = \frac{\phi}{I} = \frac{B \cdot A \cdot N}{I} = \frac{\mu_0 n \cancel{I} \cdot A \cdot N}{\cancel{I}} = \mu_0 n N A = \mu_0 \frac{N^2}{l} A$$

$$u_L = \frac{\frac{1}{2} L I^2}{A \cdot l} = \frac{\frac{1}{2}\mu_0 \frac{N^2}{l} \cdot \cancel{A} \cdot I^2}{\cancel{A} \cdot l} = \frac{1}{2}\mu_0 \frac{N^2}{l^2} I^2 = \frac{1}{2}\mu_0 \frac{B^2}{\mu_0^2}$$

$$\frac{NI}{l} = \frac{B}{\mu_0} \quad \rightarrow \boxed{u_L = \frac{1}{2\mu_0} B^2}$$

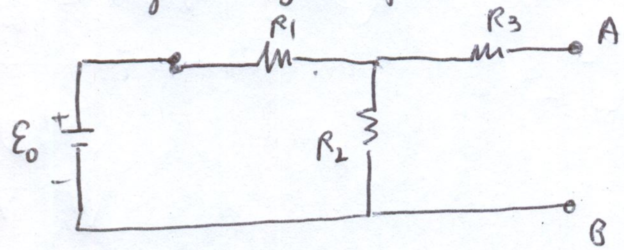
27.59



$\mathcal{E}_0 = 12V$
$R_1 = 4\Omega;\ R_2 = 8\Omega;\ R_3 = 2\Omega$
$L = 2H$

a) I_2? right after switch is closed

Since $I_3 = 0$ before switch is closed, L is an inertia to changes in current it stays 0 right after switch is closed ($t=0$) $I_L = 0$: open circuit



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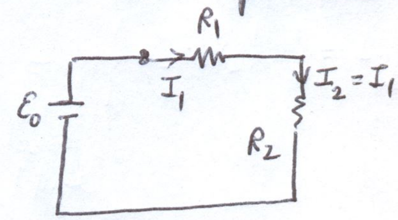
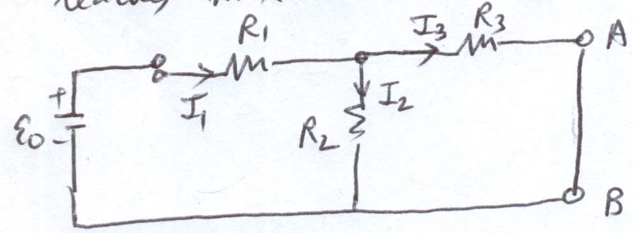


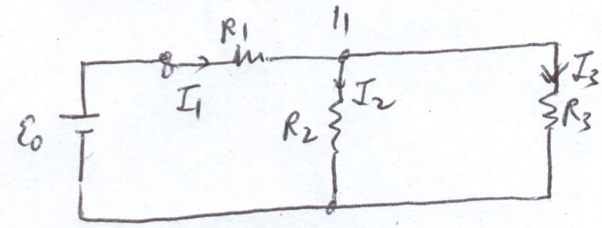
$$\boxed{I_2 = \frac{\mathcal{E}_0}{R_1+R_2} = \frac{12}{4+8} = 1A}$$

b) I_2? long after switch is closed (so current through inductor reaches max value or $V_L = 0$: short-circuit

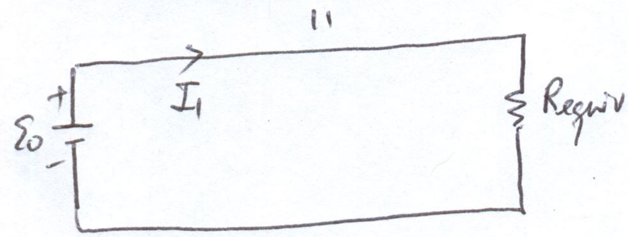


Current division: $I_2 = I_1 \cdot \frac{R_3}{R_2+R_3}$

(the larger R_3, the more I_2)



$$I_1 = \frac{\mathcal{E}_0}{R_{equiv}} = \frac{\mathcal{E}_0}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}}$$



$$I_2 = \frac{\mathcal{E}_0}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}} \cdot \frac{R_3}{R_2+R_3}$$

$$I_2 = \underbrace{\frac{12}{4 + \frac{8 \cdot 2}{10}}}_{I_1 = 2.14A} \cdot \frac{2}{10} = \frac{12}{5.6} \cdot \frac{1}{5} =$$

$$\boxed{I_2 = 0.429A}$$

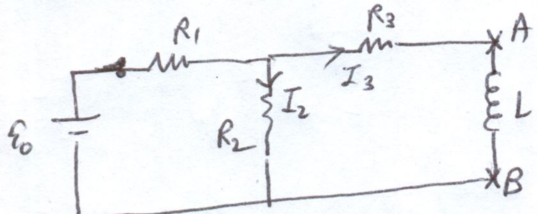
→ Check: $I_3 = I_1 \frac{R_2}{R_2+R_3} = 2.14 \cdot \frac{8}{10} = 1.71A$

$I_2 + I_3 = 1.71 + 0.429 = 2.14A = I_1$ ✓

→ Also same answers if loop analysis was used!

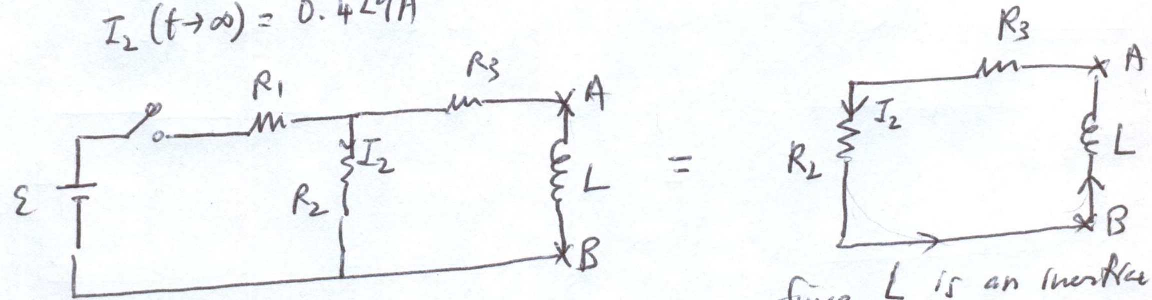
c) I_2? long after switch was closed, it is now reopened

L is an inertia to any change in current; $t \to \infty$ $\begin{cases} I_L = max \\ V_L = 0 \end{cases}$



→ acts like a short-circuit @ $t \to \infty$. but it is physically there

$I_2 (t \to \infty) = 0.429A$



→ Since L is an inertia to any change of current I_2 stays @ 0.429A. (using magnetic energy stored in L)

→ When U_L is depleted, $I_2 \to 0$

(if R_3 is a light bulb it will turn off a little bit after the main switch is open)

Ch 29 Maxwell's Equations

Maxwell's Equations

1) Gauss' Law (Ch. 21) : $\oint_{\text{Gaussian surface}} \vec{E}\cdot d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0}$

2) "Magnetic Gauss' Law" : $\oint_{\text{closed surface}} \vec{B}\cdot d\vec{A} = 0$ (no magnetic monopoles found)

3) Ampere's Law (Ch. 26) : $\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 I_{displacement}$

4) Faraday's Law (Ch. 27) : $\oint_{\text{closed loop}} \vec{E}\cdot d\vec{l} = \mathcal{E} = -\dfrac{d\phi_B}{dt} = -\int \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$

$\left(\phi_B = \int \vec{B}\cdot d\vec{A}\right)$

(i) Hints on a connection b/w $\vec{E}$ & $\vec{B}$!
A time varying $\vec{B}$ can create an electric field $\vec{E}$

(ii) Is there a viceversa? can a time varying $\vec{E}$ create a $\vec{B}$?
Maxwell demonstrated this was possible with an extra term in Ampere's Law: "displacement current"

$$\boxed{I_{displacement} = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}}$$

ϕ_E electric flux $= \int \vec{E}\cdot d\vec{A}$

↳ New Ampere's Law : $\oint_{\text{Amperian loop}} \boxed{\vec{B}}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \int \dfrac{\partial \boxed{\vec{E}}}{\partial t}\cdot d\vec{A}$

"a time-varying electric field $\vec{E}$ can create a magnetic field $\vec{B}$"

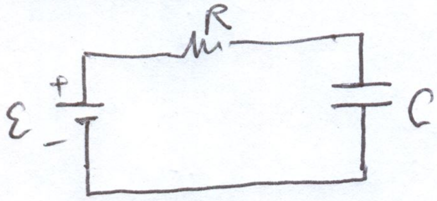
Profound conclusion:

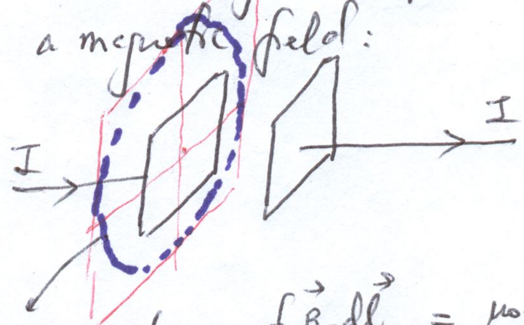
Maxwell's displacement current allows a two way connection between $\vec{E}$ & $\vec{B}$:
1) Time varying $\vec{B} \rightarrow \vec{E}$ (Faraday's)
2) Time varying $\vec{E} \rightarrow \vec{B}$

$$E(t) \rightarrow B(t) \rightarrow E(t) \rightarrow B(t) \rightarrow E(t) \rightarrow \ldots$$

This explains how EM waves propagate (w/o a medium : as opposed to sound waves, water waves which require a medium to propagate !)
- Sun light
- Cellphone signals
- Space probe signals
- Astronomy observations
- etc.

Technicality: a magnetic field can be measured around the plates of a capacitor when it is charging. It was not explained with the old Ampere's Law : which required a current enclosed by an amperian loop to have a magnetic field:





Amperian loop. $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot \underbrace{I_{enclosed}}$

0 as I does not cross the loop !

Old ampere's Law would state that $\vec{B} = 0$
→ contradicting the measurement.

→ With Maxwell's displacement current $= \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$

$\frac{\partial \vec{E}}{\partial t}$ as current is charging while C is charging would create that magnetic field.

Maxwell's equations
1) Explains propagation of EM waves in vacuum ($E(t) \to B(t) \to E(t)$ ---)
2) Both $\vec{E}$ & $\vec{B}$ are vectors : polarization of EM waves (sunglasses : pick out $\frac{1}{2}$ intensity by allowing only one direction of polarization to go through)

Maxwell's equations in vacuum :
↳ no matter, no charges, no wires, no currents

1) Gauss Law : $\oint \vec{E} \cdot d\vec{A} = 0$

2) "Magnetic Gauss Law" $\oint \vec{B} \cdot d\vec{A} = 0$

3) Modified Ampere's Law : $\oint \vec{B} \cdot d\vec{l} = \underbrace{\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}}_{\text{needs no material!}}$

Quite similar!

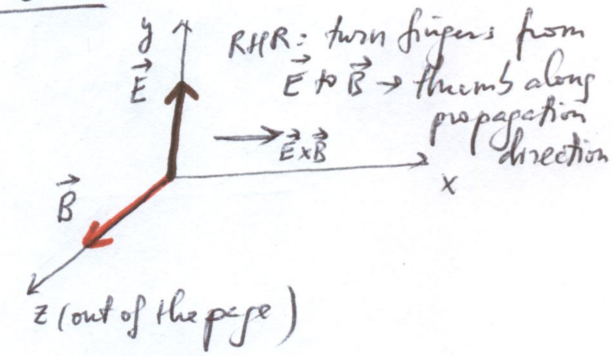
4) Faraday's Law : $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

↳ $\vec{E}(t) \to \vec{B}(t) \to \vec{E}(t) \to$ etc...

EM waves :

↳ $\vec{E}(x,t) = E_p \sin(kx - \omega t)\, \hat{j}$

$\vec{B}(x,t) = B_p \sin(kx - \omega t)\, \hat{k}$



↳ E_p & B_p : amplitudes or magnitudes

k : wave number $= \frac{2\pi}{\lambda}$ (m^{-1})
(number of waves in 2π)

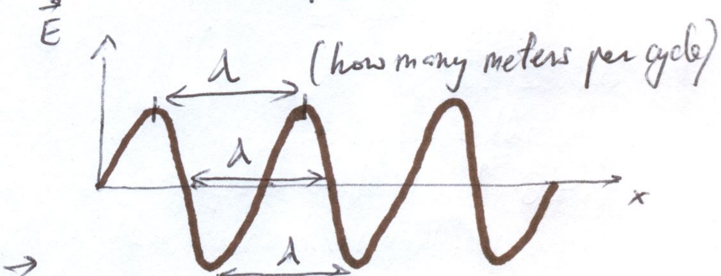
ω : angular frequency $= \frac{2\pi}{T}$ (s^{-1})

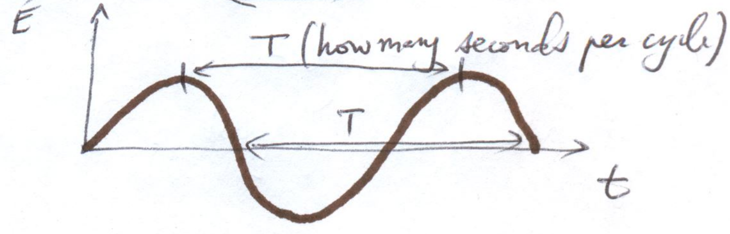
λ : wavelength (m)

T : period (s)

$\omega = \frac{2\pi}{T} = 2\pi f$

f : linear frequency (Hz)
(how many cycles per second)





Mechanical wave equation : transverse wave along a string:

$$\frac{\partial^2 y}{\partial t^2} = k \frac{\partial^2 y}{\partial x^2}$$

EM wave equation :

In vacuum (Quite similar)

3) Ampere's : $\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$

4) Faraday's : $\oint \vec{E} \cdot d\vec{l} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

Integral forms

$\Rightarrow$

3) $\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

4) $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

differential forms

$\frac{\partial}{\partial t}$ 3) $\rightarrow$ $\boxed{\frac{\partial^2 B}{\partial x \partial t}} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$

$\frac{\partial}{\partial x}$ 4) $\rightarrow$ $\frac{\partial^2 E}{\partial x^2} = -\boxed{\frac{\partial^2 B}{\partial x \partial t}}$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

electric wave equation

$\frac{\partial}{\partial x}$ 3), $\frac{\partial}{\partial t}$ 4) $\rightarrow$ $\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$ magnetic wave equation

(i) Both are similar to the <u>transverse</u> wave in a string in the sense that the perturbations (y for string or $\vec{E}$ & $\vec{B}$ for EM waves) are perpendicular to the direction of propagation. However $\vec{E}$ & $\vec{B}$ requires no medium such as the string!

(ii) Another difference is y was ~~per~~ a scalar while $\vec{E}$ & $\vec{B}$ are vectors $\rightarrow$ polarization (not present in a mechanical wave)

How fast do EM wave propagate?

wave speed : $v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}}$ (same as with mechanical waves)

Eq 4) $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ (Faraday's law in differential form)

$E = E_p \sin(kx - \omega t) \rightarrow \frac{\partial E}{\partial x} = k E_p \cos(kx - \omega t)$

$B = B_p \sin(kx - \omega t) \rightarrow -\frac{\partial B}{\partial t} = \omega B_p \cos(kx - \omega t)$

$k E_p = \omega B_p$

$$\boxed{\frac{\omega}{k} = \frac{E_p}{B_p}}$$

Eq 3) $\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ (Ampere's Law in differential form)

$\rightarrow \frac{\partial B}{\partial x} = k B_p \cos(kx - \omega t)$

$\rightarrow -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \mu_0 \varepsilon_0 \omega E_p \cos(kx - \omega t)$

$k B_p = \mu_0 \varepsilon_0 \omega E_p$

$$\boxed{\frac{k}{\mu_0 \varepsilon_0 \omega} = \frac{E_p}{B_p}}$$

$$\rightarrow \frac{\omega}{k} = \frac{k}{\mu_0 \varepsilon_0 \omega} \rightarrow \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0} \rightarrow v = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

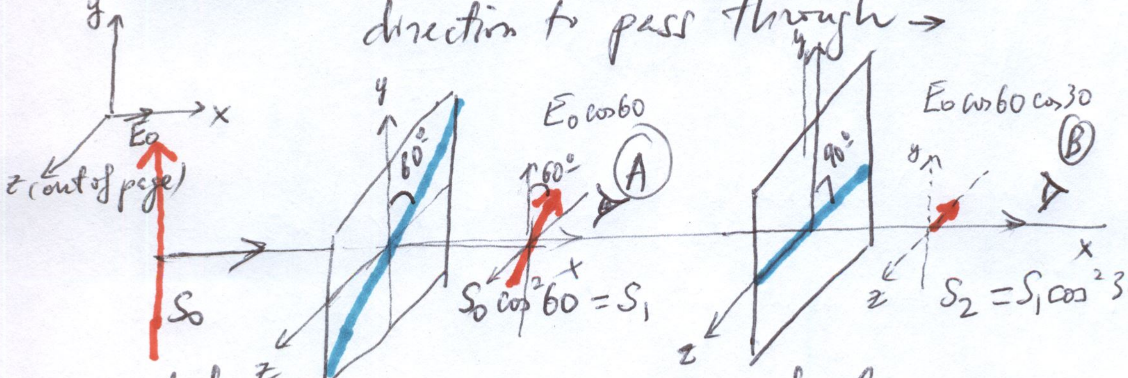
$$C = v_{EM} = \sqrt{\frac{1}{4\pi \cdot 10^{-7} \cdot 8.85 \cdot 10^{-12}}} = 3 \cdot 10^{8} \frac{m}{s}$$ (any EM waves: radio, cellphone, lights, etc..)

Also max speed any object can achieve according to Einsten's theory of speced relativity

29.43

EM waves:
1) Can propagate in vacuum $(E(t) \to B(t) \to E(t) \to \text{etc})$
2) Polarization: vector nature of $\vec{E}$ & $\vec{B}$

Polarizer: material that only allow fields along certain direction to pass through →

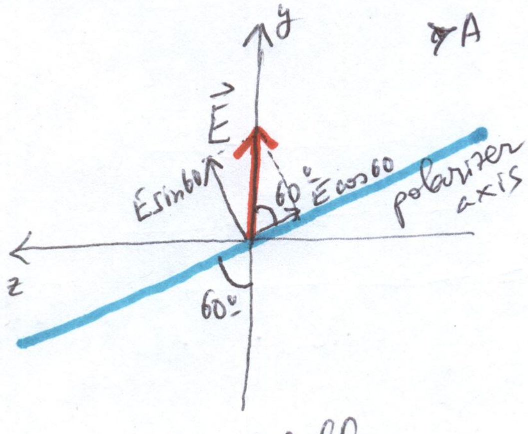


incident light is polarized along y-axis

1st polarizer allows fields point 60° to the the y-axis to pass through

2nd polarizer allows fields that points 90° to y-axis (∥ z axis) to pass through.

1) If we look at the 1st polarizer from the right (point A)



$\vec{E}$: vector can be decomposed into components:

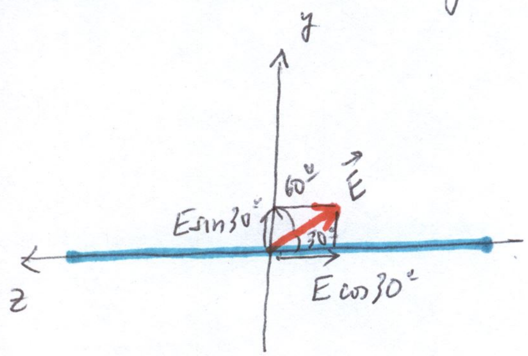
$\vec{E}$:
- $E_0 \cos 60$ points in the direction of the polarizer axis → pass through
- $E_0 \sin 60°$ points perpendicular to the polarizer axis → blocked

2) Intensity is the field amplitude squared:

↳ $S \propto E^2$ → if incident light intensity is S_0

after 1st polarizer intensity $S_0 \cos^2 60 = S_1$

3) If we look at 2nd polarizer from the right (point B)



$\vec{E}$ { $E\cos 30°$ is parallel to 2nd polarizer axis
$= E_0\cos 60°\cos 30°$ → pass through

$E\sin 30°$ is perpendicular → blocked.
$= E_0\cos 60°\sin 30°$ }

4) Intensity after 2nd polarizer: $S_2 = S_1 \cos^2 30 = S_0 \cos^2 60 \cos^2 30$

5) Fraction of light that gets through both polarizers is

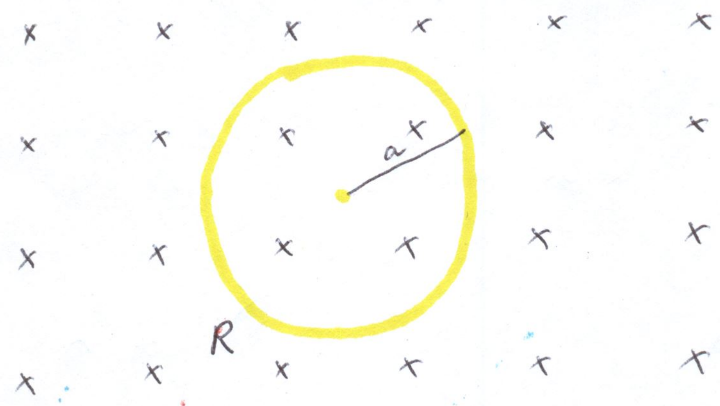
$$\frac{S_2}{S_0} = \cos^2 60 \cos^2 30 = 0.1875 \quad \text{or} \quad 18.75\%$$

6) Important: w/o the 1st polarizer : $S_2 = 0$ since $\vec{E}_0$ is along y-axis while the 2nd polarizer axis is along z-axis!

→ "We can create light by inserting the 1st polarizer in!"

27.50 Circular wire loop, resistance R, radius a, is perpendicular to a uniform magnetic field $\vec{B}$

→ Assume $\vec{B}$ goes into the page:



1) If $\vec{B}$ is constant (not varying over time), although there is a magnetic flux (since field goes through the area enclosed by the wire loop) $\Phi_B = \oint \vec{B} \cdot d\vec{A} = \vec{B} \cdot \oint d\vec{A} = B \cdot \pi a^2$, if it is not varying in time, there is no movement of charges ~~inside~~ along loop.

2) $\vec{B}$ increase from $\overset{B(0)}{B_1} \rightarrow \overset{B(t)}{B_2}$: → $\Phi_B(t) \rightarrow \boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$

Ohm's law ‖ ‖

$$\boxed{I \cdot R = -\pi a^2 \frac{dB}{dt}}$$

↓

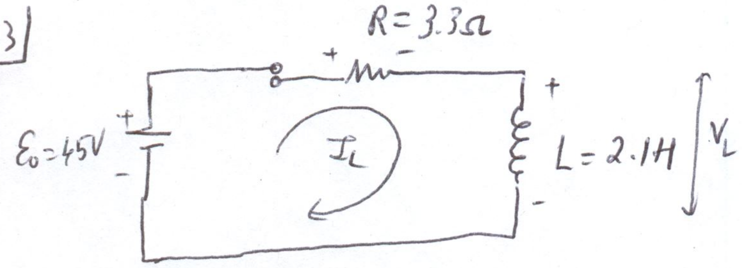
$$\frac{dq}{\cancel{dt}} \cdot R = -\pi a^2 \frac{dB}{\cancel{dt}}$$

$$\int \boxed{dq = -\frac{\pi a^2}{R} dB}$$

$$\underbrace{Q_2 - Q_1}_{\Delta Q} = -\frac{\pi a^2}{R}(B_2 - B_1) = \frac{\pi a^2}{R}(B_1 - B_2)$$

3) So only the initial and final values for the magnetic field matters to the total charge that moves around the loop. (Whether you change B_1 to B_2 slowly or quickly does not make a difference)

27.53



if $I_L(t) = 9.5A \to t?$

Inductor is an inertia to current I_L : before switch was closed $I_L \neq 0$, $I_L(0) = 0$ then it starts to build up, till it gets to the max. value when $V_L = 0$ (short circuit when $t \to \infty$)

	$t=0$	$t \to \infty$	t
$I_L =$	0	$max = \frac{\mathcal{E}_0}{R}$	$I_L(t) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{(\frac{L}{R})}}\right)$
$V_L =$	$max = \mathcal{E}_0$	0	$V_L(t) = \mathcal{E}_0 e^{-\frac{t}{(\frac{L}{R})}}$

Loop analysis: $\mathcal{E}_0 - I_L \cdot R - V_L = 0$

$$I_L(t) = \frac{\mathcal{E}_0 - V_L(t)}{R} = \frac{\mathcal{E}_0 - \mathcal{E}_0 e^{-\frac{t}{(\frac{L}{R})}}}{R}$$

$$= \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{(\frac{L}{R})}}\right)$$

$$I_L(t) = 9.5A = \frac{45}{3.3}\left(1 - e^{-\frac{t}{(\frac{2.1}{3.3})}}\right) \to e^{-\frac{t}{(\frac{2.1}{3.3})}} = 1 - \frac{9.5 \cdot 3.3}{45}$$

$$-\frac{t}{\left(\frac{2.1}{3.3}\right)} = \ln\left(1 - \frac{9.5 \cdot 3.3}{45}\right)$$

$$t = -\frac{2.1}{3.3}\ln\left(1 - \frac{9.5 \cdot 3.3}{45}\right)$$

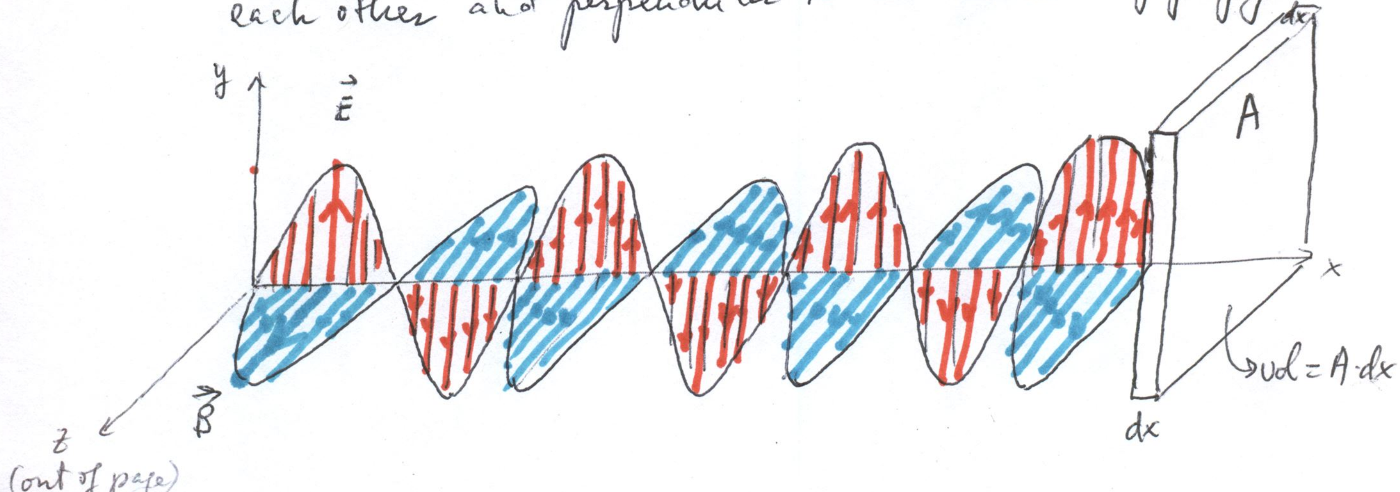
$$\boxed{t = 0.76s}$$

$\left(I_L(0) = 0 \ ; \ I_L(\infty) = \frac{\mathcal{E}_0}{R} = \frac{45}{3.3} = 13.63A\right)$

Intensity of EM waves

$\hookrightarrow S = \frac{P}{\text{Area}} = \frac{\frac{dU}{dt}}{\text{Area}}$

Wave propagation along x-axis through a rectangular slab of cross-section area A and thickness dx perpendicular to the direction of propagation; $\vec{E}$ & $\vec{B}$ are perpendicular to each other and perpendicular to the direction of propagation



→ Direction of propagation given $\underbrace{\vec{E} \times \vec{B}}_{\text{cross-product}}$ (RHR → $\hat{x}$-axis)

Total energy:

$$\frac{dU}{dt} = \frac{d}{dt}(\underbrace{u}_{\text{energy density}} \text{vol}) = \frac{d}{dt}(u \cdot A\,dx) = u \cdot A \underbrace{\frac{dx}{dt}}_{\text{how fast wave travels through slab}}$$

→ wave speed = c

$$S = \frac{\frac{dU}{dt}}{\text{Area}} = \frac{uAc}{A} = uc = \Big(\underbrace{\tfrac{1}{2}\varepsilon_0 E^2}_{\text{electric}} + \underbrace{\tfrac{1}{2}\tfrac{1}{\mu_0}B^2}_{\text{magnetic}}\Big)c$$

$c^2 = \frac{1}{\mu_0 \varepsilon_0}$ or $\varepsilon_0 = \frac{1}{c^2 \mu_0}$ or $\varepsilon_0 c^2 = \frac{1}{\mu_0}$

$\longrightarrow S = \varepsilon_0 E^2 c = \varepsilon_0 c^2 \cdot E \cdot \underbrace{\frac{E}{c}}_{B} = \frac{1}{\mu_0} E \cdot B$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$: intensity vector points in the direction of propagation