Ch 23 Electrostatic Energy & Capacitors

Ch 22 : Electric potential $\Delta V_{12} = \dfrac{\Delta U_{12}}{q'}$ → probe or test charge

U : electric potential energy (J)

V : electric potential ($\frac{J}{C} \equiv \text{Volt} = V$)

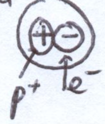
$$\Delta U_{12} = -W_{12} = -\int_1^2 \vec{F} \cdot d\vec{l}$$

↑ electric force

↑

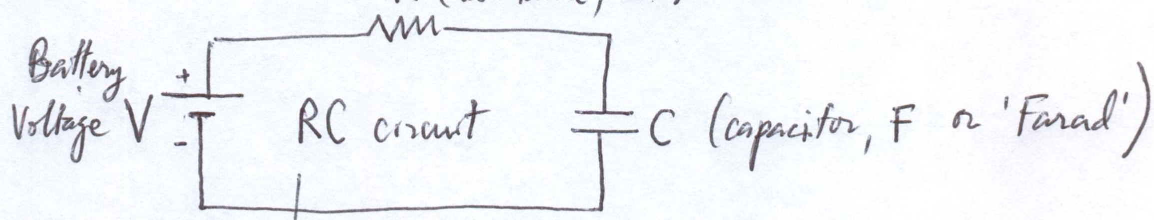
$\boxed{\text{separate these charges}}$ (~ when we pull a spring we store elastic potential energy)

$Q = 0 \rightarrow \vec{E} = 0$

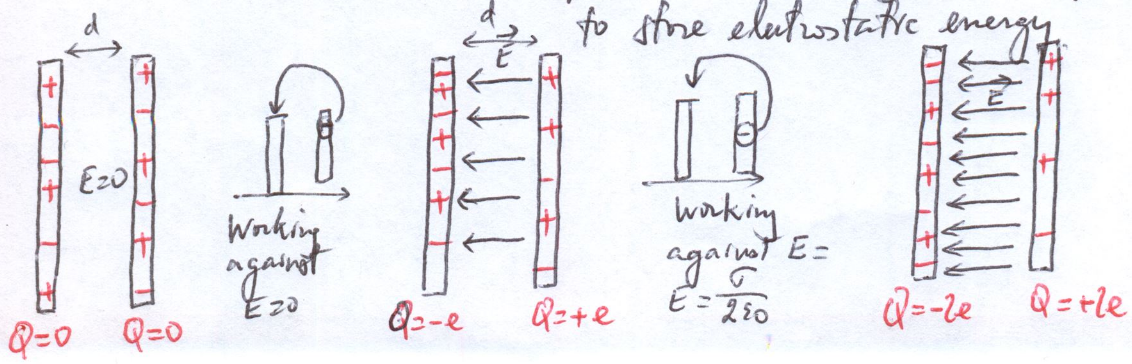


Conclusion: we can store electrostatic (potential) energy by separating charges of opposite types. → Capacitors

→ Parallel Plate Capacitors → symbol ||

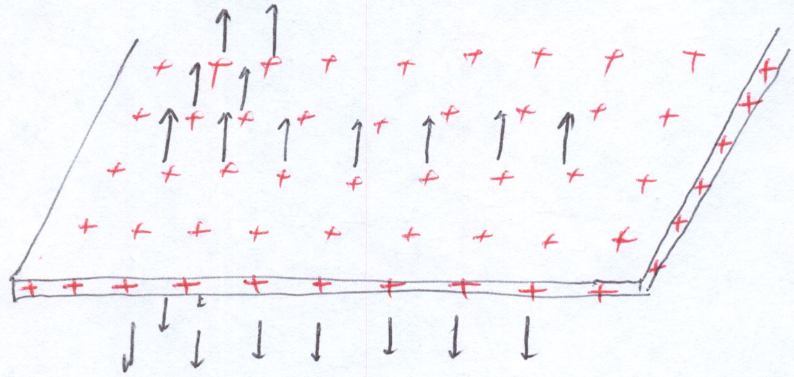


↳ Simplest circuit to charge the capacitor or to store electrostatic energy



Electric Field due to an Infinite Plate or Plane of Charge:

↳ Gauss Law : $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$

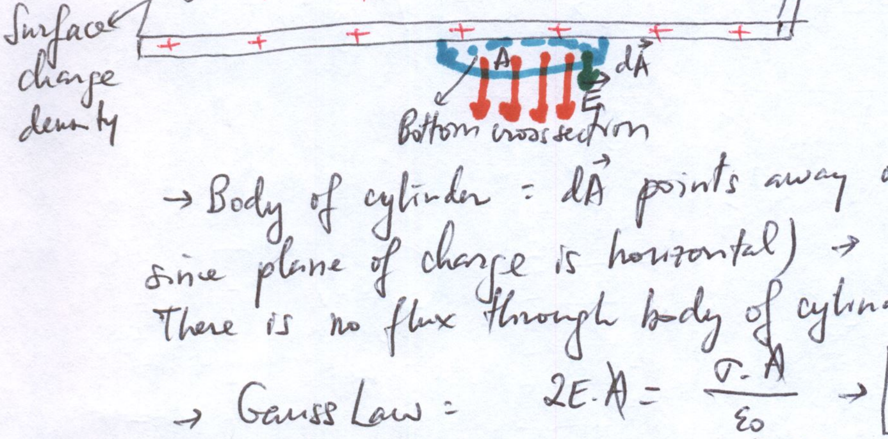


$\vec{E}$ { → By symmetry, points away & perpendicular to the plane of charge
→ Same value @ same separation (above or below) from the plane (∞ plane of charge) }

Gaussian surface: $\phi = \oint_{\boxed{\text{closed}}\ \text{surface}} \vec{E} \cdot d\vec{A}$

{ Rectangular Box / $\boxed{\text{Cylinder}}$ } such that $\vec{E}$ is constant on that surface.

↳ such that top & bottom cross-sections are parallel to plane of charge and at same separations from plane of charge



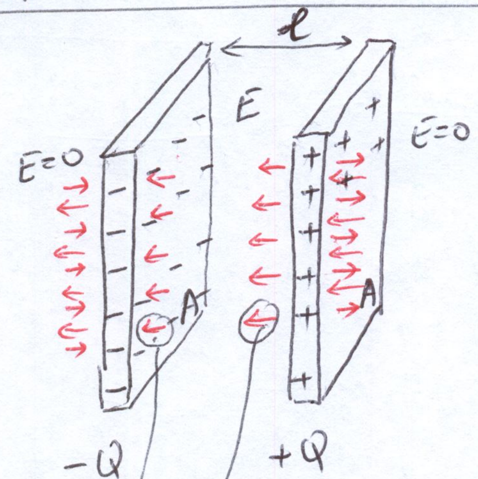
$$\phi = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \underbrace{\oint dA}_{2A} = 2EA$$

→ Body of cylinder : $d\vec{A}$ points away or perpendicular to $\vec{E}$ (vertical since plane of charge is horizontal) → $\vec{E} \cdot d\vec{A} = E\,dA \cos 90^{\circ} = 0$.
There is no flux through body of cylinder!

→ Gauss Law : $2E \cdot A = \dfrac{\sigma \cdot A}{\varepsilon_0}$ → $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Electric field due to an infinite plane of charge is the surface charge density σ divided by $2\varepsilon_0$

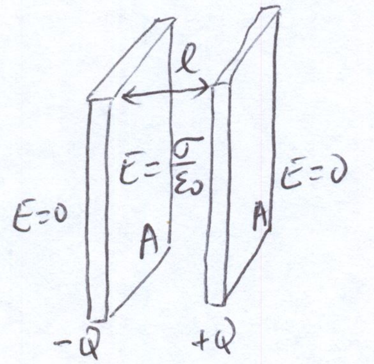
Energy Stored in a Parallel Plate Capacitor:



Assume: (ℓ: separation b/w plates)

$\ell \ll A \Rightarrow \vec{E}$ are very closed to these plates → ∞ plates of charge

$$\underbrace{\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}}_{\boxed{E = \frac{\sigma}{\varepsilon_0}}} \qquad \left(\sigma = \frac{Q}{A}\right)$$



$$V = -\int E \cdot d\vec{l} \overset{\text{E constant b/w plates}}{=}$$

b/w plates: $V = E \cdot \ell$

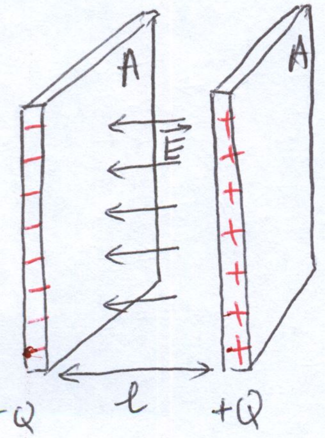
$$dU = -dW = -\underbrace{dq}_{\text{infinitesimal test charge}} \cdot V = dq \frac{\sigma}{\varepsilon_0} \ell = \boxed{dq \frac{q}{A} \frac{\ell}{\varepsilon_0}}$$

$$U = \int dU = \frac{\ell}{A\varepsilon_0} \underbrace{\int_0^Q q\, dq}_{\frac{1}{2}q^2\Big]_0^Q} = \frac{\ell}{A\varepsilon_0} \cdot \frac{1}{2} Q^2 = \frac{\ell A \varepsilon_0}{(A\varepsilon_0)^2} \frac{Q^2}{2}$$

$$\left\{ \begin{array}{l} \frac{Q^2}{(A\varepsilon_0)^2} = \left(\frac{Q}{A\varepsilon_0}\right)^2 = \left(\frac{\sigma}{\varepsilon_0}\right)^2 = E^2 \\ \ell \cdot A = \text{volume b/w the plates} \rightarrow \text{vol} \end{array} \right\} \quad U = \frac{1}{2} \varepsilon_0 E^2 \cdot \text{vol} \ (\text{J})$$

→ Electrostatic energy density: $\boxed{u = \frac{U}{\text{vol}} = \frac{1}{2} \varepsilon_0 E^2}$ $\left(\frac{J}{m^3}\right)$

Capacitance: $C \equiv \dfrac{Q}{V}$ (ratio b/w charge on either plate and the potential difference b/w plates)



$$\boxed{C \equiv \frac{Q}{E \cdot l} = \frac{Q}{\frac{\sigma}{\varepsilon_0} \cdot l} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0} \cdot l} = \frac{A\varepsilon_0}{l}}$$

→ Capacitance for a parallel plate capacitor of cross-sectional area A, separation l, and no dielectric insert b/w plates.

(A dielectric insert will increase the capacitance)

$$\begin{cases} V = -\int \vec{E} \cdot d\vec{l} = E \cdot l \\ l \ll A \rightarrow E = \dfrac{\sigma}{\varepsilon_0} \text{ b/w plates (uniform)} \\ \sigma : \text{surface charge density} \equiv \dfrac{Q}{A} \\ \varepsilon_0 : \text{dielectric constant in vacuum} \\ \quad = \dfrac{1}{4\pi k} = 8.85 \cdot 10^{-12} \dfrac{C^2}{Nm^2} \end{cases}$$

Total energy stored in a capacitor (revisited):

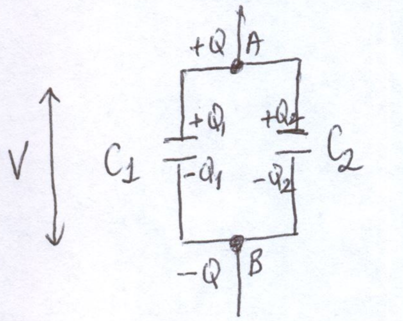
$$U = \frac{1}{2} \varepsilon_0 E^2 \cdot vol = \frac{1}{2} \varepsilon_0 E^2 \cdot \underbrace{A \cdot l}_{vol} \cdot \frac{l}{l} = \frac{1}{2} \underbrace{\frac{A\varepsilon_0}{l}}_{C} \underbrace{E^2 l^2}_{V^2} = \frac{1}{2} C V^2$$

(In Ch. 27: magnetic inductance L → total magnetic energy stored in an inductor $U = \frac{1}{2} L I^2$)

$$\begin{cases} \text{Kinetic energy} = \frac{1}{2} m v^2 & m \text{ is inertia to changes in } v \\ & \text{(the larger the mass, the harder to change the speed of that object)} \\ \text{Electric energy} = \frac{1}{2} C V^2 & \rightarrow C \text{ is an inertia to changes in } V \\ \text{Magnetic energy} = \frac{1}{2} L I^2 & \rightarrow L \text{ is an inertia to changes in } I \end{cases}$$

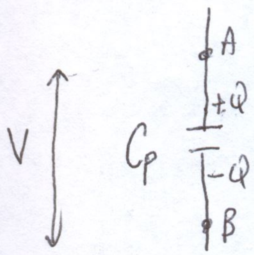
Connecting 2 Capacitors

Parallel (same potential difference V across each)



→ Statements: 1) $Q = Q_1 + Q_2$

2) Equivalent capacitor b/w A & B is delivering same potential & charges

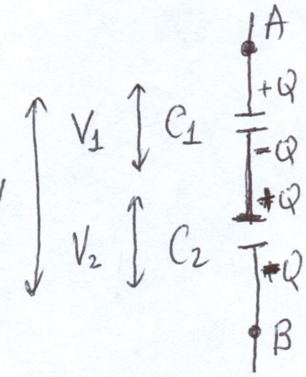


3) $C_1 = \frac{Q_1}{V}$; $C_2 = \frac{Q_2}{V}$

$$\boxed{C_p = \frac{Q}{V} \overset{1)}{=} \frac{Q_1 + Q_2}{V} = C_1 + C_2}$$

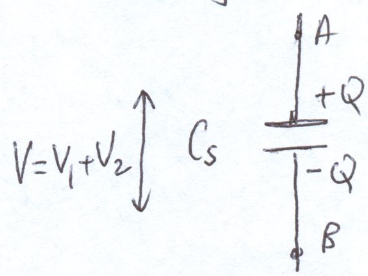
Placing capacitors in parallel connections will increase the capacitance

Series (same charge in each capacitor)



→ Statements: 1) $V_1 \neq V_2$

2) Equivalent capacitor b/w A & B delivering same potential & charges



3) $C_1 = \frac{Q}{V_1}$

$C_2 = \frac{Q}{V_2}$

$$C_s = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{1}{\frac{V_1}{Q} + \frac{V_2}{Q}}$$

$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

or $\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$

or $\boxed{C_s = \frac{1}{\frac{C_1 + C_2}{C_1 \cdot C_2}} = \frac{C_1 \cdot C_2}{C_1 + C_2}}$

If $C_1 = C_2 = C \rightarrow C_s = \frac{C \cdot \cancel{C}}{2\cancel{C}} = \frac{C}{2}$

How to increase the capacitance:

1) Connecting multiple capacitors in parallel
2) Inserting dielectric material b/w the plates:

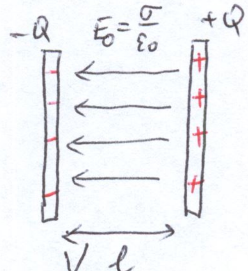
$$C = \frac{A}{l}\varepsilon = \frac{A}{l}K\varepsilon_0 = KC_0$$

a) $C_0 = \frac{A}{l}\varepsilon_0 \longrightarrow$ $\varepsilon_0 \longrightarrow \varepsilon = K \cdot \varepsilon_0$

ε_0: vacuum

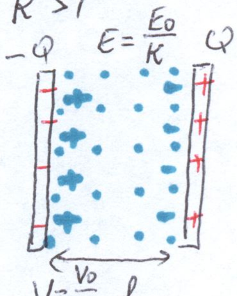
K: kappa, the dielectric coefficient. $K > 1$

b)



Vacuum ε_0

$$C_0 = \frac{Q}{V_0}$$

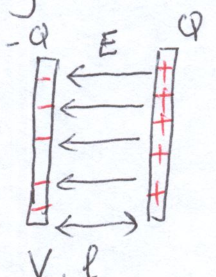


dielectric fill $\varepsilon = K\varepsilon_0$

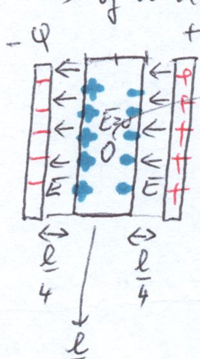
(whose e^- go ~~to the~~ toward the right plate

$$C = \frac{Q}{V} = \frac{Q}{\frac{V_0}{K}} = K\frac{Q}{V_0} = KC_0$$

3) Inserting a conducting slab b/w plates of width $< l$



$V_0 = E \cdot l$; $C_0 = \frac{Q}{V_0}$



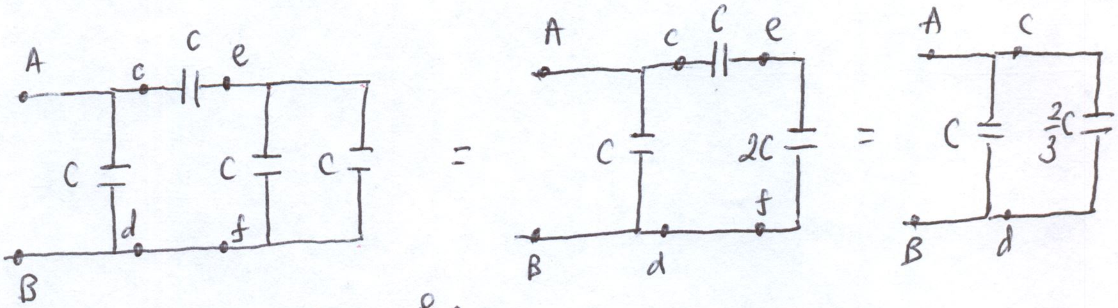
→ Conducting slab of width $\frac{l}{2}$

→ Electric field inside a conductor → charge rearrangement → same value as E but pointing in opposite direction

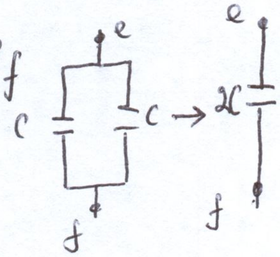
$$V = E \cdot \frac{l}{2} = \frac{V_0}{2}$$

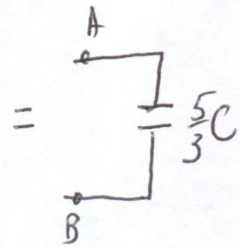
$$C = \frac{Q}{V} = \frac{Q}{\frac{V_0}{2}} = 2\frac{Q}{V_0} = 2C_0$$

23.49

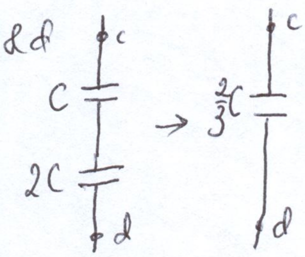


Between e & f



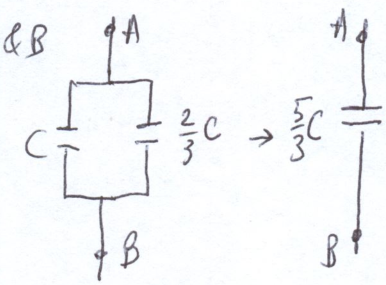


Between c & d



$$C_s = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{C \cdot 2C}{C + 2C} = \frac{2C^{\cancel{2}}}{3\cancel{C}}$$

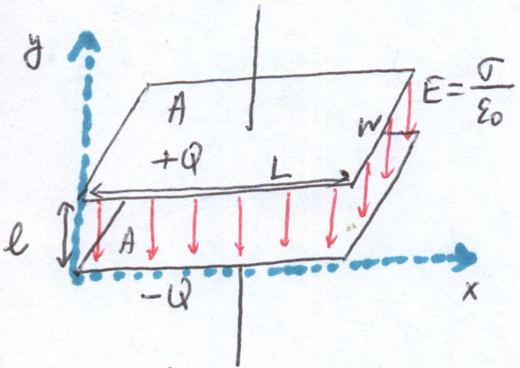
Between A & B



$$C_p = C_1 + C_2 = C + \frac{2}{3}C = \frac{5}{3}C$$

23.69]

Before (no dielectric insert)

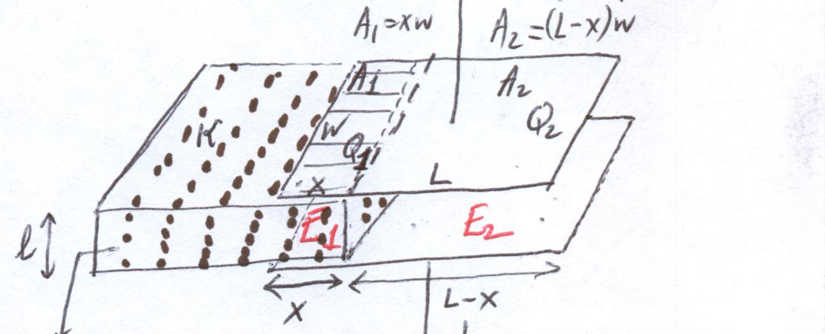


$C_0 = \frac{Q}{V_0}$

$U_0 = \frac{1}{2} C V_0^2$

After

$A = A_1 + A_2$
$Q = Q + Q_2$



dielectric slab insert

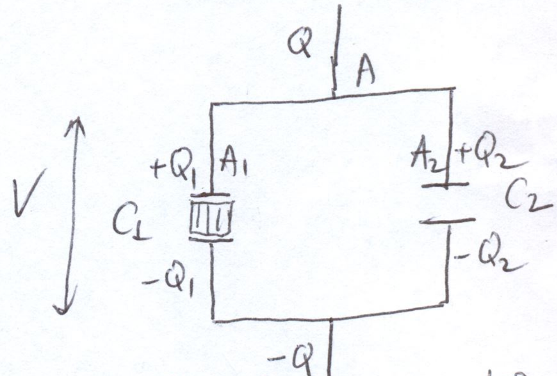
$\varepsilon = K \cdot \varepsilon_0$
↓
dielectric constant > 1

↓ with dielectric

↓ without dielectric

New capacitance C:
↓
Parallel connection of the shaded and unshaded: $C_p = C_1 + C_2$

Statement:



Would like to calculate $C = C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V} \rightarrow$ Need $\begin{cases} Q_1 \\ Q_2 \\ V \end{cases}$

Statements:

Parallel plate capacitor $\left\{ \begin{array}{l} 1) \quad E_1 = \frac{\sigma_1}{K\varepsilon_0} = \frac{\frac{Q_1}{A_1}}{K\varepsilon_0} = \frac{Q_1}{xwK\varepsilon_0} \\ 2) \quad E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{\frac{Q_2}{A_2}}{\varepsilon_0} = \frac{Q_2}{(L-x)w\varepsilon_0} \end{array} \right.$

Fronts of the dielectric ↑ x

3) Field continuity requires $E_1(x) = E_2(x)$ { at one same location there can't be two different electric fields

or field equality at $x \rightarrow \frac{Q_1}{x\cancel{w}K\cancel{\varepsilon_0}} = \frac{Q_2}{(L-x)\cancel{w}\cancel{\varepsilon_0}}$

$$\boxed{Q_1 = Q_2 \frac{Kx}{L-x}}$$

Q_1 is a fraction of Q_2 depending on how far the dielectric is inserted (x)

4) Total capacitance with dielectric inserted a distance x into spacing

$$C = \frac{Q_1 + Q_2}{V} = \frac{Q_2 \frac{Kx}{L-x} + Q_2}{V} = \frac{Q_2}{V}\left(\frac{Kx}{L-x} + 1\right)$$

5) $$\frac{Q_2}{V} = \frac{Q_2}{E_2 \cdot l} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{(L-x)w\varepsilon_0} \cdot l} = \frac{(L-x)w\varepsilon_0}{l}$$

6) $$C = \frac{(L-x)w\varepsilon_0}{l} \cdot \left(\frac{Kx}{L-x} + 1\right) = \frac{w\varepsilon_0}{l}\left(Kx + L - x\right)$$

$$C(x) = \frac{w\varepsilon_0}{l}\left[\underbrace{x(K-1) + L}_{\text{linear function of } x}\right]$$

→ Total capacitance increases linearly as the dielectric is pushed into the spacing

a) Capacitance if $x = \frac{L}{2}$:

$$C\left(x = \frac{L}{2}\right) = \frac{w\varepsilon_0}{l}\left[\frac{L}{2}(K-1) + L\right] = \frac{w\varepsilon_0 L}{2l}\left[K - 1 + 2\right]$$

$$= \frac{w\varepsilon_0 L}{2l}(K+1)$$

b) Total energy stored with dielectric pushed in x into spacing

$$U(x) = \frac{1}{2}CV^2 = \frac{1}{2}\underbrace{\frac{\cancel{w\varepsilon_0}}{\cancel{l}}\left[\cancel{x(K-1)+L}\right]}_{C(x)} \cdot \frac{Q^2 \cdot l^2}{(\varepsilon_0 w)^2 (Kx + L - x)^2}$$

$$V(x) = \begin{cases} E_1(x) l \\ \text{or} \\ E_2(x) l \end{cases} \longrightarrow V(x) = \frac{Q_1}{xwK\varepsilon_0} \cdot l$$

$E_1(x) = E_2(x)$

<u>Statement</u>: 7) $Q = Q_1 + Q_2 \longrightarrow Q = Q_1 + Q_1 \frac{L-x}{Kx} = Q_1\left(1 + \frac{L-x}{Kx}\right)$

$Q_1 \overset{3)}{=} Q_2 \frac{Kx}{L-x} \rightarrow Q_2 = Q_1 \frac{L-x}{Kx}$

$$Q_1 = \frac{Q}{1 + \frac{L-x}{Kx}} = \frac{Q}{\frac{Kx + L - x}{Kx}}$$

$$\boxed{Q_1 = \frac{Q\,Kx}{Kx + L - x}}$$

8) $V(x) = \frac{Q\,\cancel{Kx}}{Kx+L-x} \cdot \frac{l}{\cancel{K}w\cancel{K}\varepsilon_0} = \frac{Q\,l}{\varepsilon_0 w(Kx+L-x)}$

$$\boxed{U(x) = \frac{1}{2}C(x)V^2(x) = \frac{1}{2}\frac{Q^2 l}{\varepsilon_0 w(Kx+L-x)}}$$

9) Total energy stored w/o dielectric

$$\boxed{U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\cancel{C_0}\frac{Q^2}{C_0^{\cancel{2}}} = \frac{Q^2}{2C_0} = \frac{Q^2}{2\frac{\varepsilon_0 Lw}{l}} = \frac{Q^2 l}{2Lw\varepsilon_0}}$$

$C_0 = \frac{Q}{V_0}$ $\quad V_0 = \frac{Q}{C_0}$

$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{\frac{Q}{A}}{\varepsilon_0} \Rightarrow Q = \frac{\varepsilon_0 E_0 A}{\cancel{A}}$

$= \frac{Q}{A\varepsilon_0}$

$$C_0 = \frac{Q}{V_0} = \frac{\varepsilon_0 \cancel{E_0} A}{\cancel{E_0}\cdot l} = \boxed{\frac{\varepsilon_0 A}{l}} = \frac{\varepsilon_0 \cdot L \cdot w}{l}$$

$$U(x) = \underbrace{\boxed{\frac{1}{2}\frac{Q^2 l}{L\varepsilon_0 w}}}_{U_0} \frac{L}{(Kx+L-x)} = U_0\frac{L}{Kx+L-x}$$

$$x = \frac{L}{2} \rightarrow U\left(x=\frac{L}{2}\right) = U_0\frac{L}{K\frac{L}{2}+L-\frac{L}{2}} = U_0\frac{1}{\frac{K-1}{2}+1} = U_0\frac{2}{K+1}$$

$$a\ U\left(x=\frac{L}{2}\right) = \frac{C_0V_0^2}{K+1}$$

c) <u>Force on slab?</u> $\quad F = -\frac{dU}{dx}$ $\quad \left(\vec{E} = -\frac{dV}{dx} \rightarrow qE = -\frac{d(qV)}{dx}\right)$

$$U = U_0\frac{L}{(K-1)x+L}$$

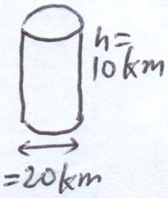
$$F = -\frac{dU}{dx} = -U_0L\frac{d}{dx}\left[(K-1)x+L\right]^{-1} = U_0L\frac{K-1}{[(K-1)x+L]^2} \rightarrow F\left(x=\frac{L}{2}\right) = \frac{U_0L(K-1)}{\left[(K-1)\frac{L}{2}+L\right]^2}$$

$$= \frac{4U_0(K-1)}{L(K+1)^2} = \frac{2C_0V_0^2(K-1)}{L(K+1)^2}$$

23.65| Lightning flash every 5 s $\begin{cases} Q = 30\,C \\ V = 30 \cdot 10^6\,V \end{cases}$ $(M \equiv 10^6)$

→ How long will it last? Total energy stored: 140 GJ

(Example 23.4: $\begin{cases} E = 10^5 \frac{V}{m} \text{ (electric field strength)} \\ \text{Thundercloud volume} \end{cases}$



$$Vol = \pi R^2 h = \pi \cdot (10^4)^2 \cdot 10^4 = \pi 10^{12}\,m^3$$

$$u = \frac{1}{2}\varepsilon_0 E^2 \rightarrow U = u \cdot vol = \frac{1}{2} \cdot 8.85 \cdot 10^{-12} \cdot 10^{10} \cdot \pi \cdot 10^{12} = \underbrace{\frac{8.85\pi}{2}}_{14} \cdot 10^{10}\,J$$

$\rightarrow U = 140 \cdot 10^9\,J = 140\,GJ.$)

$U = 140\,GJ$ (electric energy stored in the clouds) → how many flashes if each flash spends $U_F = q \cdot V = 30 \cdot 30 \cdot 10^6 = 9 \cdot 10^8\,J = 0.9\,GJ$.

$$\#\text{Flashes} = \frac{140\,GJ}{0.9\,\frac{GJ}{flash}} = 156 \text{ flashes}$$

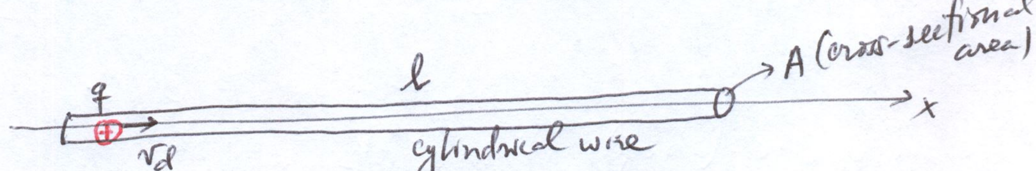
→ Storm will last: $156 \text{ flashes} \times \frac{5\cancel{s}}{flash} \times \frac{1 min}{60\cancel{s}} \simeq 13\,min$

Ch 24 Electric Current (I $\begin{cases} \frac{\Delta q}{\Delta t} & \text{(average)} \\ \text{or } \frac{dq}{dt} & \text{(instantaneous)} \end{cases}$

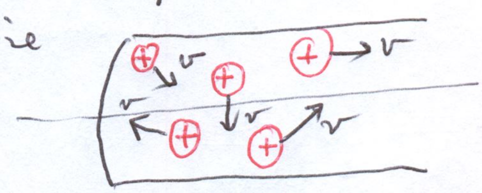
Unit in SI: $\frac{C}{s}$ or A (Amp)

<u>Current</u>: describes motion of charges, a macroscopic quantity that arises from microscopic behavior of charges:

Electrical wire (copper, aluminum)



v_d: drift velocity = average velocity along wire axis
actual velocity v can point in any direction
closer look at the wire



v: very large
v_d: normally small
(average over quasi-random distribution of actual velocities)
n: number of charge per unit volume $\left(\frac{N}{vol} = n\right)$
A: cross-sectional area of wire

Average Current $\underset{\text{macroscopic}}{I} = \frac{\Delta q}{\Delta t} = \frac{n \cdot A \cdot \ell \cdot q}{\frac{\ell}{v_d}} = \underbrace{nqAv_d}_{\text{microscopic}}$

Let's calculate the drift velocity v_d for charges in a copper wire ($A = 1mm^2$; $I = 5A$; each atom of copper Cu contributes 1.3e charge)

$$v_d = \frac{I}{nqA} \quad \begin{cases} n: \text{number \# charge density} \xrightarrow{\text{in Cu}} \begin{cases} \text{mass density of copper} \\ \rho = 8920 \frac{kg}{m^3} \\ \text{periodic table: mass of one Cu atom: } 63.55 \text{ a.u} \\ \downarrow \times 1.66 \cdot 10^{-27} \frac{kg}{a.u} \\ m_{Cu} = 63.55 \cdot 1.66 \cdot 10^{-27} kg \end{cases} \\ q = 1.3e = 1.3 \cdot 1.6 \cdot 10^{-19} C \\ A = 1mm^2 = 10^{-6} m^2 \end{cases}$$

$$\begin{matrix}\text{\# Cu Atoms} \\ \text{per unit} \\ \text{volume}\end{matrix} = \frac{\rho}{m_{Cu}} = \frac{8920 \frac{kg}{m^3}}{63.55 \cdot 1.6 \cdot 10^{-27} kg} = 8.5 \cdot 10^{28} \frac{\text{Cu Atoms}}{m^3}$$

Since each atom of copper contributes a charge $q = 1.3e$, this also the number of charge per unit volume n.

$$v_d = \frac{I}{nqA} = \frac{5}{8.5 \cdot 10^{28} \cdot 1.3 \cdot 1.6 \cdot 10^{-19} \cdot 10^{-6}} = 0.283 \frac{mm}{s}$$

Drift velocity for charges in the copper wire

Actual velocity: brute approx using the ideal gas model for these charges: $KE_{average} = d.o.f. \times \frac{1}{2} kT$

(equipartition theorem)

$$\frac{1}{2} \underset{\substack{\uparrow \\ \text{mass of} \\ \text{one charge}}}{m} v^2 = \frac{3}{2} kT$$

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 298.16}{9.11 \cdot 10^{-31}}}$$

$$\sim 10^3 \frac{m}{s} \gg v_d.$$

Ohm's Law : $\begin{cases} I = \dfrac{V}{R} & \text{or current is the voltage divided by the resistance } (\Omega \text{ or Ohm}) \\ \vec{J} = \sigma \vec{E} & (\sigma\text{: conductivity; } \vec{E} \text{ electric field}) \end{cases}$

→ Resistance R = depending on the material of the wire, its length, and the cross-sectional area of the wire

$$R = \rho \cdot \frac{l}{A}$$ (longer wire → larger cross-sectional area for a same resistance R)

ρ ↓ resistivity of material (low in metals → copper wire)

Power : $P = I \cdot V$ $\begin{cases} \dfrac{V^2}{R} \\ IR^2 \end{cases}$

I ↓ current, V ↓ voltage

Power ↓ Electrical power

Current in a wire → feels warm as a current is pushed through: charges colliding with the wire transferring their KE in form of heat: "dissipation", it's a loss. P can be related to the heat loss per unit time.

Current density : $J \equiv \dfrac{I}{\text{Area}}$ $\left(\dfrac{A}{m^2}\right)$

24.43

$I_{max} = 20A$

Cylindrical wires : $A = \pi \cdot \left(\frac{2.1 \cdot 10^{-3}}{2}\right)^2 m^2$

a) Current density $J = \frac{20\,A}{\pi \cdot (1.05 \cdot 10^{-3})^2 m^2} =$

b) Microscopic Ohm's law : $J = \sigma E \rightarrow E = \frac{J}{\sigma}$

$\sigma_{Cu} = \frac{1}{\rho_{Cu}} \rightarrow E = \rho_{Cu} J = 1.68 \cdot 10^{-8}\,\Omega \cdot m \cdot \frac{20A}{\pi (1.05 \cdot 10^{-3})^2 m^2}$

$= (\qquad) \frac{V}{m}$

conductivity is the inverse of the resistivity → Table 24.1: $\rho_{Cu} = 1.68 \cdot 10^{-8}\,\Omega \cdot m$

$\frac{A \cdot \Omega}{m} = \frac{\overbrace{C \cdot \Omega}^{V}}{s \cdot m} = \frac{V}{m}$

24.60

Similar to the previous example but now using an aluminum wire

- 12-gauge wire → $A = \pi \cdot (1.05 \cdot 10^{-3})^2$
- $I = 20A$
- Each atom of Al contributes a charge $q = 3.5e = 3.5 \cdot 1.6 \cdot 10^{-19} C$

$v_d = \frac{I}{n q A}$ → need to find the # of charge per unit volume n, which is also the # atoms of Al per unit volume

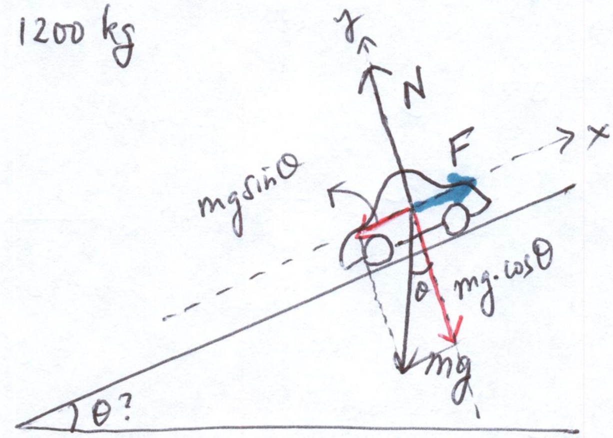
$$n = \frac{\rho_{AL}}{m_{AL}} = \frac{2702 \frac{kg}{m^3}}{\underbrace{26.98}_{a.u.} \cdot 1.66 \cdot 10^{-27} kg}$$

$$\rightarrow v_d = \frac{20}{\frac{2702}{26.98 \cdot 1.66 \cdot 10^{-27}} \cdot 3.5 \cdot 1.6 \cdot 10^{-19} \cdot \pi (1.05 \cdot 10^{-3})^2} = 0.171 \frac{mm}{s}$$

(same order of magnitude as that for Cu)

24.65 How steep a slope (θ) a hybrid car can go up @ $v = 60 \frac{km}{h}$ (ignoring friction) using <u>only</u> electrical power (battery provides 360V @ 180A max.)?

$m_{car} = 1200$ kg



Statements

1) Car is pulled downhill by $mg\sin\theta$
2) To bring it uphill @ constant speed ($a=0$) it needs F

$F - mg\sin\theta = m \cdot a = 0$

$F = mg\sin\theta$

$$\Rightarrow P = \text{Power} \begin{cases} \dfrac{\text{Work}}{\Delta t} = \dfrac{F \cdot \Delta x}{\Delta t} = F \cdot v = mg\sin\theta \cdot v \\ \text{Electrical: } \dfrac{U}{\Delta t} = \dfrac{q \cdot V}{\Delta t} = I \cdot V \end{cases}$$

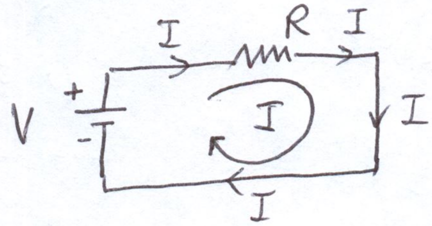
$$\underbrace{IV}_{max} = mg\sin\theta \cdot v \rightarrow \sin\theta \leq \frac{IV}{mg \cdot v}$$

$$\rightarrow \theta \leq \sin^{-1}\left(\frac{I \cdot V}{m \cdot g \cdot v}\right) = \sin^{-1}\left(\frac{180 \cdot 360}{1200 \cdot 9.81 \cdot 16.6667}\right) = 0.3366 \text{ rad} \xrightarrow{\times \frac{180}{\pi}} \boxed{19.29^\circ}$$

$$v = 60 \frac{\cancel{km}}{\cancel{h}} \cdot \frac{10^3 m}{\cancel{km}} \cdot \frac{1\cancel{h}}{3600 s} = \frac{60}{3.6} \frac{m}{s} = 16.6667 \frac{m}{s}$$

Ch 25 Electrical Circuits:

Circuits: batteries & resistors (later will add capacitor, inductor)

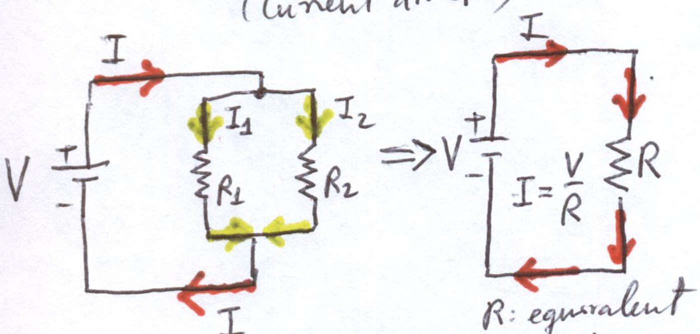
Simplest: 1) 

$I = \frac{V}{R}$ (Ohm's Law)

linear circuits follow Ohm's Law

2) One battery & two resistors:

Parallel Connection
(Current division)



R: equivalent resistance for R_1 & R_2

1) $\boxed{I = I_1 + I_2}$ Current division

If we replace R_1 & R_2 by R, battery can't tell the difference as it draws the same current I out of the battery

2) Same voltage V is applied to R_1 & R_2

$$\left\{\begin{array}{l} I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ I = \frac{V}{R} \end{array}\right\} \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

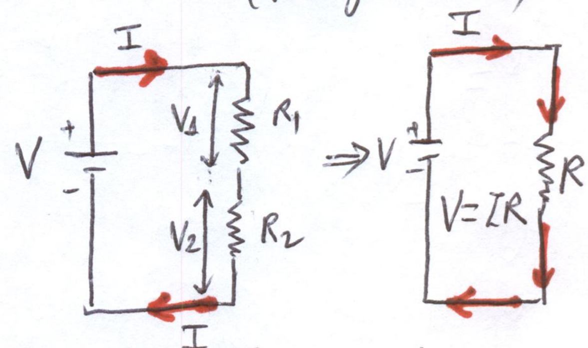
$$\boxed{R = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

Current Division:

$$\boxed{I_1 = \frac{V}{R_1} = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) R_1} = I \frac{1}{\frac{R_1 + R_2}{\cancel{R_1} \cdot R_2} \cancel{R_1}} = I \cdot \frac{R_2}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{V}{R_2} = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) R_2} = I \frac{1}{\frac{R_1 + R_2}{R_1 \cdot \cancel{R_2}} \cancel{R_2}} = I \cdot \frac{R_1}{R_1 + R_2}}$$

Series Connection
(Voltage division)



1) $\boxed{V = V_1 + V_2}$ Voltage division

~~$V = IR_1$~~

2) Same current I goes through R_1 & R_2

$$\left.\begin{array}{l} V = IR_1 + IR_2 = I(R_1 + R_2) \\ V = IR \end{array}\right\} \boxed{R = R_1 + R_2}$$

Voltage Division:

$$\boxed{\begin{array}{l} V_1 = I \cdot R_1 = \frac{V}{R_1 + R_2} \cdot R_1 = V \cdot \frac{R_1}{R_1 + R_2} \\ V_2 = I \cdot R_2 = V \frac{R_2}{R_1 + R_2} \end{array}}$$