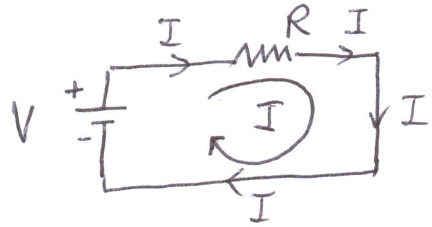
Ch 25 Electrical Circuits:

Circuits: batteries & resistors (later will add capacitor, inductor)

Simplest: 1)

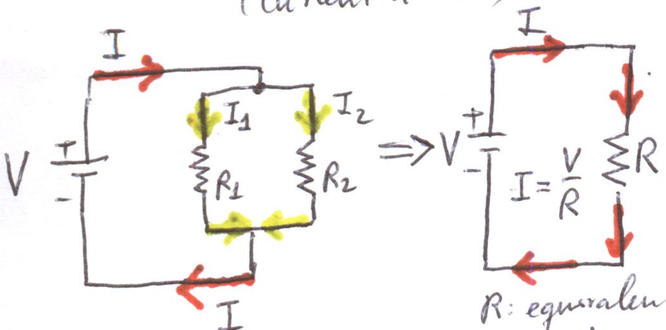


$I = \frac{V}{R}$ (Ohm's Law)

linear circuits follow Ohm's Law

2) One battery & two resistors:

Parallel Connection (Current division)



R: equivalent resistance for R_1 & R_2

1) $\boxed{I = I_1 + I_2}$ Current division

If we replace R_1 & R_2 by R, battery can't tell the difference as it draws the same current I out of the battery

2) Same voltage V is applied to R_1 & R_2

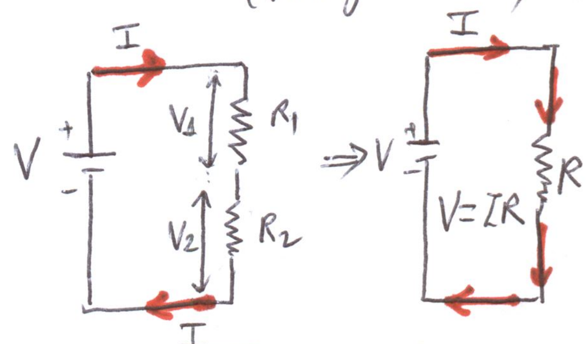
$$\begin{cases} I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ I = \frac{V}{R} \end{cases} \quad \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}} \quad \boxed{R = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

Current Division:

$$\boxed{I_1 = \frac{V}{R_1} = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) R_1} = I \frac{1}{\frac{R_1 + R_2}{\cancel{R_1} \cdot R_2} R_1} = I \cdot \frac{R_2}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{V}{R_2} = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) R_2} = I \frac{1}{\frac{R_1 + R_2}{R_1 \cdot \cancel{R_2}} R_2} = I \cdot \frac{R_1}{R_1 + R_2}}$$

Series Connection (Voltage division)



1) $\boxed{V = V_1 + V_2}$ Voltage division

~~$V = IR_1$~~

2) Same current I goes through R_1 & R_2

$V = IR_1 + IR_2 = I(R_1 + R_2)$ $\boxed{R = R_1 + R_2}$

$V = IR$

Voltage Division:

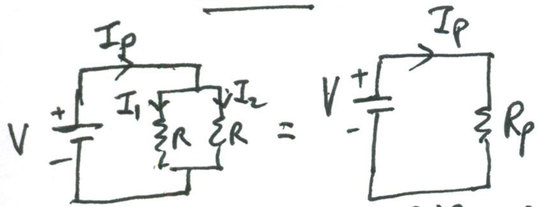
$$\boxed{\begin{aligned} V_1 &= I \cdot R_1 = \frac{V}{R_1 + R_2} \cdot R_1 = V \cdot \frac{R_1}{R_1 + R_2} \\ V_2 &= I \cdot R_2 = V \frac{R_2}{R_1 + R_2} \end{aligned}}$$

Power Consumption $R_1 = R_2 \equiv R$

$$P = I \cdot V = \frac{V^2}{R} = \boxed{I^2 R}$$

$$I = \frac{V}{R} \text{ or } V = IR$$

Parallel



$$R_p = \frac{R \cdot \cancel{R}}{2\cancel{R}} = \frac{R}{2}$$

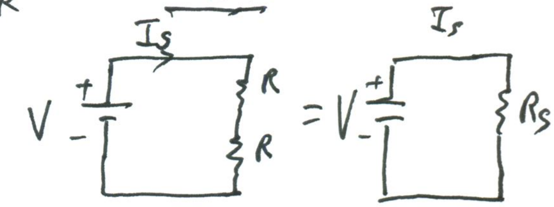
→ Power consumption can be calculated on either the original or the equivalent circuit:

$$\left[P_p = I_p \cdot V = \frac{V}{\frac{R}{2}} \cdot V = 2\frac{V^2}{R} \right]$$

→ $I_1 = I_p \frac{R}{2R} = \frac{I_p}{2} = I_2$

Since $I_p = \frac{2V}{R}$ → $\left\{ \boxed{I_1 = \frac{V}{R}}, \boxed{I_2 = \frac{V}{R}} \right.$

Series



$$R_s = 2 \cdot R$$

$$\left[P_s = I_s \cdot V = \frac{V}{2R} \cdot V = \frac{V^2}{2R} \right]$$

→ $I_s = \frac{V}{2R}$ (current through each R)

(half compared to parallel connection)

↓

$\left(P_s = \frac{P_p}{4} \right)$ since $P = I^2 \cdot R$

→ light bulbs as resistors → they are brighter in parallel connections!

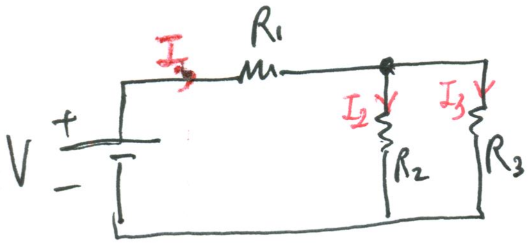
Analysis of Circuits with Resistors Only

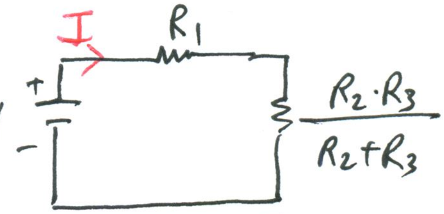
↳ 1) Can use parallel/series equivalent to reduce to simplest

$I = \frac{V}{R}$

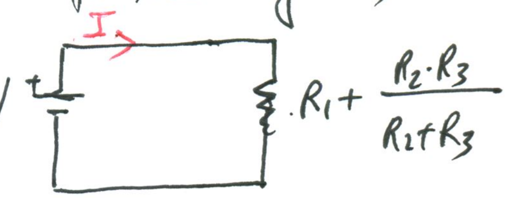
Then find currents through each resistor.

2) Can't use parallel/series, there are more than one battery. Must use Loop or Node Analysis

1) 

= 

same V, same I
(this circuit is equivalent to the original)

= 

same V, same I
(this is also equivalent to the original circuit)

$$I = \frac{V}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}}$$

Now we can find currents through individual resistors:

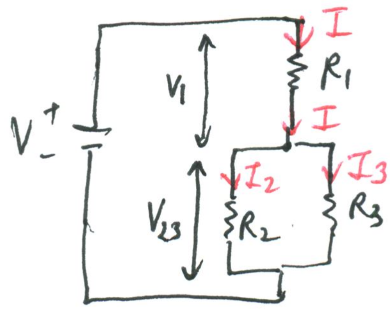
a) Current through R_1 is I

b) Current through R_2 & R_3 2 different ways:

(i) Use current division:

$$\begin{cases} I_2 = I \cdot \frac{R_3}{R_2+R_3} = \frac{V}{R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}} \cdot \frac{R_3}{R_2+R_3} \\ I_3 = I \cdot \frac{R_2}{R_2+R_3} \end{cases}$$

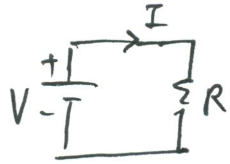
(ii) Use voltage division:



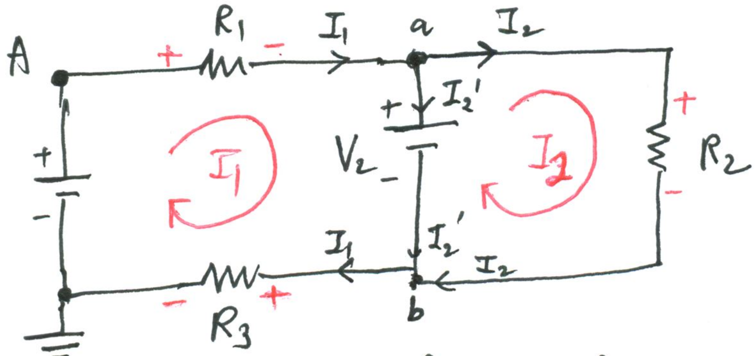
$$V = V_1 + V_{23}$$

& Ohm's Law

$$\begin{cases} I_2 = \frac{V_{23}}{R_2} = \frac{V \cdot \frac{\cancel{R_2} \cdot R_3}{R_2+R_3} \Big/ \left(R_1 + \frac{R_2 \cdot R_3}{R_2+R_3}\right)}{\cancel{R_2}} \\ I_3 = \frac{V_{23}}{R_3} = \end{cases}$$

2) Reduction to 

using only parallel & series equivalent is not possible → use Loop or Node Analysis or Kirchoff's Laws



Ground or zero potential (critical to know for Loop/Node analysis)

Note: 1) R_1 & R_2 are not in series as not same current going through both

2) R_1 & R_3 are not in parallel as not same voltage across each of them

Loop Analysis

Kirchoff's Law: total voltage difference across elements in a closed loop (in the circuit) is 0

$(\sum_i V_i = 0)$

↳ Signs: first assume a direction for the current in each closed loop

↓

1) If this current goes through battery from $-$ to $+$ → battery voltage is positive. Negative if the opposite happens.

2) Voltage difference across any resistor is always negative.

Node Analysis

Kirchoff's Law: total current at any node in circuit is 0

$(\sum_i I_i = 0)$

↳ Signs:

1) Any current going into node is positive

2) Any current leaving node is negative

Loop Analysis

→ By visual inspection there are 2 loops

→ Assume CW currents in each loop I_1 & I_2

Loop 1: $+V_1 - I_1 \cdot R_1 - V_2 - I_1 \cdot R_3 = 0$

($I = \frac{V}{R}$ or $V = I \cdot R$)
Ohm's Law

Loop 2: $+V_2 - I_2 \cdot R_2 = 0$

→ Find currents I_1 & I_2

$$I_1 = \frac{V_1 - V_2}{R_1 + R_3}$$

$$I_2 = \frac{V_2}{R_2}$$

$$I_1 - I_2 = I_2'$$

Node Analysis

→ Visual inspection: 2 nodes a, b

Node a: $I_1 - I_2 - I_2' = 0$

Node b: $-I_1 + I_2 + I_2' = 0$ (same equation)

→ Only one independent node!

→ Write currents in term of data (V_1, V_2, R_1, R_2, R_3) based on the location of the ground in circuit

$$\begin{cases} V_A = V_1 \\ V_a = V_2 + I_1 \cdot R_3 \end{cases} \quad I_1 = \frac{V_A - V_a}{R_1}$$

$$\rightarrow I_1 = \frac{V_1 - (V_2 + I_1 \cdot R_3)}{R_1}$$

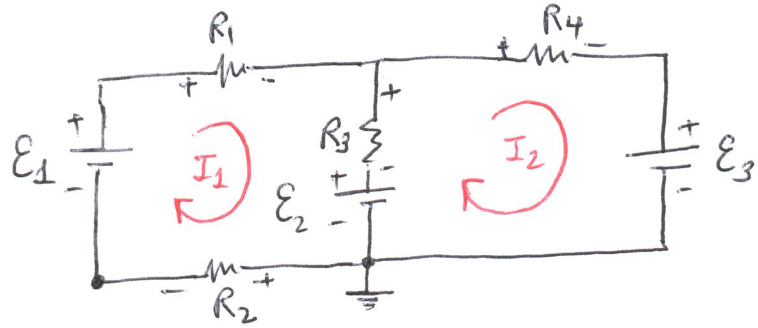
$$I_1 \cdot R_1 = V_1 - V_2 - I_1 \cdot R_3$$

$$I_1 (R_1 + R_3) = V_1 - V_2$$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

also $\boxed{I_2 = \frac{V_2}{R_2}}$ (Ohm's law)

Node a: $\frac{V_1 - V_2}{R_1 + R_3} - \frac{V_2}{R_2} = I_2'$



$\mathcal{E}_1 = 6V$

$\mathcal{E}_2 = 1.5V$

$\mathcal{E}_3 = 4.5V$

$R_1 = 270\Omega$; $R_2 = 150\Omega$

$R_3 = 560\Omega$; $R_4 = 820\Omega$

→ Find current through R_3 with direction (up or down)

<u>Loop Analysis:</u>

Visual inspection: 2 loops, assume CW currents I_1 & I_2

<u>Loop 1</u>: $+\mathcal{E}_1 - I_1 \cdot R_1 \underbrace{-(I_1 - I_2)\cdot R_3}_{\text{net current thru } R_3} - \mathcal{E}_2 - I_1 \cdot R_2 = 0$ (1)

<u>Loop 2</u>: $+\mathcal{E}_2 - (I_2 - I_1)\cdot R_3 - I_2 \cdot R_4 - \mathcal{E}_3 = 0$ (2)

Algebraic manipulations to solve for currents I_1 & I_2

(1) + (2) → $\mathcal{E}_1 - \mathcal{E}_3 - I_1 \cdot (R_1 + R_2) - I_2 \cdot R_4 = 0$

Solve for I_1 ⟶ $I_1 = \dfrac{\mathcal{E}_1 - \mathcal{E}_3 - I_2 \cdot R_4}{R_1 + R_2}$ (3)

Eq(2): $\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_3 + R_4) + I_1 \cdot R_3 = 0$

$$\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_3 + R_4) + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} R_3 - I_2 \frac{R_3 \cdot R_4}{R_1 + R_2} = 0$$

$$\mathcal{E}_2 - \mathcal{E}_3 + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} \cdot R_3 = I_2 \left(R_3 + R_4 + \frac{R_3 \cdot R_4}{R_1 + R_2} \right)$$

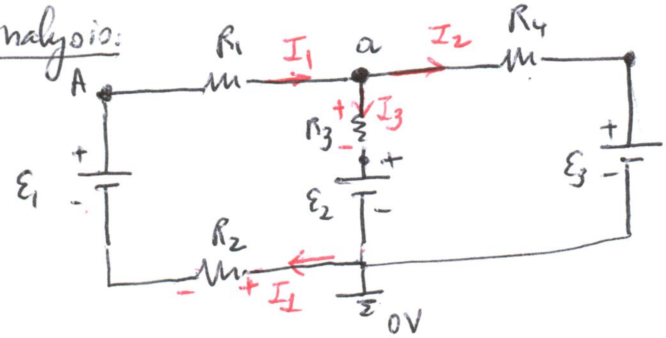
$$I_2 = \frac{\mathcal{E}_2 - \mathcal{E}_3 + \frac{\mathcal{E}_1 - \mathcal{E}_3}{R_1 + R_2} R_3}{R_3 + R_4 + \frac{R_3 \cdot R_4}{R_1 + R_2}}$$

$$I_2 = \frac{-3 + \frac{1.5}{420} \cdot 560}{1380 + \frac{560 \cdot 820}{420}} = -0.4 \cdot 10^{-3} A = -0.4 \text{ mA}$$

Eq(3): $I_1 = \dfrac{1.5 - (-0.4 \cdot 10^{-3}) 820}{420} = 4.36 \text{ mA}$

Current through R_3 is $I_1 - I_2 = 4.36 - (-0.4) = 4.76$ mA downward (+)

Node Analysis:



Same circuit
Current through R_3?

Visual inspection: (i) one independent node : a
(ii) assign directions for I_1, I_2, I_3 on circuit.
(iii) locate ground in circuit.

Node a: $\quad I_1 - I_2 - I_3 = 0$

Now write these currents in terms of ε's & R's (data)

a) $$I_1 = \frac{V_A - V_a}{R_1} = \frac{(\varepsilon_1 - I_1 \cdot R_2) - (I_3 R_3 + \varepsilon_2)}{R_1} = \frac{\varepsilon_1 - I_1 \cdot R_2 - V_a}{R_1}$$

Ohm's Law $\qquad I_1 \cdot R_1 = \varepsilon_1 - I_1 \cdot R_2 - V_a \rightarrow \boxed{I_1 = \frac{\varepsilon_1 - V_a}{R_1 + R_2}}$

b) $$\boxed{I_2 = \frac{V_a - \varepsilon_3}{R_4}}$$ (Ohm's Law through R_4)

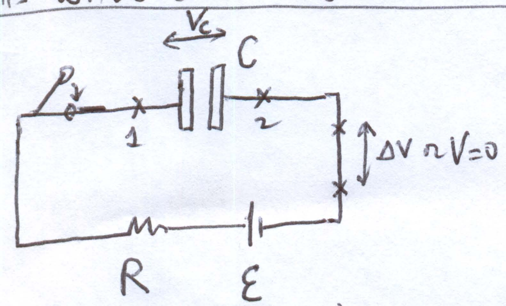
c) $$\boxed{I_3 = \frac{V_a - \varepsilon_2}{R_3}} \Rightarrow V_a = I_3 R_3 + \varepsilon_2$$

$$\frac{\varepsilon_1 - V_a}{R_1 + R_2} - \frac{V_a - \varepsilon_3}{R_4} - \frac{V_a - \varepsilon_2}{R_3} = 0$$

$$\frac{6 - V_a}{420} - \frac{V_a - 4.5}{820} - \frac{V_a - 1.5}{560} = 0 \Rightarrow V_a = 4.17\text{ V}$$

c) $$I_3 = \frac{4.17 - 1.5}{560} = 4.76\text{ mA}$$ (positive downward)

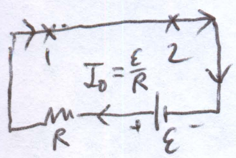
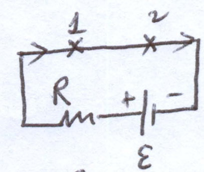
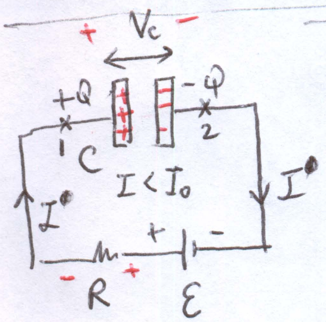
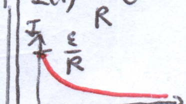
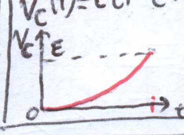
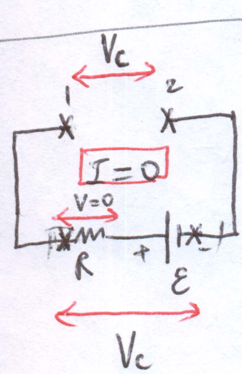
Circuits with both resistors & capacitors:



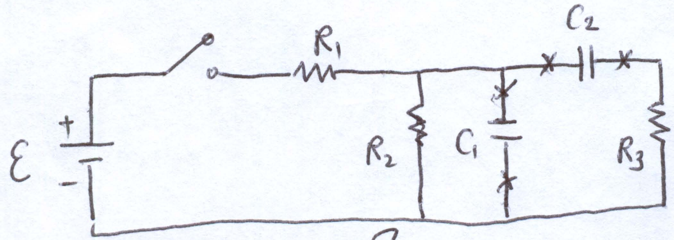
What happens if an uncharged capacitor of capacitance C is connected to rest of this circuit?

(e.m.f)

(electro motive force)

	t	Q	V_c (potential across plates)	Capacitor Behaves like	I
Starts connection to circuit 	0	0	0	a wire "short circuit"  $I_0 = \frac{\mathcal{E}}{R}$	$\frac{\mathcal{E}}{R} = I_0$ (max)
	$t > 0$	Charges are moved by the current in the circuit. (harder to move next charge as it has to go against the new electric field…	$\neq 0$ $\frac{d}{dt}(\mathcal{E} - IR - V_c) = 0$ $V_c = \frac{Q}{C} \rightarrow \frac{dV_c}{dt} = \frac{I}{C}$ $\frac{d\mathcal{E}}{dt} = 0$ $\rightarrow -R\frac{dI}{dt} - \frac{I}{C} = 0$ $\frac{dI}{dt} = -\frac{I}{RC} \rightarrow \boxed{\frac{dI}{I} = -\frac{dt}{RC}}$ $\rightarrow \ln I = -\frac{t}{RC} + \text{constant}$ $I(t) = I_0 e^{-\frac{t}{RC}}$	current gets smaller! I^* $I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$  $V_c(t) = \mathcal{E}(1 - e^{-\frac{t}{RC}})$ 	Loop equation: (W current I^*: $\boxed{+\mathcal{E} - I^*R - V_c = 0}$ $I^*R = \mathcal{E} - V_c$ $I^* = \frac{\mathcal{E} - V_c}{R}$ $= \frac{\mathcal{E}}{R} - \frac{V_c}{R} < I_0$
	$t \rightarrow \infty$	Capacitor is fully charge Max charge ↓ Max E ↓ max V_c	max $V_c = \mathcal{E}$	open circuit	$I = 0$

25.60



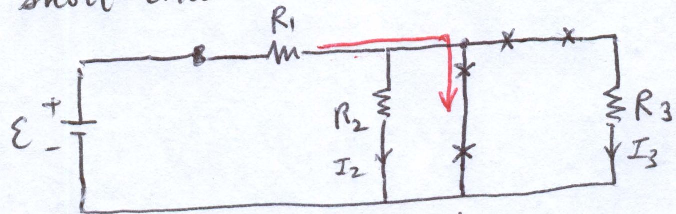
$R_1 = R_2 = R$

Capacitors initially uncharged

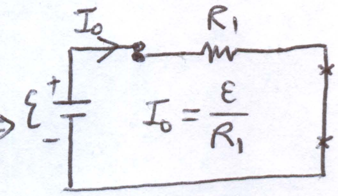
Current in R_2 @ $t=0$ & @ $t \to \infty$?

Statements:

1) @ $t=0$ (just after switch is closed)
$Q=0 \to V_c = 0 \to$ capacitors behave like a piece of wire or short-circuit.



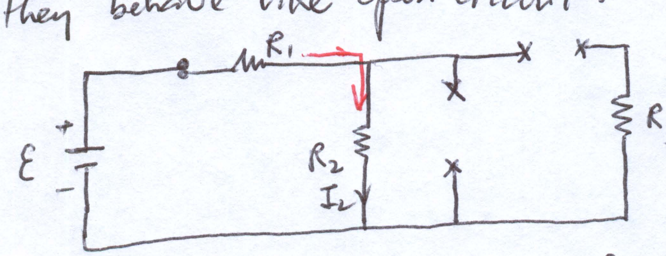
$\Rightarrow$



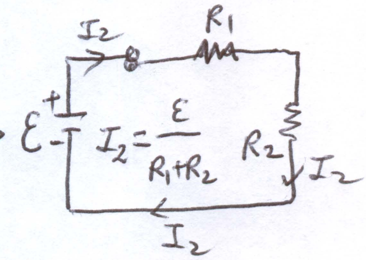
↓
wire: all current coming in from R_1 will go down this wire (path of least resistance)
↓
$\boxed{I_2 = 0} = I_3$

Current in R_2 @ $t=0$ is 0

2) @ $t \to \infty$: sufficiently long after, ~~the~~ both capacitors get fully charged? They don't allow any further current $I=0$
$\to$ they behave like open circuit:

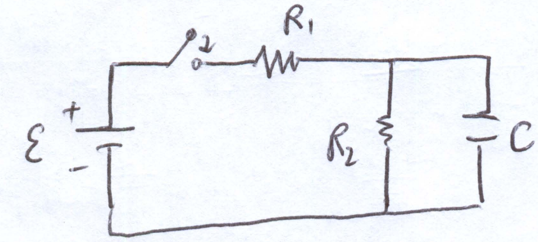


$\Rightarrow$



Current in R_2 @ $t \to \infty$ is $I_2 = \dfrac{\mathcal{E}}{R_1 + R_2}$

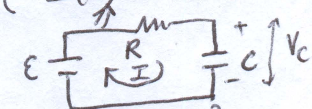
$\boxed{25.75}$



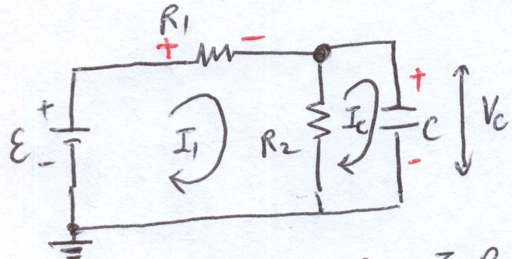
Write Loop & Node equation & find time constant

(tau)

Time constant τ time for current across capacitor to decay by a factor of e ($I(t) = I_0 e^{-\frac{t}{RC}} \to \tau = R\cdot C$)



$\mathcal{E} - IR - \frac{Q}{C} = 0$



Loop equation: 1) $\mathcal{E} - I_1 \cdot R_1 - (I_1 - I_c)\cdot R_2 = 0$

2) $-(I_c - I_1)\cdot R_2 - V_c = 0$ $\quad \left(V_c = \frac{Q}{C} \right)$

$-V_c = (I_c - I_1)\cdot R_2$

$\boxed{-\frac{Q}{C} = -(I_1 - I_c)\cdot R_2}$ (a)

$\rightarrow$ 1) $\mathcal{E} - I_1 \cdot R_1 - \frac{Q}{C} = 0 \rightarrow \boxed{I_1 = \frac{\mathcal{E} - \frac{Q}{C}}{R_1}}$ (b)

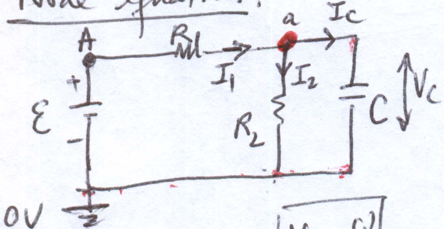
(b) in (a) $+\frac{Q}{C} = I_1 \cdot R_2 - I_c R_2$

$= \left(\mathcal{E} - \frac{Q}{C}\right)\frac{R_2}{R_1} - I_c \cdot R_2$

$\frac{d}{dt}\left[\frac{Q}{C}\left(1 + \frac{R_2}{R_1}\right) = \mathcal{E}\frac{R_2}{R_1} - I_c \cdot R_2\right]$

$I_c\left(\frac{1 + \frac{R_2}{R_1}}{C}\right) = -\frac{dI_c}{dt} R_2 \quad -\frac{\left(1 + \frac{R_2}{R_1}\right)}{R_2 C} dt = \frac{dI_c}{I_c} \Rightarrow -\frac{R_1 + R_2}{R_1 R_2}\frac{dt}{C} = \frac{dI_c}{I_c} \Rightarrow$

Node equation:



0V

$V_A = \mathcal{E}$

$V_a = V_c$

$\boxed{V_c = \frac{Q}{C}}$

$I_1 - I_2 - I_c = 0 \quad \rightarrow \quad \boxed{\frac{\mathcal{E} - V_c}{R_1} - \frac{V_c}{R_2} - I_c = 0}$

$I_1 = \frac{V_A - V_a}{R_1} = \frac{\mathcal{E} - V_c}{R_1}$

$I_2 = \frac{V_c}{R_2}$

$\boxed{I_c = \frac{dQ}{dt}}$

$\frac{\mathcal{E}}{R_1} - V_c\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - I_c = 0$

$\frac{d}{dt}\left[\frac{\mathcal{E}}{R_1} - \frac{Q}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - I_c = 0\right]$

$-\frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{dQ}{dt} - \frac{dI_c}{dt} = 0$

$\frac{dI_c}{dt} = -\frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) I_c \rightarrow \frac{dI_c}{I_c} = -\frac{dt}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow I_c(t) = I_0\, e^{-\frac{t}{\boxed{C\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}}} \rightarrow \tau$

$\rightarrow \boxed{\tau = \frac{C}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{C R_1 R_2}{R_1 + R_2}}$

Cont. from Node Equation for 25.75 :

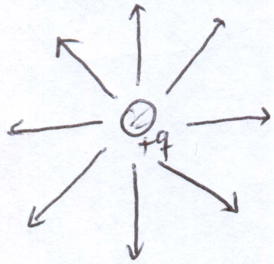
$$-\frac{dt}{\left(\frac{R_1R_2}{R_1+R_2}\right)\cdot C} = \frac{dI_c}{I_c} \rightarrow \ln I_c = -\frac{t}{\frac{R_1R_2}{R_1+R_2}\cdot C}$$

$$I_c(t) = I_0\, e^{-\frac{t}{\boxed{\frac{R_1R_2}{R_1+R_2}\cdot C}}} \quad \tau$$

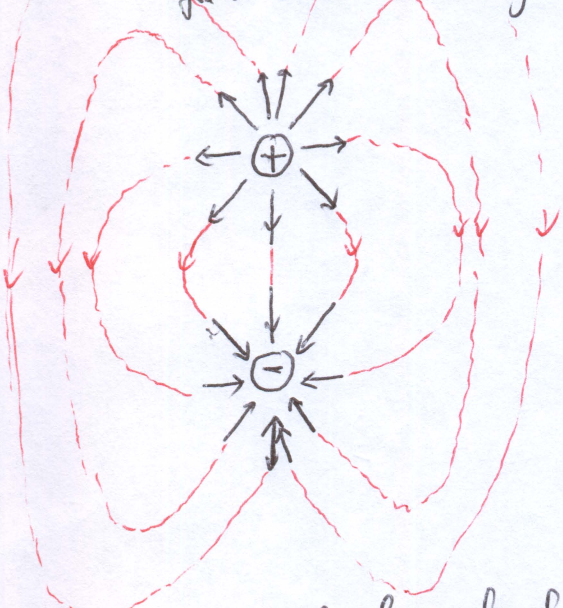
$$\tau = \frac{R_1R_2}{R_1+R_2}\cdot C = \frac{C}{\left(\frac{R_1+R_2}{R_1R_2}\right)} = \frac{C}{\left(\frac{1}{R_2}+\frac{1}{R_1}\right)}$$

Ch 26 Magnetic Field

Electric field



→ Field lines : higher density closer to charge ⟺ stronger field closer to charge

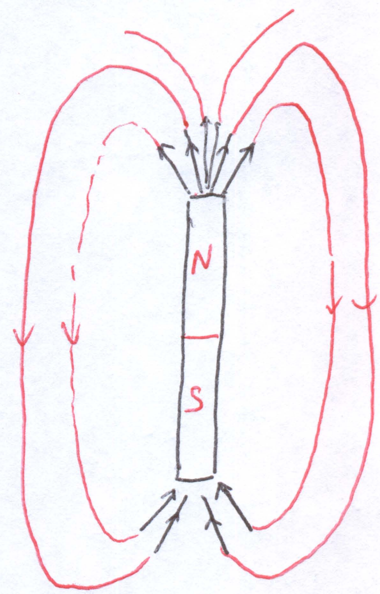


→ Electric dipoles : closed field lines

→ Effect : in printing, $\vec{E}$ pushed the charged ink droplet off its original trajectory

Magnetic field

No magnetic monopole has been found.



→ Magnets : always with 2 poles → Magnetic field lines are always closed)

Effect: keep charged and moving particles in circular trajectories

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

(Lorentz force by a magnetic field $\vec{B}$ on a moving particle of charg q & velocity $\vec{v}$)

$\vec{F} = q\vec{v} \times \vec{B}$ → $F = qvB\sin\theta$ (θ angle b/w $\vec{v}$ & $\vec{B}$)

cross product → use RHR to find direction for $\vec{F}$ which is perpendicular to both $\vec{v}$ & $\vec{B}$

A particle won't feel the magnetic force if:

a) If it is neutral!

b) If it is charged but static ($\vec{v} = 0$)

c) If it is charged and moving along direction of the magnetic field (aurora borealis)

$\vec{F} = q\vec{v} \times \vec{B}$ → $F = qvB\sin\theta$ (θ angle b/w $\vec{v}$ & $\vec{B}$)

$\boxed{\text{cross product}}$ → use RHR to find direction for $\vec{F}$ which is perpendicular to both $\vec{v}$ & $\vec{B}$

A particle won't feel the magnetic force if:

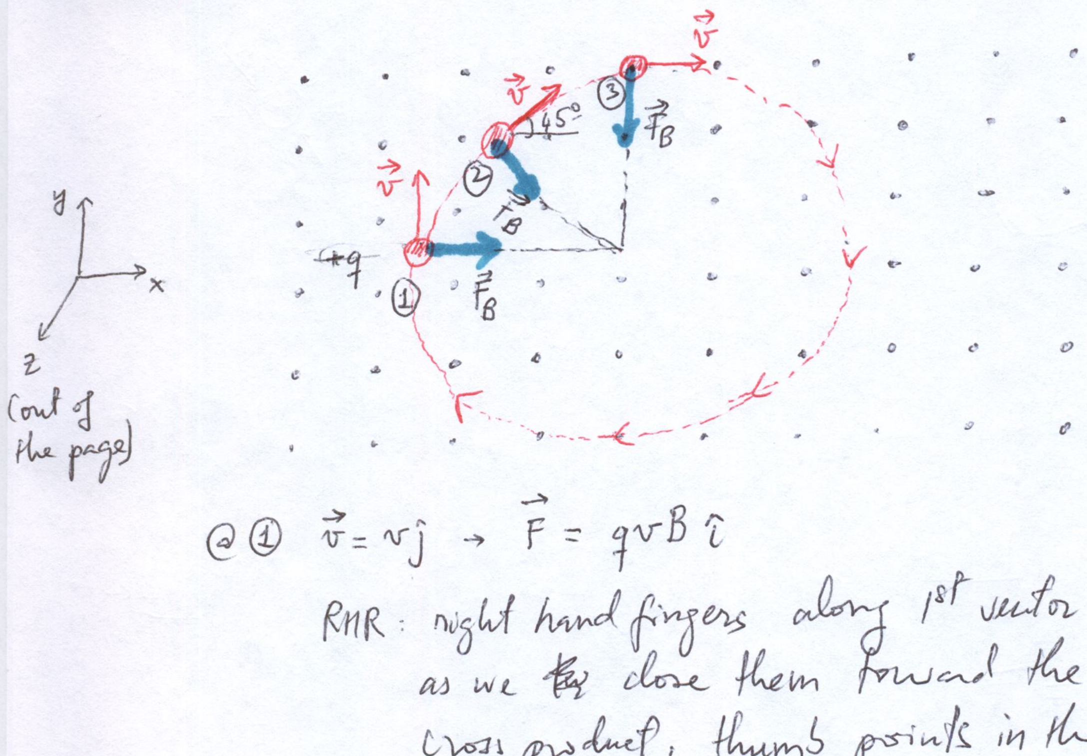
a) If it is neutral!

b) If it is charged but static ($\vec{v} = 0$)

c) If it is charged and moving along direction of the magnetic field (aurora borealis)

Region of magnetic field, uniform (not varying from point to point), pointing out of the page $\odot$

Dots are equally spaced in both directions



$\vec{B} = B\hat{k}$ ↑ unit vector in direction z

@ ① $\vec{v} = v\hat{j}$ → $\vec{F} = qvB\,\hat{i}$

RHR: right hand fingers along 1st vector in the cross product, as we ~~to~~ close them toward the 2nd vector of the cross product, thumb points in the direction of the cross product

As long as the particle moves only in the XY plane, $\vec{v}$ will always be perpendicular to the magnetic field $\vec{B}$, $\sin\theta = 1$ → $F = qvB$

@ 2 $\quad \underbrace{\vec{v} = v\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)}_{1^{st}\text{ quadrant}} \rightarrow \vec{F} = qvB \underbrace{\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)}_{4^{th}\text{ quadrant}}$

@ 3 $\quad \vec{v} = v\hat{i} \quad \rightarrow \vec{F} = qvB\,(-\hat{j})$

Conclusions:

a) Effect of $\vec{B}$ was to confine the moving charged particle to go along a circular trajectory. In this case ($\vec{B} = B\hat{k}$ out of page) $\begin{cases} \text{CW as charge is } + \\ \text{CCW if charge is } - \end{cases}$

$\downarrow r\,?$

b) Even in the presence of $\vec{B}$ particle is subject to 2^{nd} Newton's Law: $\vec{F}_{net} = m\cdot\vec{a}$

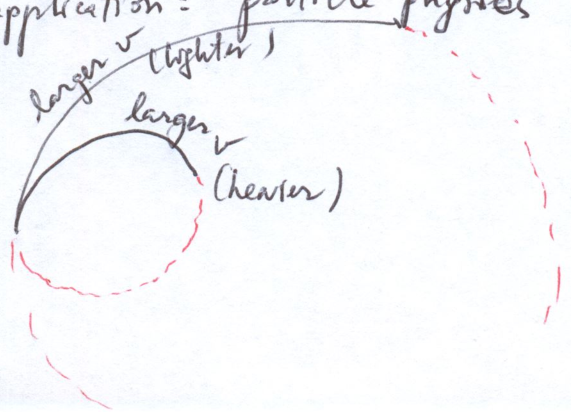
(i) Our charge follows a UCM (Uniform Circular Motion) = since $\vec{F}_B$ is always perpendicular to $\vec{v}$, it only changes its direction but not its magnitude $\rightarrow v$ (speed) stays the same throughout the circular orbit.

(ii) The acceleration then is not tangential but radial (toward center of curvature) to changes the direction of velocity $\vec{v} \rightarrow a = \frac{v^2}{r}$)

$$\Rightarrow q\cancel{v}B = m\cdot\frac{v^{\cancel{2}}}{r} \rightarrow \boxed{r = \frac{mv}{qB}}$$

$\begin{cases} \rightarrow \text{Larger } v \rightarrow \text{larger orbit} \\ \rightarrow \text{Larger } B \rightarrow \text{smaller orbit.} \end{cases}$

one application: particle physics detectors; plasma fusion



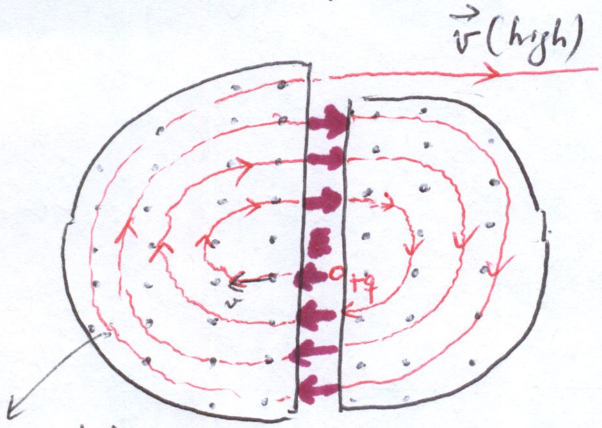
→ Orbital period: T: how long per orbit:

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi\frac{m\cancel{v}}{qB}}{\cancel{v}} = \frac{2\pi m}{qB}}$$

Cyclotron
(accelerate charged particles to high speed by running them through circular orbits multiple times)

→ We can also accelerate charged particles to high speed using an electric field & a very long tunnel

→ But cyclotron uses $\vec{B}$ & $\vec{E}$ can do the job in a smaller area like a hospital lab.



largest orbit
highest speed

Use radius of Cyclotron R

Cyclotron

1) Two dee's with a gap b/w them, where an alternating electric field is applied: left, right, left, etc..

2) A uniform $\vec{B}$ fill the dee's pointing out of page

3) $KE_{max} = \frac{1}{2} m v_{max}^2$

$$r = \frac{mv}{qB} \rightarrow R = \frac{m v_{max}}{qB}$$

$$\rightarrow v_{max} = \frac{qBR}{m}$$

$$\rightarrow KE_{max} = \frac{1}{2} m \left(\frac{qBR}{m}\right)^2 = \frac{(qBR)^2}{2m}$$

Note: for $v \ll c = 3 \cdot 10^8 \frac{m}{s}$: Newtonian mechanics applies

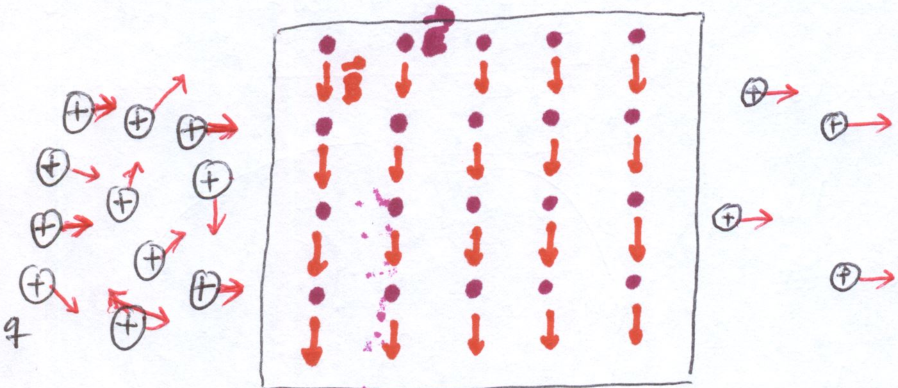
but if $v \rightarrow c$: we need relativistic corrections:

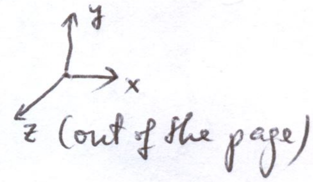
→ synchrotron

Velocity Selector:

We can pick among those ions with different velocities, particular ones with a selected velocity by running them through a region filled with both $\vec{E}$ & $\vec{B}$.

For example, to pick among ions with velocities in random direction, those with velocities in the x-direction:





$\vec{E} = E\hat{k}$ & $\vec{B} = B(-\hat{j})$

For those ions with $\vec{v} = v\hat{\imath}$
$$\begin{cases} \vec{F}_E = q\vec{E} = qE\hat{k} \\ \vec{F}_B = qvB\,\underbrace{(-\hat{\imath}\times\hat{\jmath})}_{-\hat{k}} = -qvB\,\hat{k} \end{cases}$$

$\rightarrow \vec{F}_{net} = \vec{F}_E + \vec{F}_B = (qE - qvB)\hat{k}$ could be zero

If $\vec{F}_{net} = 0 \rightarrow$ ions go through unaffected:

$$E - vB = 0 \quad \text{or} \quad \boxed{v = \frac{E}{B}}$$

Those ions with $\vec{v} = \frac{E}{B}\hat{\imath}$ they are "selected".

Calculation of the Magnetic Field (Sources of the Magnetic field):

Electric Field

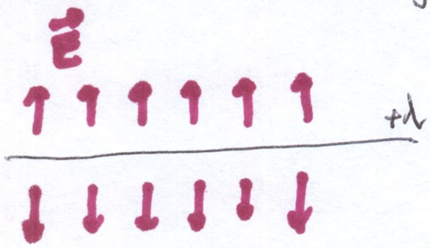
Source → Charge

$$d\vec{E} = \frac{k\,dq}{r^2}\,\hat{r}$$

→ Coulomb's Law or Inverse-square law
(field $\sim \frac{1}{r^2}$)

$\hat{r}$: radial unit vector

→ $\vec{E}$ due to a line of charge



$$E = \frac{2k\lambda}{r}$$

r: separation from line of charge

Magnetic Field

Source → Moving charge or current

$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\,\boxed{d\vec{l} \times \hat{r}}}{r^2}$$

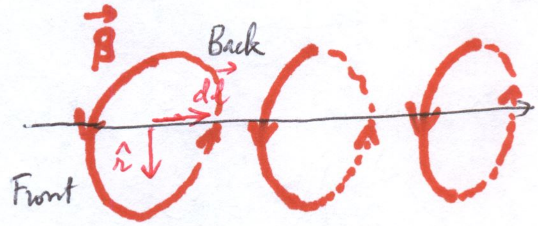
→ Biot-Savart Law
Inverse-square law

$$\mu_0 = 4\pi \times 10^{-7}\,\frac{N}{A^2}$$

permeability in vacuum

→ $d\vec{l} \times \hat{r}$: magnetic field due to a line of current wraps around the current:

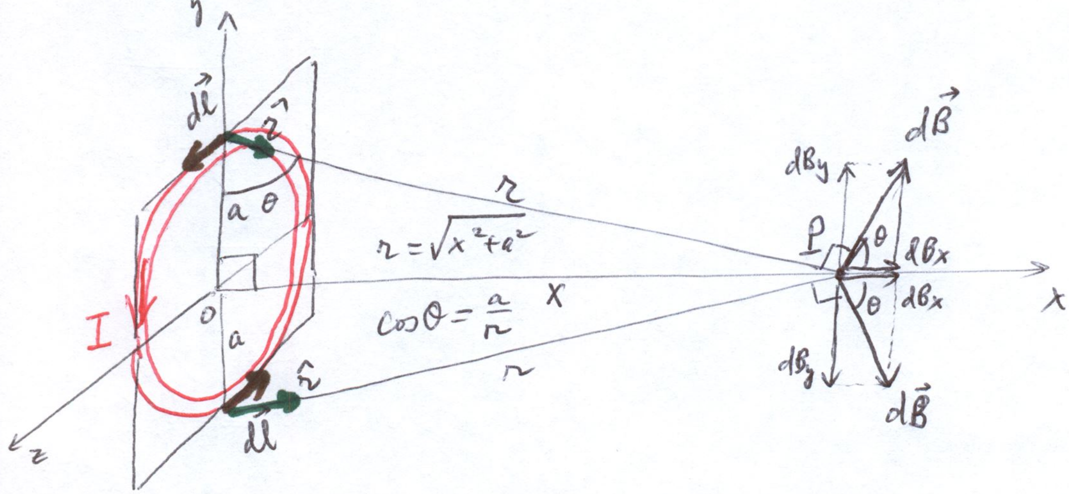
→ $\vec{B}$ due to a line of current.



$\vec{B}$ is related to its source I by RHR: If your RH thumb points along the current I (source), $\vec{B}$ wraps around I like the RH fingers

Magnetic Field due to a Loop of Current :

$\downarrow$ along x-axis $\qquad\qquad$ $\downarrow$ in YZ plane



$d\vec{B}$: field @ P by an element of current $I d\vec{l}$, direction given by $d\vec{l} \times \hat{r}$

dB_y's cancel each other

dB_x's are adding

$$dB_x = dB \cos\theta = \frac{dB . a}{r}$$

→ Magnetic field dB due to top + bottom elements of currents is

$$\begin{cases} d\vec{B} = 2\, dB_x\, \hat{i} = \dfrac{2\, \overset{\to \text{Biot-Savart}}{dB}\, a}{r}\, \hat{i} = 2\, \dfrac{\mu_0}{4\pi} \dfrac{I\, dl}{r^2} \dfrac{a}{r}\, \hat{i} \\ |d\vec{l} \times \hat{r}| = dl \end{cases}$$

$$= \frac{2\mu_0}{4\pi} \frac{Ia}{r^3}\, dl\, \hat{i}$$

→ Total $\vec{B}$ due to whole ring : $\vec{B} = \int_{\text{half ring}} d\vec{B} = \frac{2\mu_0}{4\pi} \frac{Ia}{r^3}\, \hat{i} \underbrace{\int_{\text{Half Ring}} dl}_{\pi a}$

$$\boxed{\vec{B} = \frac{\mu_0}{2} \frac{Ia^2}{r^3}\, \hat{i} = \frac{\mu_0}{2} \frac{Ia^2}{(x^2+a^2)^{3/2}}\, \hat{i}}$$

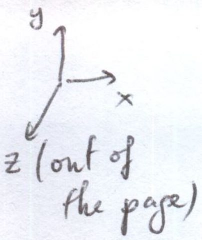
<u>Observation</u>: $\quad x \gg a$ (very far from ring)

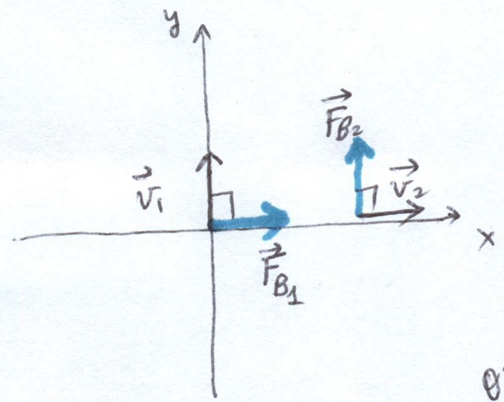
$$x^2 + a^2 \sim x^2 \rightarrow \frac{1}{(x^2+a^2)^{3/2}} \longrightarrow \frac{1}{x^3}$$

$$B \sim \frac{\mu_0 I a^2}{2x^3} \quad \text{(inverse - cube law)}$$

<u>Recall</u>: $E_{Dipole} \sim \frac{1}{x^3}$ (far away from dipole)

$\boxed{26.44}$





Data:
- $\vec{v}_1 = 3.6\times10^{4}\,\frac{m}{s}\,\hat{j}$
- $\vec{F}_{B_1} = 7.4\times10^{-16}\,N\,\hat{i}$
- $\vec{v}_2 = \pm v_2\,\hat{i}$ $\boxed{v_2\,?}$
- $\vec{F}_{B_2} = 2.8\cdot10^{-16}\,N\,\hat{j}$
- $\vec{B}$ is uniform: $\boxed{\vec{B}\,?}$

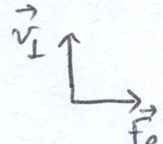
$\theta = 90^\circ$

Magnetic force on a moving charge q and velocity $\vec{v}$:

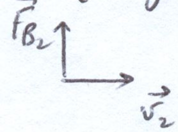
$$\vec{F}_B = q\vec{v}\times\vec{B} = qvB\,\hat{k}$$

cross-product → RHR for direction; magnitude is $qvB\sin\theta$

Discussions: 1)

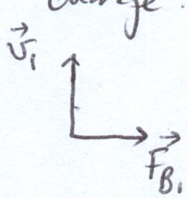


&

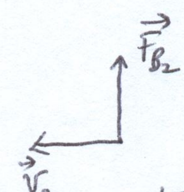


are consistent with a negative charge only and $\vec{B} = -B\hat{k}$

2) Since we have two protons, which carry + charge:



&



with $\vec{B} = B\hat{k}$

Conclusions: $\vec{v} = v_2(-\hat{i})$ & $\boxed{\vec{B} = B\hat{k}}$

angle θ is 90° since $\vec{B}\perp\vec{v}$

proton #1: $7.4\cdot10^{-16} = 1.6\cdot10^{-19}\cdot3.6\cdot10^{4}\cdot B$

$$B = \frac{7.4\cdot10^{-16}}{1.6\cdot10^{-19}\cdot3.6\cdot10^{4}} = \frac{7.4\cdot10^{-1}}{1.6\cdot3.6}\ \mathrm{T}\ \text{(Tesla)}$$

$$B = 0.128\ \mathrm{T} = 128\ \mathrm{mT}$$

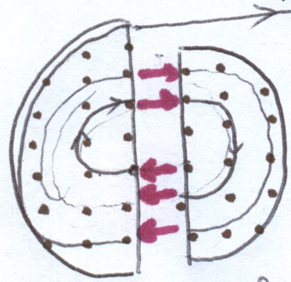
proton #2: $2.8\cdot10^{-16} = 1.6\cdot10^{-19}\,v_2\cdot0.128$

$$v_2 = \frac{2.8\cdot10^{-16}}{1.6\cdot10^{-19}\cdot0.128} = \frac{2.8\cdot10^{3}}{1.6\cdot0.128}\,\frac{m}{s} = 13.7\cdot10^{3}\,\frac{m}{s}$$

26.48 Cyclotron $\begin{cases} d = 0.9\,m \\ B = 2\,T \\ \text{deuterium} = p + n \begin{cases} q = +e \\ m = 2 \cdot 1.67 \cdot 10^{-27}\,kg \end{cases} \end{cases}$

a) what is the alternating frequency for the $\vec{E}$ in the gap

$KE_{max} = \frac{1}{2} m v_{max}^2$



$\vec{E}$ cycle should coincide with deuterium cycle: $T = \frac{2\pi m}{qB}$

$$f = \frac{1}{T} = \frac{qB}{2\pi m} = \frac{1.6 \cdot 10^{-19} \cdot 2}{2\pi \cdot 2 \cdot 1.67 \cdot 10^{-27}} = 1.53 \cdot 10^{7}\,Hz$$

$\begin{cases} T: \text{\# of seconds per cycle} \\ f: \text{\# of cycles per second} \end{cases}$

b) $KE_{max} = \frac{1}{2} m v_{max}^2 \longrightarrow \frac{(qBR)^2}{2m} = \frac{(1.6 \cdot 10^{-19} \cdot 2 \cdot 0.45)^2}{2 \cdot 2 \cdot 1.67 \cdot 10^{-27}}\,J$

$= 3.1 \cdot 10^{-12}\,J$

$r = \frac{mv}{qB} \rightarrow R = \frac{m v_{max}}{qB}$

R ↑ Cyclotron radius

c) $V = 1500\,V$ $\quad V = \frac{U}{q}$ (potential was potential energy per unit charge)

$$\left(E \cdot d = \frac{F \cdot d}{q} \right)$$

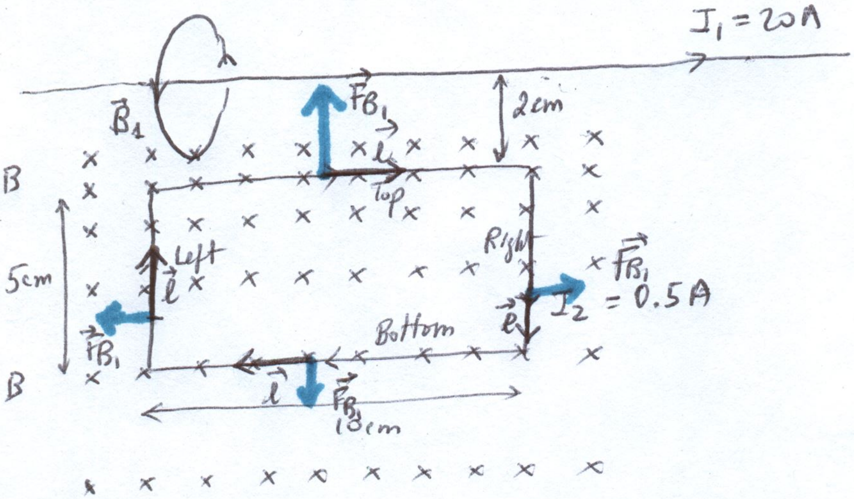
Each crossing, deuterium will receive eV

\# crossings is $\frac{KE_{max}}{eV} = \frac{3.1 \cdot 10^{-12}}{1.6 \cdot 10^{-19} \cdot 1500} = 12934$

\# orbits is 6467 (2 crossings per orbit)

86

26.63



line of current
↓
$\vec{B}_1$ wraps around, direction by RHR: in region below I_1 it goes into the page ×

$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\hat{\theta}$

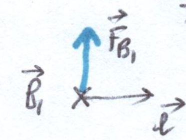
loop is form by 4 straight wires carrying I_2, each in principle will feel the effect of $\vec{B}_1$ as given by:

$$\vec{F}_{B_1} = q_2 \vec{v} \times \vec{B}_1 = q_2 \underbrace{\frac{\vec{l}}{\Delta t}}_{\vec{v}} \times \vec{B}_1 = I_2 \underbrace{\vec{l} \times \vec{B}_1}_{l B_1 \sin 90}$$

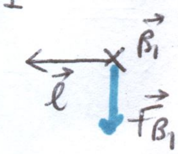
& direction by RHR

→ Direction of $\vec{F}_{B_1}$ by $\vec{l} \times \vec{B}_1$:

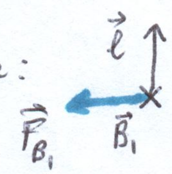
Top wire:



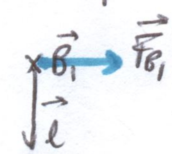
Bottom wire:



Left wire:



Right wire



→ Magnitudes: $F_{B_1 \, Top} > F_{B_1 \, Bottom}$; $F_{B_1 \, Left} = F_{B_1 \, Right}$

y, x

$$\vec{F}_{B_1 \, net} = F_{B_1 \, Top}\hat{j} - F_{B_1 \, Bottom}\hat{j} = \left(I_2 l \frac{\mu_0 I_1}{2\pi \cdot 0.02} - I_2 l \frac{\mu_0 I_1}{2\pi \cdot 0.07} \right)\hat{j}$$

$$= \frac{\mu_0 l I_1 I_2}{2\pi}\left(\frac{1}{0.02} - \frac{1}{0.07}\right)\hat{j} = \frac{\cancel{4\pi}^{2} \cdot 10^{-7} \cdot 0.1 \cdot 20 \cdot 0.5}{\cancel{2\pi}}\left(\frac{1}{0.02} - \frac{1}{0.07}\right)\hat{j}$$

$$= 7.14 \cdot 10^{-6}\, N \,\hat{j}$$ (toward long straight line of current)

Calculation of Fields

Electric

→ Vector addition using Coulomb's law

→ Gauss' Law

$$\underbrace{\oint \vec{E}\cdot d\vec{A}}_{\text{Electric flux } \phi} = \frac{q_{enclosed}}{\varepsilon_0}$$

over a closed Gaussian surface

→ Electric potential V

(Scalar → arithmetic addition

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Magnetic

- Vector addition using Biot-Savart law

→ Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enclosed}$$

over a closed Amperian loop

→ Vector potential $\vec{A}$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

{ Rotational of $\vec{A}$
Curl of $\vec{A}$

Ampere's Law : $\displaystyle\oint_{\substack{\text{Amperian}\\ \text{loop}}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enclosed}$

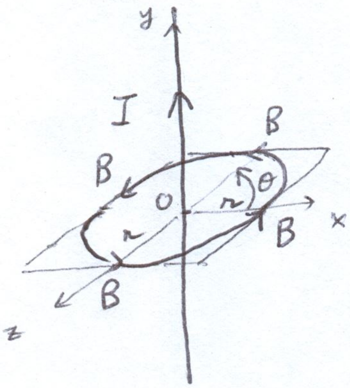
1) Determine the Amperian loop : so $\vec{B}$ would be constant along that loop so it can be factored out.

$$\vec{B}\cdot \underbrace{\oint d\vec{\ell}}_{\text{simple on simple loop}}$$

{ circumference
perimeter

2) Find current enclosed by that loop : $I_{enclosed}$.

Magnetic field $\vec{B}$ due to a long line of current I:



1) Amperian loop is a circle of radius r centered @ current

θ is angle from x-axis on XZ plane

$\rightarrow \hat{\theta}$ goes around loop $\rightarrow \vec{B} = B(r)\,\hat{\theta}$

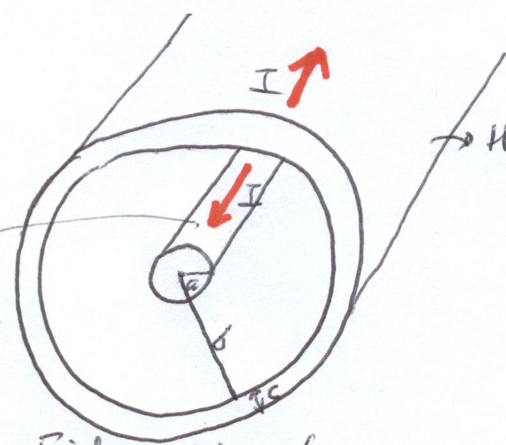
$$\underset{\text{Along Amperian loop}}{\oint \vec{B}\cdot d\vec{l}} = B(r)\cdot \underbrace{\oint dl}_{\substack{\text{Circumference} \\ 2\pi r}}$$

2) Current enclosed by this loop is I

$\rightarrow$ Ampere's Law: $B\cdot 2\pi r = \mu_0 I \rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}}$$

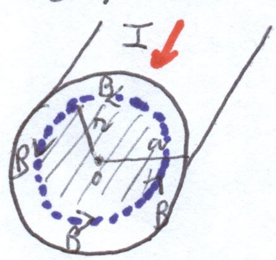
26.66



Hollow outer shell

solid inner conductor

a) Find $B(r)$ when $r < a$ (inside inner conductor):



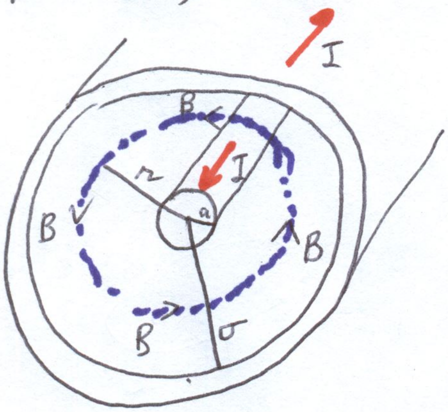
Ampere's Law:

$$B\cdot 2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{a^2} \rightarrow \boxed{B(r) = \frac{\mu_0 I}{2\pi a^2}\,r}$$

I goes through solid inner conductor cross sectional area πa^2

$$I_{enclosed} = \frac{\cancel{\pi} r^2}{\cancel{\pi} a^2}\,I$$

b) Find $B(r)$ when $a < r < b$



Ampere's law:

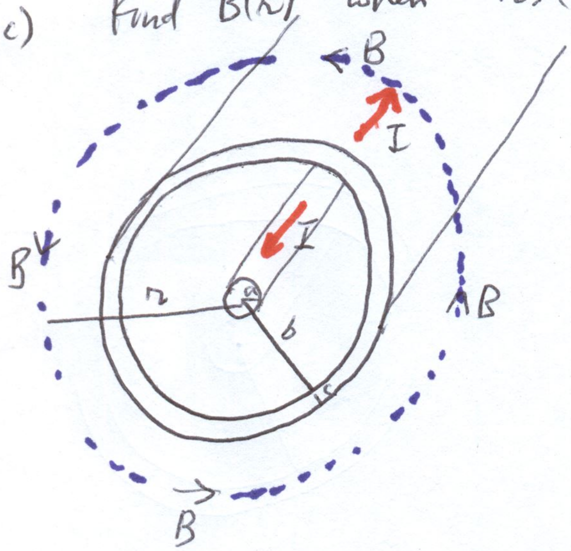
$$\underbrace{B \cdot 2\pi r}_{\text{circumference of Amperian loop}} = \mu_0 I$$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

Field due to a long line of current.

In this region $a < r < b$ outer shell has no effect.

c) Find $B(r)$ when $r > (b+c)$



Ampere's law:

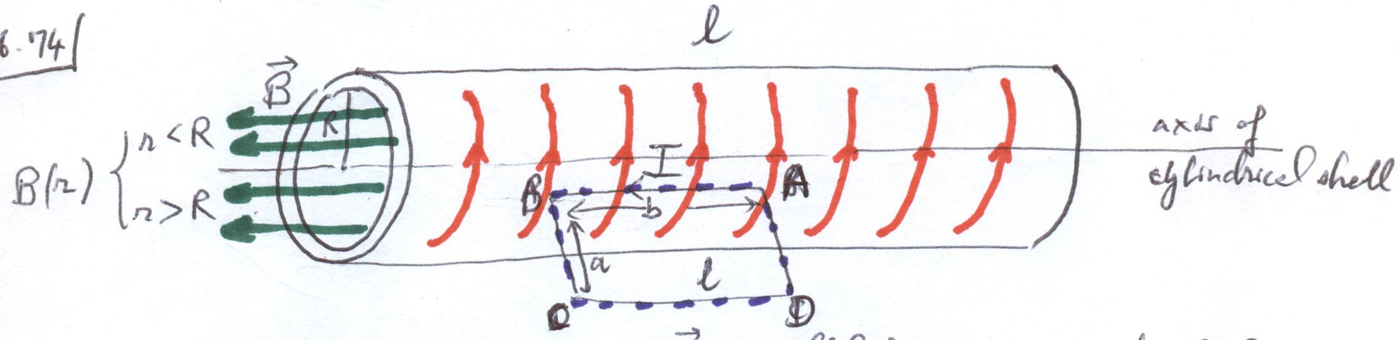
$$B \cdot 2\pi r = \mu_0 \cdot (\underbrace{I - I}_{0})$$

$$B(r) = 0$$

↓

Coaxial cable shields outside environment from the magnetic field due to the inner conductor

26.74



Magnetic field: ring of current $\rightarrow$ $\vec{B}$ parallel to axis given by RHR

1) $B(r)$ $r<R$:

Ampere's Law $\qquad \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

Amperian Loop: ABCD : $\displaystyle\int_{AB} \vec{B}\cdot d\vec{l} + \underbrace{\cancel{\int_{BC} \vec{B}\cdot d\vec{l}}}_{\cos 90=0} + \underbrace{\cancel{\int_{DA} \vec{B}\cdot d\vec{l}}}_{\cos 90^\circ} + \int_{CD} \overset{0}{\vec{B}}\cdot d\vec{l}$

$= B\cdot b$

Current enclosed: $I_{enclosed} = I\dfrac{b}{l} \rightarrow B\cdot \cancel{b} = \mu_0 I \dfrac{\cancel{b}}{l}$

$$\boxed{B = \frac{\mu_0 I}{l}}$$

2) $B(r)=0$ $r>R$:

26.26