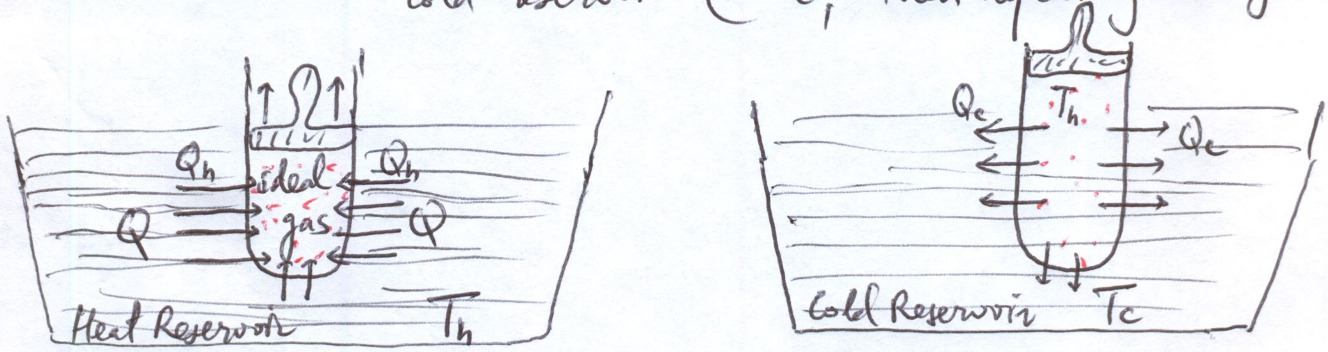
Ch 19 2nd Law of Thermodynamics

Heat Reservoir: Source of heat, at constant temperature

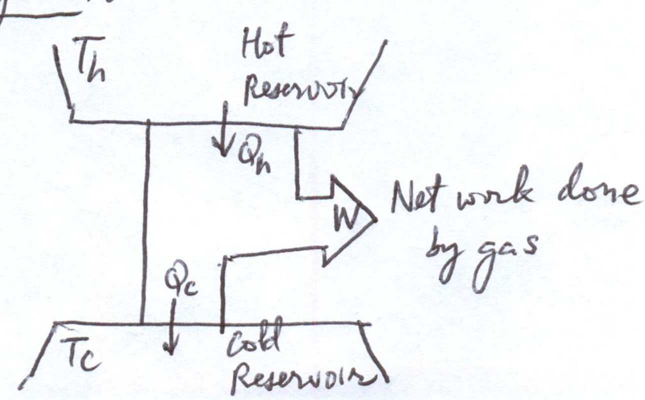
↳ **Heat Engine**: placing an ideal gas in a cylinder/piston in thermal contact with a hot heat reservoir @ T_h, then in thermal contact with a cold reservoir @ T_c, then repeating this cycle.



→ Heat transfer in, increasing KE of gas molecules → increasing its temperature & pressure, gas expands piston is pushed up, work done by gas W is positive. This continues until gas reaches TD equilibrium with reservoir @ T_h

→ Heat transfer out, decreasing KE of gas molecules, decreasing its temperature & pressure, gas compresses, piston comes back down, work done by gas W is negative. This continues until gas reaches TD equilibrium with cold reservoir @ T_c

Heat Engine Diagram:



Efficiency of Heat Engine:

1) 1st Law of T.D: $\Delta U_{\text{Heat Engine}} = Q - W$ (Heat absorbed minus work done by gas)

$$= Q_h - Q_c - W$$

2) Heat reservoir → Heat transfers at constant temperature or isothermal processes → ideal gas $\begin{cases} \text{Monoatomic}: \Delta U = \frac{3}{2} k \Delta T N \\ \text{Diatomic}: \Delta U = \frac{5}{2} k \Delta T N \end{cases}$

$\xrightarrow{\Delta T = 0} \Delta U = 0 \Rightarrow 0 = Q_h - Q_c - W$ or $\boxed{Q_h - Q_c = W}$

3) Efficiency of a heat engine using ideal gas $\left(\frac{1}{2} m \overline{v^2} = dof \times \frac{1}{2} kT\right)$

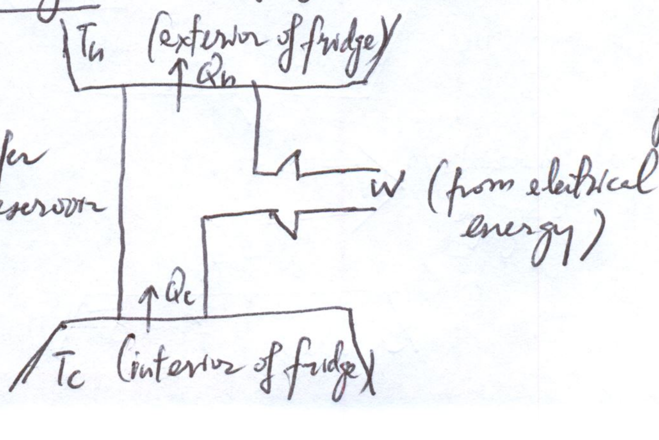
$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \underbrace{\frac{Q_c}{Q_h}}_{(e<1)} \rightarrow 1 - \frac{|Q_c|}{|Q_h|}$$

$$|Q_c| < |Q_h| \rightarrow \frac{|Q_c|}{|Q_h|} < 1 \rightarrow e < 1$$

2nd Law of TD: it is impossible to build a heat engine working in cycles that extracts heat from a hot reservoir (and returning some of it to a cold reservoir) that can deliver 100% of work (e < 1)

Reversed Heat Engine: Refrigerators:

2nd Law of TD: It is impossible to transfer heat from a cold reservoir to a hot reservoir without requiring any work W



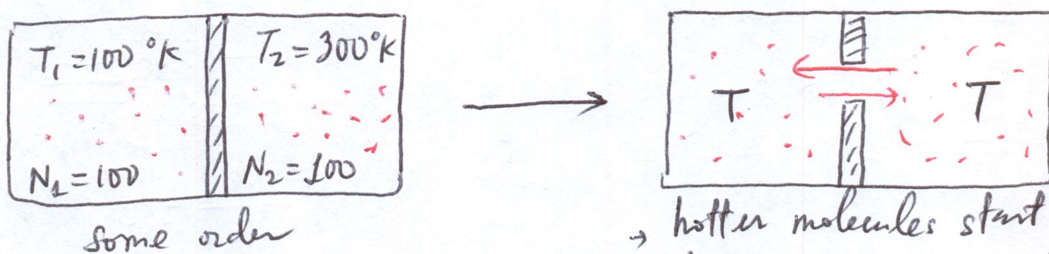
Coefficient of performance: $C.O.P = \frac{Q_c}{W}$

3rd Law of TD:

Entropy: $\Delta S \equiv \int_1^2 \frac{dQ}{T}$ (~ degree of dis-order)

→ The entropy of a closed system (without external assistance) can never decrease or $\Delta S \geq 0$

Example: two gases @ different temperatures once mixed up would never separate back to their original temperatures



→ hotter molecules start to go left, then some of the cold molecules get pushed to right, they will arrive @ same final temperature T.

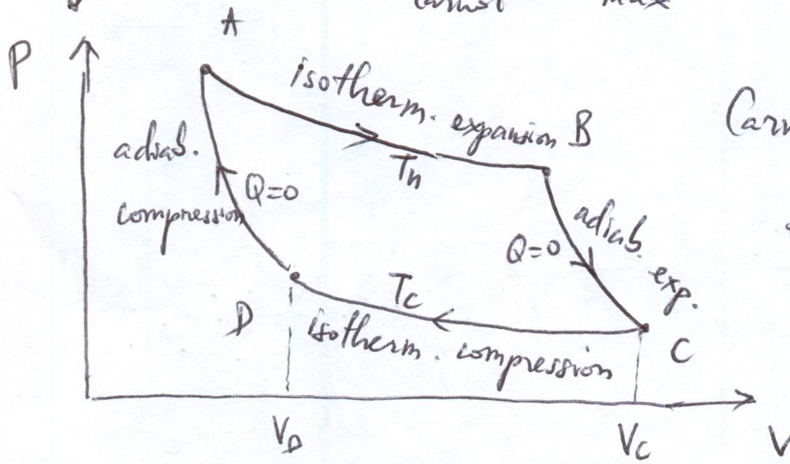
→ higher level of disorder

Carnot Engines: special type of Heat Engines that follow

→ four reversible processes (two isothermal & two adiabatic)

→ efficiency for Carnot Engines e_{Carnot} is the maximum achievable efficiency for Heat Engines

$e_{carnot} = e_{max}$.



Carnot cycle: $A \to B \to C \to D \to A$

$$e_{max} = 1 - \frac{|Q_c|}{|Q_h|}$$

Q_h = heat absorbed from hot reservoir during isothermal expansion $A \to B$ (reservoir @ constant temperature)

$$\underset{\substack{\uparrow \\ \Delta U = Q - W = 0 \text{ (isothermal)}}}{Q_h = W} = nRT_h \cdot \ln\left(\frac{V_B}{V_A}\right)$$

Q_c = heat ejected to cold reservoir during isothermal compression $C \to D$

$$Q_c = nRT_c \ln\left(\frac{V_D}{V_C}\right)$$ (note $V_D < V_C \to Q_c < 0$ = heat loss)

$B \to C$: adiabatic expansion: $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1} \to \left(\frac{V_B}{V_C}\right)^{\gamma-1} = \frac{T_C}{T_B} = \frac{T_c}{T_h}$

$D \to A$: adiabatic compression: $T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1} \to \left(\frac{V_D}{V_A}\right)^{\gamma-1} = \frac{T_A}{T_D} = \frac{T_h}{T_c}$

$\to \frac{V_B}{V_C} = \frac{V_A}{V_D} \to \boxed{\frac{V_B}{V_A} = \frac{V_C}{V_D}}$

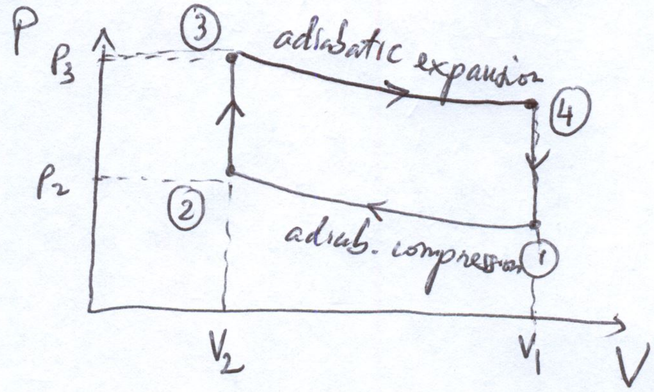
$$\boxed{e_{max} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{|nRT_c \ln\left(\frac{V_D}{V_C}\right)|}{|nRT_h \ln\left(\frac{V_B}{V_A}\right)|} = 1 - \frac{T_c\left|\ln\left(\frac{V_C}{V_D}\right)\right|}{T_h\left|\ln\left(\frac{V_B}{V_A}\right)\right|} = 1 - \frac{T_c}{T_h}}$$

Carnot Engines : 2 isothermal + 2 adiabatic $e_{max} = 1 - \frac{T_c}{T_h}$

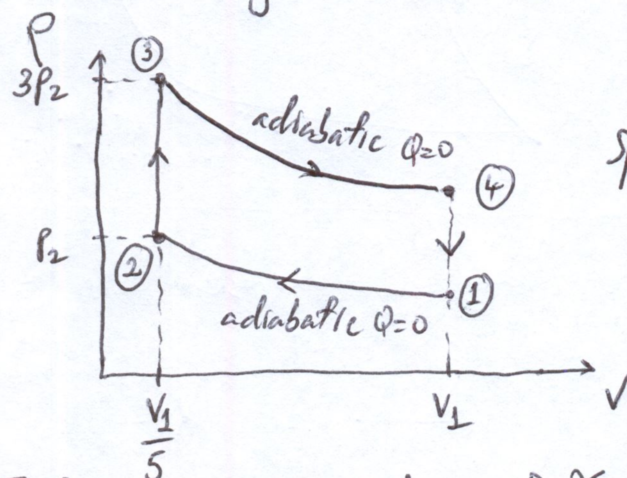
Otto Cycle Engines : 2 adiabatic + 2 isovolumic

~~only 2 vol~~

$e_{Otto} < e_{Carnot} = e_{max}$



19.53 Gasoline engine under Otto Cycle : 4 reversible processes
2 adiabatic & 2 isovolumic



Specific heat ratio $\gamma = \frac{C_p}{C_V}$

a) Find efficiency e in term of γ

↳ Heat engine = $e \equiv \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|} < 1$

Q_h → Heat reservoir @ constant temperature

$0 = \Delta U = Q_h - Q_c - W$

→ Need Q_h & Q_c : Otto Cycle → look at isovolumic processes 23 & 41

23 absorbs heat: $P_3 = 3P_2$

41 ejecting heat

$$\left\{\begin{array}{l} Q_h = Q_{23} \underset{\text{isovolumic}}{=} n c_V (T_3 - T_2) \\ Q_c = Q_{41} = n c_V (T_1 - T_4) \end{array}\right\} \rightarrow e = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|}$$

→ Need to express e_{otto} in term of γ: when we relate the temperatures T_1 & T_2 and T_3 & T_4 via the 2 adiabatic processes

1) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ $\left\{ \text{comes from } P_1 V_1^{\cancel{\gamma-1}\,\gamma} = P_2 V_2^{\gamma} \text{ and } \left\{ P_1 = \frac{nRT_1}{V_1},\ P_2 = \frac{nRT_2}{V_2} \right. \right\}$

2) $T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$

3) $\left\{ \begin{array}{l} V_4 = V_1 \\ V_3 = V_2 = \frac{V_1}{5} \end{array} \right\}$ Given information

$$\left.\begin{array}{l} \to 1)\ T_1 V_1^{\gamma-1} = T_2 \left(\frac{V_1}{5}\right)^{\gamma-1} \\ 2)\ T_4 V_1^{\gamma-1} = T_3 \left(\frac{V_1}{5}\right)^{\gamma-1} \end{array}\right\} \Rightarrow \boxed{\frac{T_1}{T_4} = \frac{T_2}{T_3}}$$

→ Simplify the efficiency $e_{otto} = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|} = 1 - \frac{T_4 \left| \frac{T_1}{T_4} - 1 \right|}{T_3 \left| 1 - \frac{T_2}{T_3} \right|}$

$T_4 \cancel{V_1^{\gamma-1}} = T_3 \cancel{V_1^{\gamma-1}} \cdot \frac{1}{5^{\gamma-1}}$

$\frac{T_4}{T_3} = \frac{1}{5^{\gamma-1}}$

$$\boxed{e_{otto} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{5^{\gamma-1}} = 1 - 5^{1-\gamma}}$$

$\frac{1}{a} = a^{-1}$

b) Find T_{max} in term of T_{min}

From PV diagram $\begin{cases} \text{state 1 largest volume + lowest pressure} \rightarrow T_{min} = T_1 \\ \text{state 3 smallest volume + largest pressure} \rightarrow T_{max} = T_3 \end{cases}$

↳ We want to relate T_3 to T_1

1) From adiabatic 34 (part a) → $\boxed{T_3 = T_4 \, 5^{\gamma-1}}$ $\left(V_3 = \frac{V_1}{5} \text{ \& } V_4 = V_1 \right)$

2) From adiabatic 34 & 12 (part a) : $\frac{T_1}{T_4} = \boxed{\frac{T_2}{T_3}} = \frac{1}{3}$

Ideal gas equation : $PV = nRT$

$$\frac{T_2}{T_3} = \frac{\frac{P_2 V_2}{\cancel{nR}}}{\frac{P_3 V_3}{\cancel{nR}}} = \frac{P_2 V_2}{P_3 V_3} = \frac{P_2}{P_3} = \frac{\cancel{P_2}}{3\cancel{P_2}} = \frac{1}{3}$$

given

$\boxed{\begin{array}{c} V_2 = V_3 \\ \text{23 isovolume} \end{array}}$

$\frac{T_1}{T_4} = \frac{1}{3} \rightarrow \boxed{T_4 = 3T_1}$

$$\boxed{T_3 = 3 \cdot T_1 \cdot 5^{\gamma-1} = 3 \cdot 5^{\gamma-1} \cdot T_1}$$

or $\boxed{T_{max} = 3 \cdot 5^{\gamma-1} \cdot T_{min}}$

c) Efficiency for a Carnot engine b/w T_{max} & T_{min} :

$$\boxed{e_{carnot} = e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{1}{3 \cdot 5^{\gamma-1}} = 1 - \frac{5^{1-\gamma}}{3}}$$

$$e_{otto} \underset{(a)}{=} 1 - 5^{1-\gamma} < e_{carnot}$$

Entropy Calculation:

$$\Delta S \equiv \int_1^2 \frac{dQ}{T}$$

isothermal process $= \Delta S = \frac{1}{T}\int_1^2 dQ = \frac{Q_2 - Q_1}{T} = \frac{\Delta Q}{T}$

(Heat transfer per unit temperature)

isovolumic process : $dQ = n c_v \, dT$ $\left(c_v \equiv \frac{dQ}{n\, dT} \right.$

$$\Delta S = \int_1^2 \frac{n c_v \, dT}{T} = n c_v \underbrace{\int_1^2 \frac{dT}{T}}_{\ln T} = n c_v \ln\left(\frac{T_2}{T_1}\right)$$

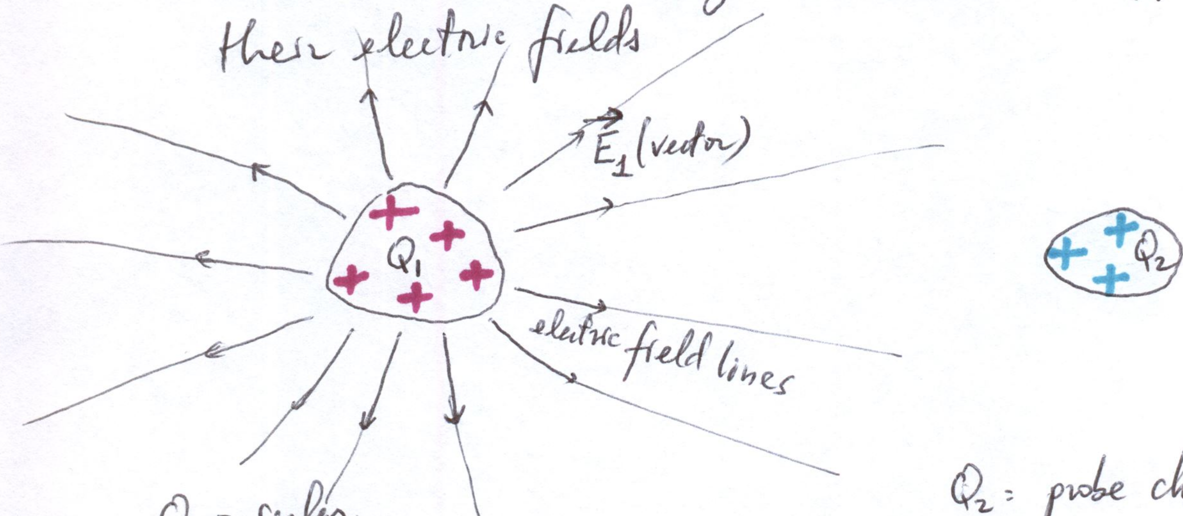
closed system

isobaric process: $\Delta S = n c_p \ln\left(\frac{T_2}{T_1}\right)$

Ch 20 Electric Charge, Force, Field

→ Electric charge distribution → electric field → when a probe (another charge) comes to its proximity it will feel the electric force by the original charge distribution

→ Interaction between charge distributions happens through these electric fields



Q_1 = scalar

$\vec{E}_1$ = electric field by charge distribution Q_1, is a vector

→ Electric field lines indicate directions & intensity of the electric field. Intensity or strength is represented by density of field lines

Q_2 : probe charge distribution (C for Coulomb)

$$\vec{F}_{12} = Q_2 \vec{E}_1$$

$\vec{F}_{12}$ = force by 1 on 2 (N)

→ Unit for electric field is $\frac{N}{C}$

→ Q_2 has its own electric field $\vec{E}_2$

↳ Q_1 feels force $\vec{F}_{21} = Q_1 \vec{E}_2$

→ Both guest charge & host charge will feel the electric interaction!

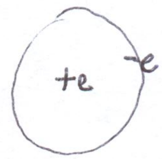
Charges

- Types: 2 $\left\{ \begin{matrix} + \\ - \end{matrix} \right\}$ charge unit = charge of the electron
  $e = 1.6 \times 10^{-19}\ C$ (Coulomb)

  Proton charge is $+e$

  Deuteron charge is $+2e$ (charge superposition)

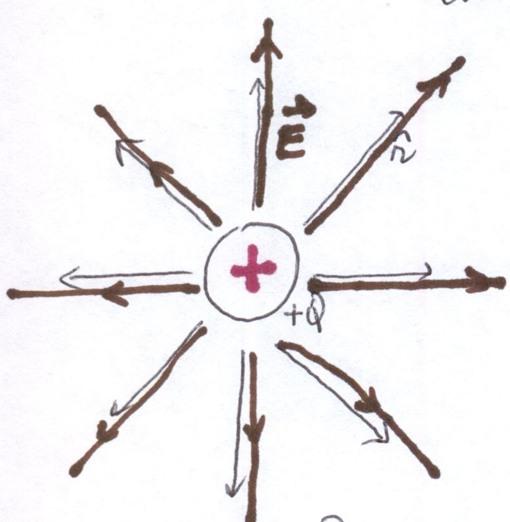
  Hydrogen atom charge is 0



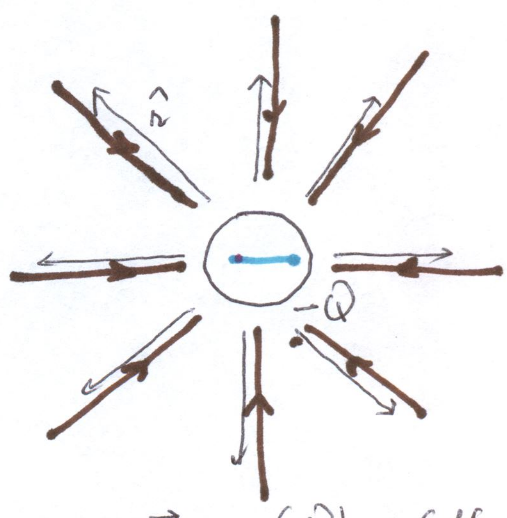
  Hydrogen ion H^+ charge $= +e$, etc.

- Sources of electric field: $\boxed{\vec{E} = k \frac{Q}{r^2} \hat{r}}$

  - k = electric constant $= 9 \times 10^9 \frac{Nm^2}{C^2}$
    (k : not spring constant, not Boltzmann, not thermal conductivity!)
  - Q : net charge of the charge distribution
  - r = separation from the charge Q to the field point (where electric field is measured)
  - $\hat{r}$: radial unit vector, points away from the charge distribution

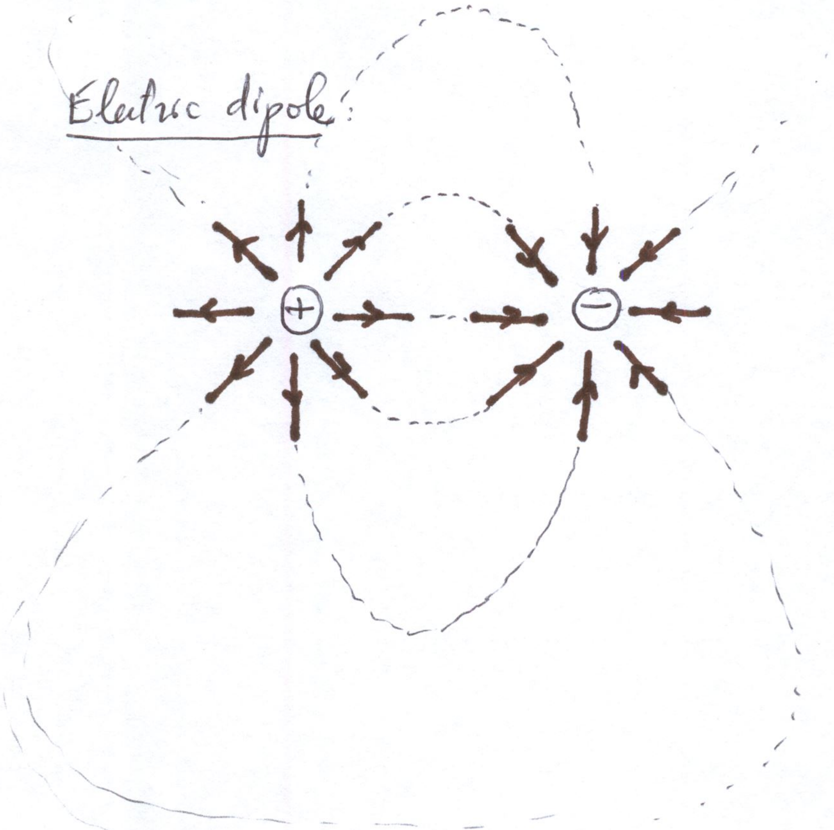


$\vec{E} = k \frac{+Q}{r^2} \hat{r}$ (Repulsive) away from charge



$\vec{E} = k \frac{(-Q)}{r^2} \hat{r}$ (attractive) (toward the charge)

Electric dipole:



Electric

- Two types of charge

  ↳ Repulsive

→ Fields { Repulsive / Attractive }

→ $\vec{E} = k \frac{Q}{r^2} \hat{r}$

$k = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\frac{k}{G} \sim 10^{20}$

Gravitational

→ One type of mass

↳ Field is attractive only

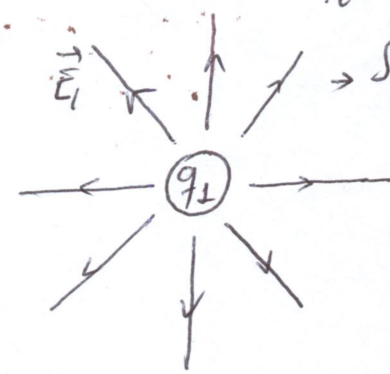
$\vec{g} = -G \frac{M}{r^2} \hat{r}$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Electric interaction are huge! Like charges repel!
However...it's not on if the net charge is 0 while gravitational interaction is always on although 10^{20} times weaker

Calculation of the electric field:

Single charge $q_1 \rightarrow \vec{E}_1 = k\dfrac{q_1}{r^2}\hat{r}$

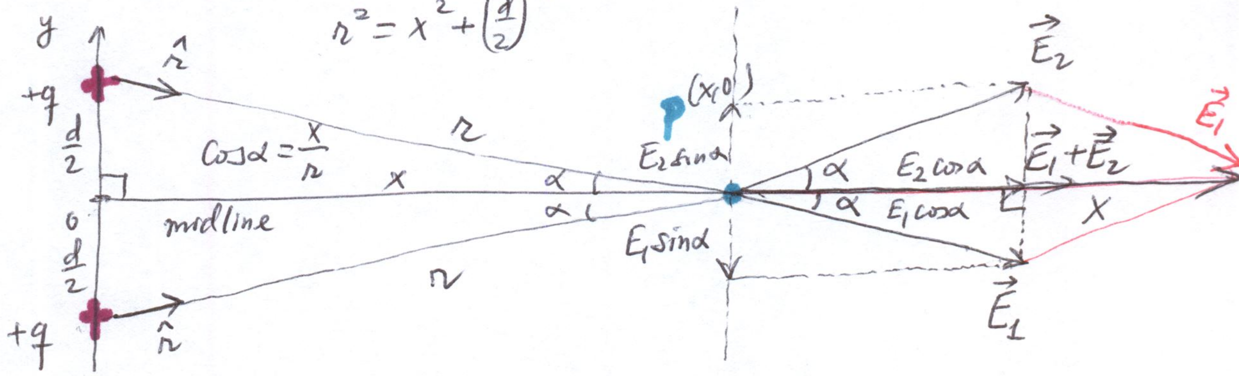


→ Strength of electric field is proportional to k, q_1 but inversely proportional to the separation r^2 (another inverse squared law)

Two positive charges : **Principle of Superposition** $\vec{E} = \vec{E}_1 + \vec{E}_2$

↳ Charges along y-axis, $\vec{E}$ along x-axis (midline b/w the two charges)

$r^2 = x^2 + \left(\frac{d}{2}\right)^2$



$\vec{E}_1$ & $\vec{E}_2$ have equal strength $\frac{kq}{r^2}$ (same vector lengths) but points along different directions

↳ superposition
- ↳ Graphically (diagonal of parallelogram formed by the 2 vectors)
- ↳ Mathematically using Cartesian components

$$\vec{E}_1 = E_1\cos\alpha\,\hat{\imath} - E_1\sin\alpha\,\hat{\jmath}$$
$$+\ \vec{E}_2 = E_1\cos\alpha\,\hat{\imath} + E_1\sin\alpha\,\hat{\jmath}$$

$$\vec{E} = 2E_1\cos\alpha\,\hat{\imath}$$ (diagonal of parallelogram)

$$\vec{E} = 2E_1 \cos\alpha \, \hat{i} = 2\frac{kq}{r^2}\frac{x}{r}\,\hat{i} = \frac{2kqx}{r^3}\,\hat{i} = \frac{2kqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\,\hat{i}$$

$$\left\{\begin{array}{l} E_1 = E_2 = \dfrac{kq}{r^2} \\ \cos\alpha = \dfrac{x}{r} \end{array}\right.$$

$\left(\frac{N}{C}\right)$

$$\vec{E} = 2E_1 \cos\alpha\, \hat{i} = 2\frac{kq}{r^2}\frac{x}{r}\,\hat{i} = \frac{2kqx}{r^3}\,\hat{i} = \frac{2kq\,x}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\,\hat{i}$$

$$\begin{cases} E_1 = E_2 = \dfrac{kq}{r^2} \\ \cos\alpha = \dfrac{x}{r} \end{cases} \qquad \left(\frac{N}{C}\right)$$

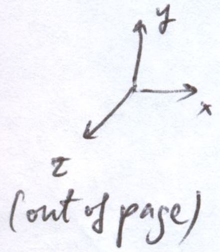
→ Electric field due to two positive charges separated by distance d @ a point $P(x,0)$ along the midline between the two charges points away along the midline. Field strength is $\dfrac{2kq\,x}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}$

→ When P is very far away from the two charges ($x \to \infty$)

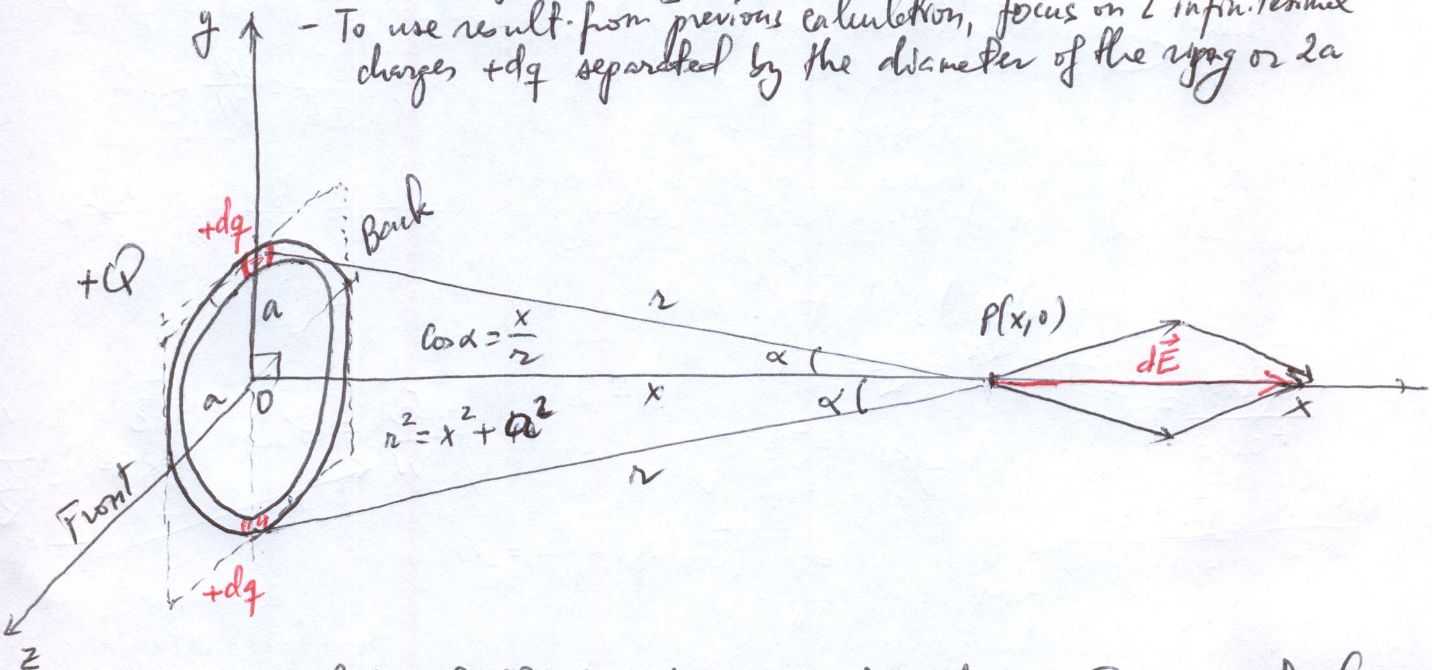
$$E \xrightarrow{x\to\infty} \frac{2kqx}{\left(x^2\right)^{3/2}} = \frac{2kq}{x^2} = \frac{k(2q)}{x^2}$$

Very far away the field is that of a point charge of value $2q$!

Electric field due to a continuous ring of charge, @ a point P along its axis:



- Ring on YZ plane, its center axis is the x-axis
- Center of ring is also ~~coo~~ origin of coordinates
- Total charge on ring is Q.
- To use result from previous calculation, focus on 2 infinitesimal charges +dq separated by the diameter of the ring or 2a



From the electric field due to 2 positive charges @ a point along their midline:
$$d\vec{E} = \frac{2k\,dq\,x}{(x^2+a^2)^{3/2}}\,\hat{\imath}$$
$\hat{\imath}$ ↳ unit vector in x-direction

Field due to the entire ring: superposition of fields ~~fo~~

$$\boxed{\vec{E} = \int_{\text{Half Ring}} d\vec{E} = \frac{2kx\,\hat{\imath}}{(x^2+a^2)^{3/2}} \underbrace{\int_{\text{Half Ring}} dq}_{\frac{Q}{2}} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{\imath}}$$

If P is very far away from the ring $x \to \infty$ $\begin{cases} x^2+a^2 \to x^2 \\ \vec{E} = \dfrac{kQx}{(x^2)^{3/2}}\,\hat{\imath} \\ \boxed{\vec{E} = \dfrac{kQ}{x^2}\,\hat{\imath}} \end{cases}$

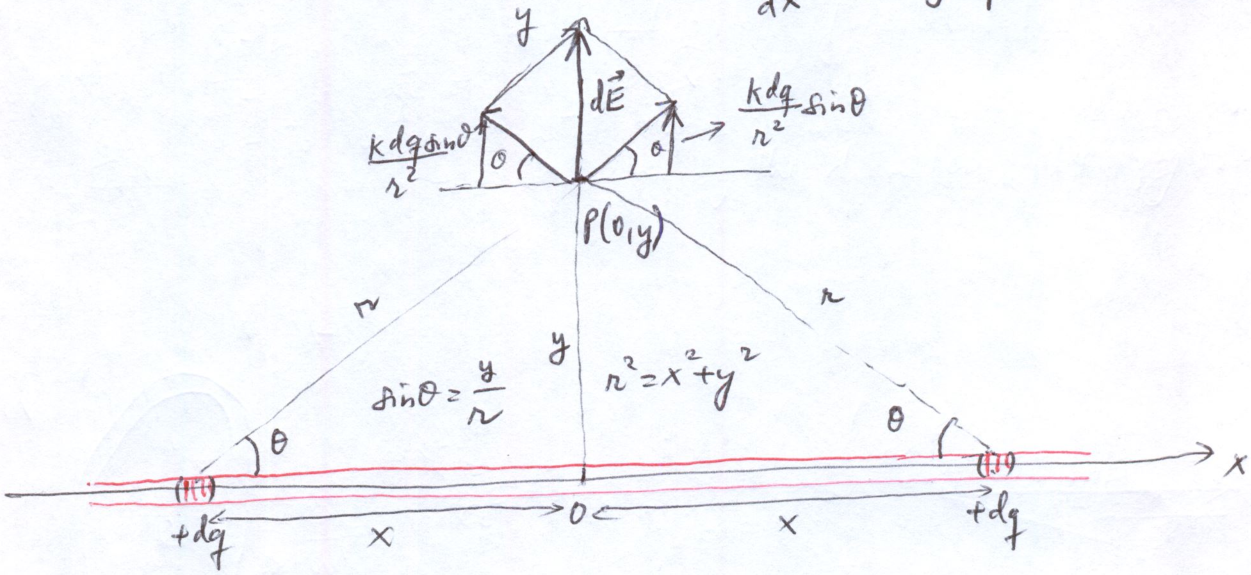
↓

Ring acts like a point charge of value Q

Electric field due to a ∞ long line of charge

linear charge density

$$\lambda \equiv \frac{dq}{dx} \quad \text{(charge per unit length)}$$



To apply previous result, we focus on 2 infinitesimal charges $+dq$ at $\pm x$ from the origin of coordinates:

$$d\vec{E} = \frac{2k\,dq}{r^2}\sin\theta\,\hat{j} = \frac{2k\,dq\,y}{r^3}\hat{j} = \frac{2k\,\overset{\lambda\cdot dx}{dq}\,y}{(x^2+y^2)^{3/2}}\hat{j}$$

$\hat{j}$ ↳ unit vector in y-direction

Field due to whole line of charge → superposition:

$$\vec{E} = \int_{\substack{\text{Half}\\\text{line}}} d\vec{E} = 2k\lambda y\hat{j}\underbrace{\int_{\substack{\text{Half}\\\text{line}}}\frac{dx}{(x^2+y^2)^{3/2}}}\underset{a\equiv y}{=} 2k\lambda y\hat{j}\underbrace{\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_{x=0}^{x=\infty}}_{\text{Half line}}$$

(integral in x)

Table: $\displaystyle\int\frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

$x\to\infty$: $(x^2+y^2)^{1/2} \longrightarrow (x^2)^{1/2} \simeq x$

$$\boxed{\vec{E} = 2k\lambda\hat{j}\,\frac{\cancel{y}\cancel{x}}{y^{\cancel{2}}\cancel{x}} = \frac{2k\lambda}{y}\hat{j}}$$

Note: inverse linear law (not inverse square law)

Ch 21 Gauss Law

We are learning how to calculate electric fields

1) Superposition or vector addition (Ch. 20)
2) Symmetry & Gauss Law (Ch. 21)
3) Derivatives & Electric Potential (Ch. 22)

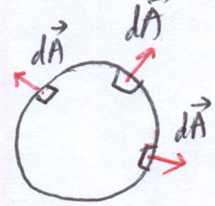
Electric Flux: $\phi = \oint_{\substack{\text{closed}\\ \text{surface}\\ \text{integral}}} \vec{E} \cdot d\vec{A}$

'Phi'

1) 'scalar product'

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\left\{ \begin{array}{l} \theta \text{ is the angle b/w } \vec{A} \,\&\, \vec{B} \\ \theta = 0 \text{ if } \vec{A} \parallel \vec{B} \rightarrow \vec{A}\cdot\vec{B} = AB \end{array} \right.$



2) $d\vec{A}$: vector element of area, direction is perpendicular and away from the element of area

3) $\vec{E}$ is the electric field.

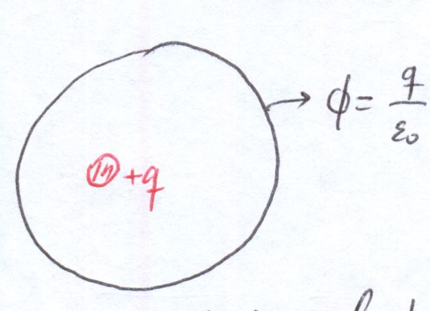
$\vec{E}\cdot d\vec{A}$ is the element of flux → integral over entire surface gives total electric flux through that surface

Gauss Law:

$$\boxed{\phi_{\substack{\text{closed}\\ \text{surface}}} = \frac{q_{\substack{\text{enclosed}\\ \text{by surface}}}}{\varepsilon_0}}$$

ε_0 = dielectric constant $= \dfrac{1}{4\pi \cdot k} = 8.85 \times 10^{-12} \dfrac{C^2}{Nm^2}$

$k = 9 \cdot 10^9 \dfrac{Nm^2}{C^2}$

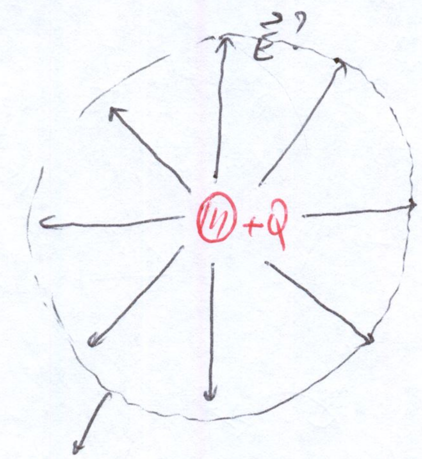


Why is it useful to calculate the electric field?

→ In situations with high symmetry (spheres, cylinder, rectangular boxes, ...) the surface integral can be simplified to $E \cdot A$ ⟶

$\rightarrow$ with Gauss law $\quad E \cdot A = \dfrac{q}{\varepsilon_0} \rightarrow E = \dfrac{q}{\varepsilon_0 A}$

Calculation of $\vec{E}$ due to a point charge using Gauss law



Gaussian surface

Electric flux ϕ through a spherical surface of radius r, centered at the charge:

$$\phi = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \cdot 4\pi r^2$$

1) $\hat{r} \cdot \hat{r} = 1$
$\downarrow$
$\vec{E} \cdot d\vec{A} = E\,dA$

$\vec{E}$ away from charge $+Q$, in radial directions $\vec{E} = E \cdot \hat{r}$

$d\vec{A}$: also in radial directions for this spherical surface centered at $+Q$: $d\vec{A} = dA\,\hat{r}$

2) $\vec{E}$ at different points on this spherical surface is the same! (Charge at center of sphere!) and the $\oint$ is over that surface

$$\oint E\,dA = E \underbrace{\oint dA}_{4\pi r^2}$$

3) Gauss law: $\quad \phi = \dfrac{q_{enclosed}}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0}$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

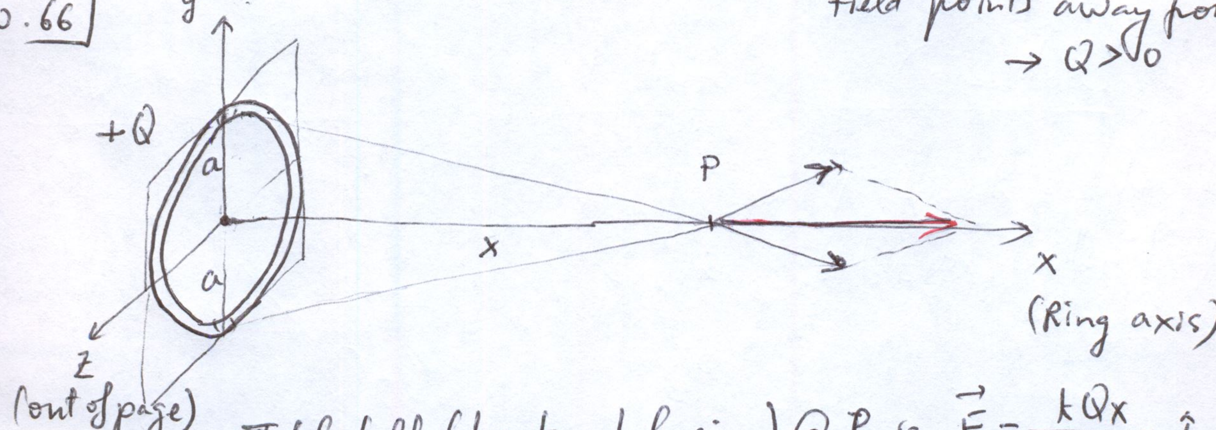
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

$\varepsilon_0 = \dfrac{1}{4\pi k} \rightarrow k = \dfrac{1}{4\pi\varepsilon_0}$

Yes, it is the field due to a point charge Q

20.66

Field points away from ring $\rightarrow Q > 0$



Total field (due to whole ring) @ P is $\vec{E} = \underbrace{\frac{kQx}{(x^2+a^2)^{3/2}}}_{E} \hat{\imath}$

Data $\left\{ \begin{array}{l} x = 0.05\,m \,;\; E = 380\,\frac{kN}{C} \\ x = 0.15\,m \,;\; E = 160\,\frac{kN}{C} \end{array} \right\} \rightarrow$ a) a? b) Q?

a) $$\frac{38\cancel{0}}{16\cancel{0}} = \frac{E_{0.05m}}{E_{0.15m}} = \frac{\frac{\cancel{kQ} \cdot 0.05}{(0.05^2+a^2)^{3/2}}}{\frac{\cancel{kQ} \cdot 0.15}{(0.15^2+a^2)^{3/2}}} \rightarrow \frac{19}{8} = \underbrace{\frac{0.05}{0.15}}_{\frac{1}{3}} \cdot \frac{(0.15^2+a^2)^{3/2}}{(0.05^2+a^2)^{3/2}}$$

$$\rightarrow \left(\frac{57}{8}\right)^{2/3} = \frac{0.15^2+a^2}{0.05^2+a^2} \rightarrow \left(\frac{57}{8}\right)^{2/3}(0.05^2) - (0.15^2) = \left[1 - \left(\frac{57}{8}\right)^{2/3}\right] a^2$$

$$a^2 = \frac{\left(\frac{57}{8}\right)^{2/3} 0.05^2 - 0.15^2}{1 - \left(\frac{57}{8}\right)^{2/3}}$$

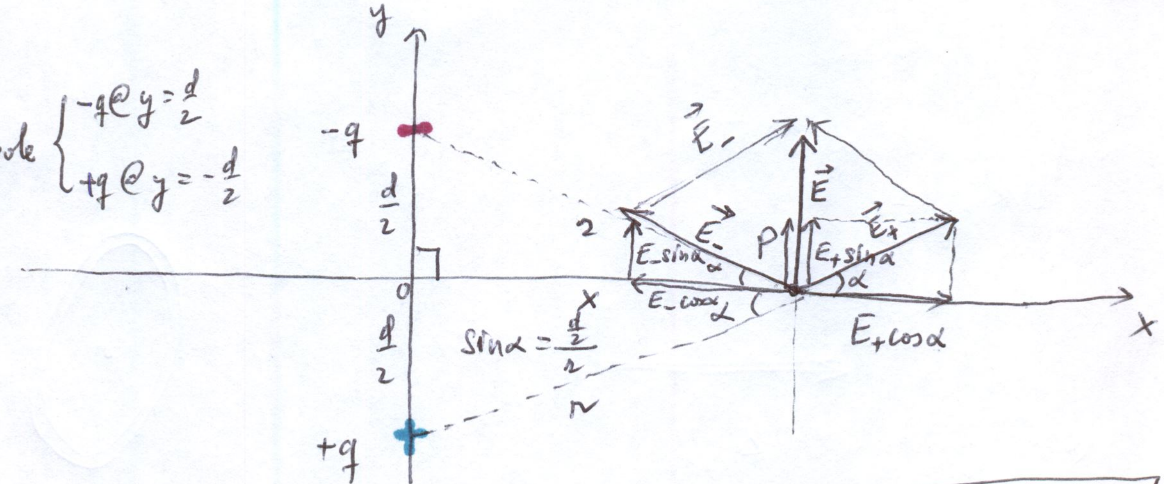
$$a = \cancel{0.0213}\,m \quad 0.07\,m$$

b) $$380 \cdot 10^3 = E_{0.05} = \frac{kQ\, 0.05}{(0.05^2 + \underset{0.07}{\cancel{0.0213}}^2)^{3/2}} \rightarrow Q = \frac{380 \cdot 10^3 (0.05^2 + \overset{0.07^2}{\cancel{0.0213}^2})^{3/2}}{9 \cdot 10^9 \cdot 0.05}$$

$$= 5.38 \cdot 10^{-7}\,C$$

20-49

Dipole $\begin{cases} -q @ y = \frac{d}{2} \\ +q @ y = -\frac{d}{2} \end{cases}$



Statements: 1) $E_+ = E_-$ (same values q & $\boxed{r = \sqrt{x^2 + \left(\frac{d}{2}\right)^2}}$)

2) x-components cancel out : 0

3) y-components add up : $2E_+ \sin\alpha$

$\rightarrow \vec{E} = 2E_+ \sin\alpha \, \hat{j}$ $\begin{cases} \sin\alpha = \frac{d/2}{r} = \frac{d}{2r} \\ E_+ = \frac{kq}{r^2} \text{ (field due to a point charge } +q \text{ @ separation } r) \end{cases}$

$$\boxed{\underline{\text{Dipole}}: \vec{E} = \frac{\cancel{2} kq \, d/\cancel{2}}{r^3} \hat{j} = \frac{kqd}{r^3} \hat{j}}$$

$\rightarrow$ Dipole field is an inverse-cube law

$\rightarrow$ Data $\begin{cases} \frac{d}{2} = 0.6 \text{ nm} = 0.6 \cdot 10^{-9} \text{ m} \; ; \; q = 1.6 \cdot 10^{-19} \text{ C} \\ \text{a) } x = 0 \rightarrow E? \quad \text{b) } x = 2\text{nm} \rightarrow E? \quad \text{c) } x = -20\text{nm} \; E? \end{cases}$

a) $E = \frac{9 \cdot 10^{9} \cdot 1.6 \cdot 10^{-19} \cdot 1.2 \cdot 10^{-9}}{\left[0 + (0.6 \cdot 10^{-9})^2\right]^{3/2}} = 8 \cdot 10^{9} \frac{\text{N}}{\text{C}}$

b) $E = \frac{9 \cdot 10^{9} \cdot 1.6 \cdot 10^{-19} \cdot 1.2 \cdot 10^{-9}}{\left[(2 \cdot 10^{-9})^2 + (0.6 \cdot 10^{-9})^2\right]^{3/2}} = \ldots$

c) $E = \frac{9 \cdot 10^{9} \cdot 1.6 \cdot 10^{-19} \cdot 1.2 \cdot 10^{-9}}{\left[(20 \cdot 10^{-9})^2 + (0.6 \cdot 10^{-9})^2\right]^{3/2}} = \ldots$

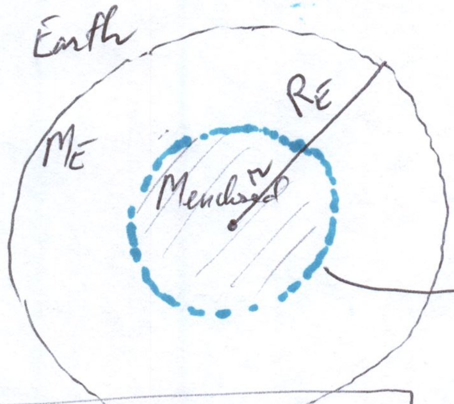
21.70

Gauss Law $= \quad \phi \equiv \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0}$

↳ For Gravitational Field $\vec{g}$ $\quad \boxed{\phi_g \equiv \oint \vec{g} \cdot d\vec{A} = -4\pi G \cdot M_{enclosed}}$

$\left(\dfrac{1}{\varepsilon_0} = 4\pi k\right)$

↳ Calculate $\vec{g}(r)$ $(r < R_E)$ interior of Earth



1) Gaussian surface: a sphere of radius $r < R_E$, centered @ Earth's center.

Assume Earth is homogeneous or same density throughout:

$\dfrac{M_{enclosed}}{M_E} = \dfrac{r^3}{R_E^3} \rightarrow \boxed{M_{enclosed} = M_E \dfrac{r^3}{R_E^3}}$

2) $\vec{g}$ also radial, toward center as with outside of Earth.

3) Since the Gaussian surface is a sphere, $d\vec{A}$ also points in the radial but away from center → angle θ b/w $\vec{g}$ & $d\vec{A}$ is $180°$ $(\cos 180° = -1)$

4) $\vec{g}$ is constant on the Gaussian surface.

$$\phi_g = \oint \vec{g} \cdot d\vec{A} \underset{1,2,3}{=} -\oint g\,dA \underset{4}{=} -g \underbrace{\oint dA}_{\substack{\text{Surface area} \\ \text{of Gaussian surface} \\ 4\pi r^2}} = -g\,4\pi r^2$$

↳ "Gauss Law for grav. field": $\phi_g = -g\,4\pi r^2 = -4\pi G \cdot M_{enclosed}$

$$\boxed{g = \frac{G M_{enclosed}}{r^2}}$$

$$g(r) = \frac{G}{\cancel{r^2}} M_E \frac{r^{\cancel{3}}}{R_E^3} = \underbrace{\left[\frac{G M_E}{R_E^2}\right]}_{g_0} \frac{r}{R_E}$$

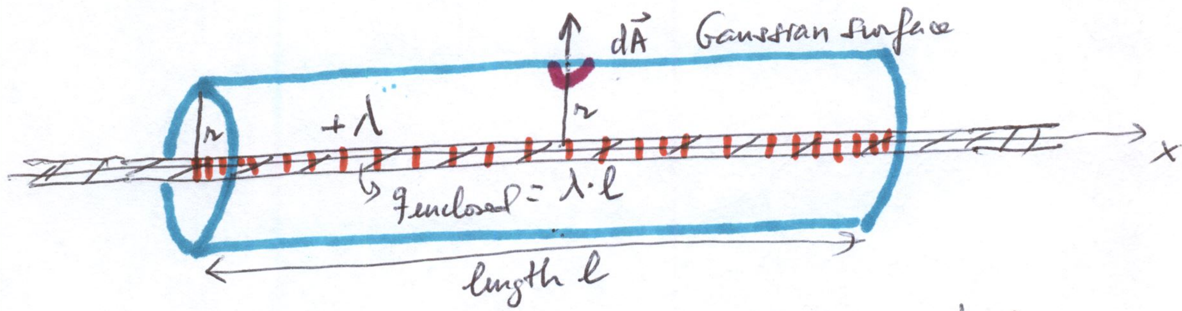
5) At surface: $\boxed{g_0 = \dfrac{G M_E}{R_E^2}}$

$$\Rightarrow \boxed{g(r) = g_0 \cdot \frac{r}{R_E}}$$

Linear charge density $\lambda \equiv \frac{dq}{dx}$

Calculation of $\vec{E}$ due to an ∞-long line of charge using Gauss Law

Gauss Law : symmetry of charge $\rightarrow$ cylindrical gaussian surface whose axis is the line of charge



Statements: 1) $d\vec{A}$ in radial & away from cylinder

2) $\vec{E}$ also radial and away from line of charge

$\rightarrow \vec{E} \cdot d\vec{A} = E dA$

3) E is constant on this Gaussian surface (same separation r from line of charge)

$$\rightarrow \phi = \oint \vec{E} \cdot d\vec{A} \underset{1,2}{=} \oint E dA \underset{3}{=} E \underbrace{\oint dA}_{\text{Surface of Gaussian cylinder } 2\pi r \ell \text{ (Body only)}}$$

4) What left & right sides $A = \pi r^2$

Why they are not included in ϕ?

Since $d\vec{A}$ for these sides point along x-axis (left : $-x$, right $+x$) they form an ~~of~~ angle of $90°$ to $\vec{E}$ which is always radial.

$\rightarrow$ For the left & right ~~side~~ cross-sections $\vec{E} \cdot d\vec{A} = 0$

5) Gauss Law : $\phi = E \cdot 2\pi r \ell = \dfrac{\lambda \cdot \ell}{\varepsilon_0} \rightarrow E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2\lambda}{4\pi\varepsilon_0 r}$

$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$ $\boxed{E = \frac{2k\lambda}{r}}$

(Gauss Law is another way to calculate the electric field!)

Ch 22: Electric Potential

(3rd Method to calculate the electric field $\vec{E}$)

Potential energy difference b/w points A & B

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$

↳ scalar product

→ Electrical potential energy → $\vec{F} = \underbrace{q'}_{\text{probe or test charge}}\vec{E}$

(' is not a derivative)

$$\Delta U_{AB} = -q' \int_A^B \vec{E} \cdot d\vec{l}$$

↳ scalar product

→ Electric potential is the electric potential energy per unit test charge: $\Delta V_{AB} = \dfrac{\Delta U_{AB}}{q'} = -\int_A^B \vec{E} \cdot d\vec{l}$

→ Unit: $\dfrac{J}{C}$

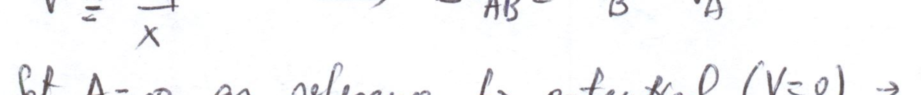
"gradient"

$$\rightarrow \vec{E} = -\frac{d\,\Delta V_{AB}}{dl} = -\nabla V_{AB}$$

Electric field due to a point charge using potential difference

→ Electric potential due to a point charge:

$d\vec{l} = dx\,\hat{i}$



1) $\Delta V_{AB} = -\int_A^B \dfrac{kq}{x^2} \underbrace{\hat{i} \cdot \hat{i}}_{1} dx = -kq \int_A^B \dfrac{dx}{x^2} = kq\left[\dfrac{1}{x}\right]_A^B = kq\left[\dfrac{1}{x_B} - \dfrac{1}{x_A}\right]$

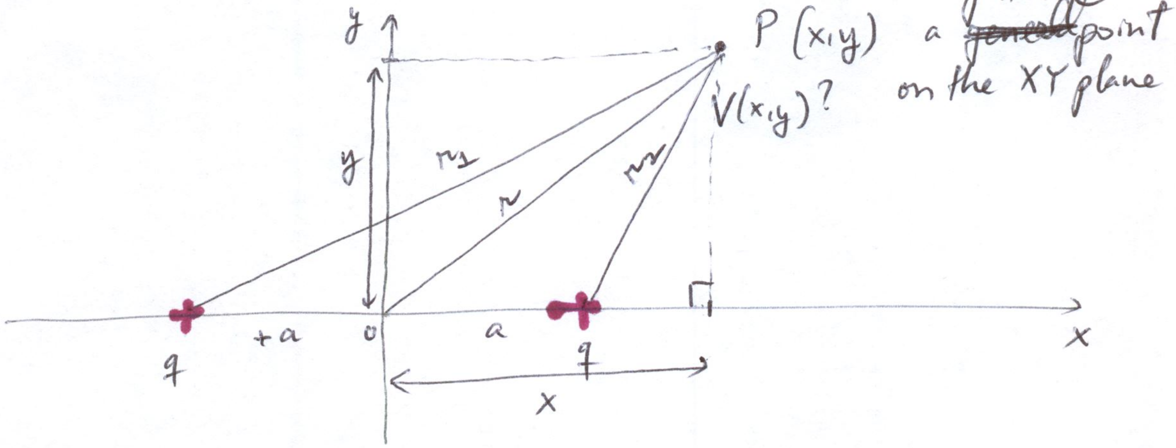
$V \equiv \dfrac{kq}{x} \longrightarrow \Delta V_{AB} = V_B - V_A$

2) Set $A = \infty$ as reference for potential $(V=0) \rightarrow V_\infty = \dfrac{kq}{\infty} = 0$

$\Delta V_{\infty B} = V_B - 0 = V_B$ → no further need to talk about potential differences, just potential

$\boxed{\text{E-potential due to a point charge } V(r) = \dfrac{kq}{r}} \longrightarrow E(r) = -\dfrac{d}{dr}\left(\dfrac{kq}{r}\right) = \dfrac{kq}{r^2}$

22.52 Electric potential is a scalar → superposition is simple arithmetic addition



1) Due to one charge : $V(r) = \frac{kq}{r}$ → need separation r in addition to q

$$r_1 = \sqrt{(x+a)^2 + y^2} \quad ; \quad r_2 = \sqrt{(x-a)^2 + y^2}$$

2) Total electric potential $V(x,y) = \frac{kq}{r_1} + \frac{kq}{r_2} = kq\left[\frac{1}{\sqrt{(x+a)^2+y^2}} + \frac{1}{\sqrt{(x-a)^2+y^2}}\right]$

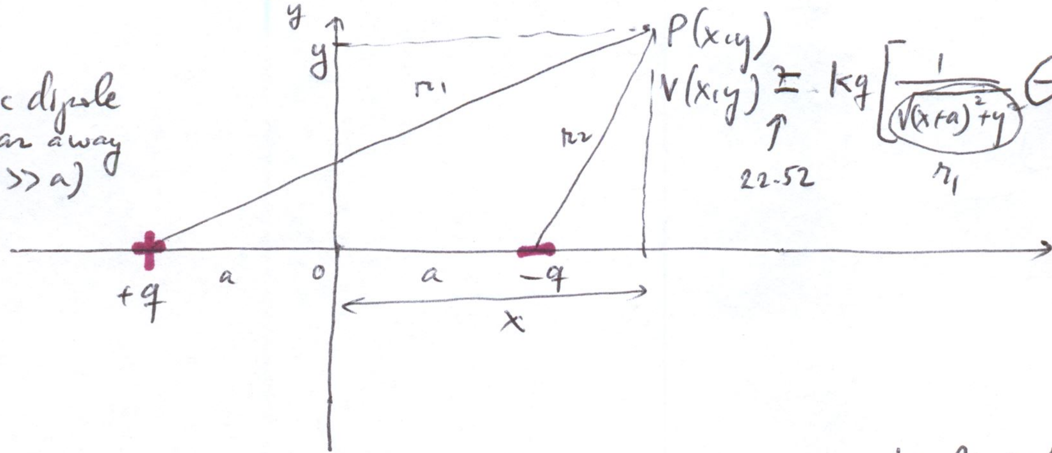
3) Very far away from these two charges: $x >> a \begin{cases} x+a \sim x \\ x-a \sim x \end{cases}$

$$V(x,y) = \frac{k\,2q}{\sqrt{x^2+y^2}} = \frac{k\,2q}{r}$$

→ Potential is that of one point charge of value $2q$ located at their midpoint which is the origin of coordinates.

22.53

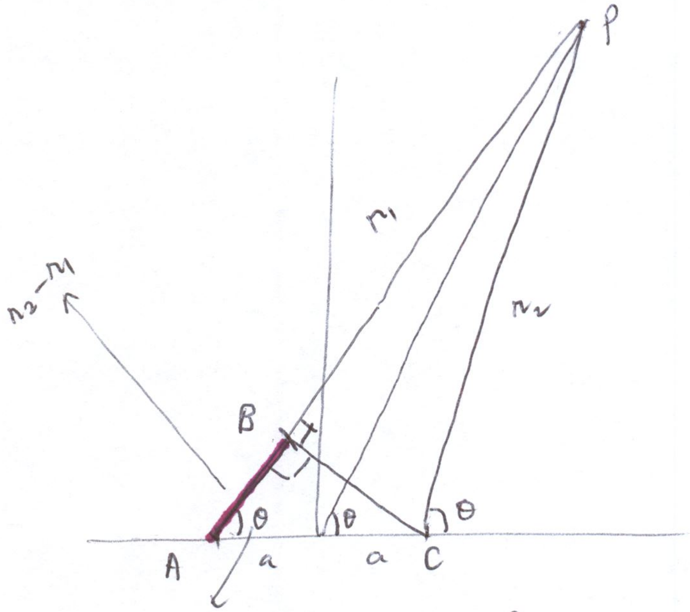
→ Electric dipole very far away (10 cm >> a)



$$V(x,y) = kq\left[\frac{1}{\underbrace{\sqrt{(x+a)^2+y^2}}_{r_1}} - \frac{1}{\underbrace{\sqrt{(x-a)^2+y^2}}_{r_2}}\right]$$

↑ 22.52

$$V(x,y) = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = kq\,\frac{r_2 - r_1}{r_1 \cdot r_2} = \frac{kq\,2a\cos\theta}{r^2} = \frac{kp\cos\theta}{r^2}$$



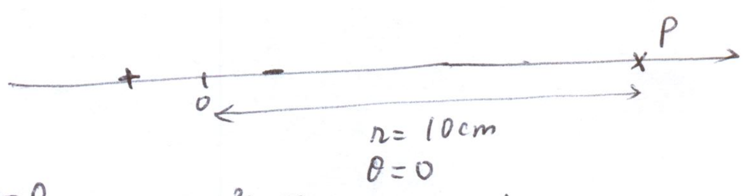
$(r_1 \sim r_2 \sim r)$

Approximation: P very far away lines r_1, r, r_2 are parallel to each other.

ΔABC: hypothenuse $2a$ → $\boxed{r_2 - r_1 = AB = 2a\cos\theta}$

Define: dipole moment $p \equiv q \cdot 2a$

$p = 2.9\,nCm$ a) 10 cm along axis



$$V = \frac{9 \cdot 10^9 \cdot 2.9 \cdot 10^{-9} \cdot \cos 0}{0.1^2} = 9 \cdot 2.9 \cdot 10^2\ \frac{J}{C} \text{ or } V \text{ (Volt)}$$

$$= 26.1 \cdot 10^2\ V$$

b) $r = 10\,cm$
$\theta = 45°$ → $V = 26.1 \cdot 10^2 \cos 45°\ V$

c) $r = 10\,cm$
$\theta = 90°$ → $\cos 90° = 0$ → $V = 0$ (along perpendicular bisector)

22.31

$$V(x,y,z) = 2xy - 3zx + 5y^2 \quad (\text{in V or volts})$$

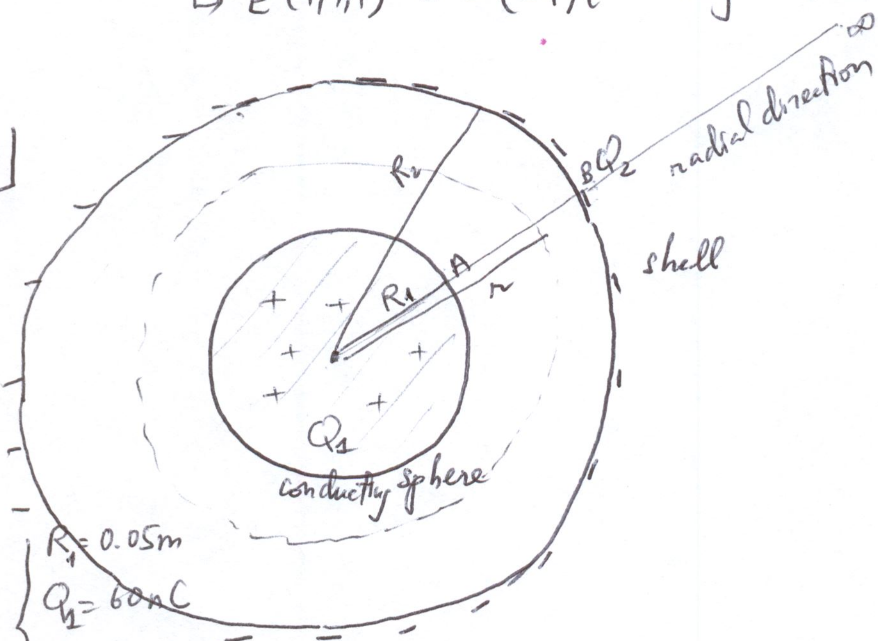
$$\rightarrow V(1,1,1) = 2 - 3 + 5 = 4V$$

$$\rightarrow \vec{E}(x,y,z) = -\vec{\nabla} V = -\left(\frac{d}{dx}V\right)\hat{i} - \left(\frac{d}{dy}V\right)\hat{j} - \left(\frac{d}{dz}V\right)\hat{k}$$

$$= -(2y - 3z)\hat{i} - (2x + 10y)\hat{j} - (-3x)\hat{k}$$

$$\rightarrow \vec{E}(1,1,1) = -(-1)\hat{i} - 12\hat{j} - (-3)\hat{k} = \hat{i} - 12\hat{j} + 3\hat{k} \ \left(\frac{N}{C}\right)$$

22.67



Data
- $R_1 = 0.05m$
- $Q_1 = 60nC$
- $R_2 = 0.15m$
- $Q_2 = -60nC$

→ Reference potential ($V=0$) @ ∞ → Potential V @ surface of sphere is

$$\Delta V_{\infty A} = V_A - V_\infty = V_A$$

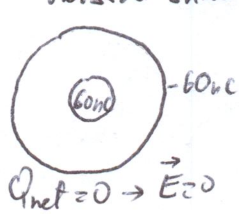
$$V_A = \Delta V_{\infty A} = -\int_\infty^A \vec{E}\cdot d\vec{l} = -\int_\infty^B \vec{E}\cdot d\vec{l} - \int_B^A \vec{E}\cdot d\vec{l} = -\int_B^A \frac{kQ_1}{r^2}\,dr$$

0 — outside shell

b/w sphere & shell

$$\vec{E} = \frac{kQ_1}{r^2}\hat{r}$$

$$d\vec{l} = dr\,\hat{r}$$



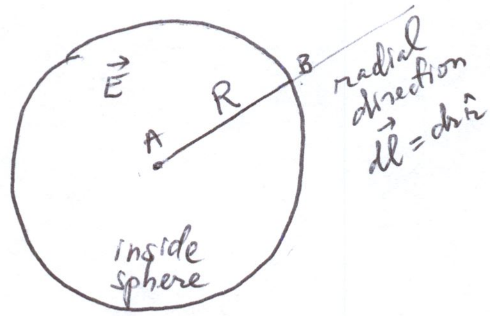
$Q_{net} = 0 \rightarrow \vec{E} = 0$

$$V_A = -kQ_1\int_{0.15m}^{0.05m} \frac{dr}{r^2} = kQ_1\left[\frac{1}{r}\right]_{0.15m}^{0.05m} = 9\cdot 10^{9}\cdot 60\cdot 10^{-9}\left[\frac{1}{0.05} - \frac{1}{0.15}\right] = 7.2\ kV$$

22.65

Sphere of radius R with electric field $\underbrace{\vec{E} = E_0\left(\frac{r}{R}\right)^2 \hat{r}}_{\text{in the sphere}}$

(E_0 a constant)



$A \to r = 0$
$B \to r = R$

Potential difference surface to center $\Delta V_{BA} = -\int_B^A \vec{E}\cdot\vec{dl}$

$$\Delta V_{BA} = -E_0 \int_B^A \frac{r^2}{R^2}\,\underbrace{\hat{r}}_{\text{from } \vec{E}}\cdot\underbrace{\hat{r}\,dr}_{\text{from } \vec{dl}} = -\frac{E_0}{R^2}\underbrace{\int_B^A r^2\,dr}_{\left[\frac{r^3}{3}\right]_B^A} = -\frac{E_0}{R^2}\left[0 - \frac{R^3}{3}\right]$$

$$\rightarrow \boxed{\Delta V_{BA} = \frac{E_0 R}{3}}$$