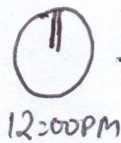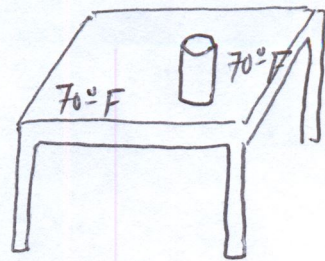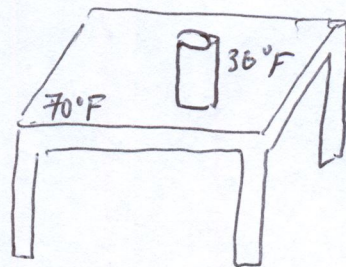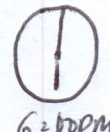# Ch 16   Temperature & Heat

<u>Temperature</u>: a <u>macroscopic</u> property (we can sense & measure), that is a consequence of the <u>microscopic</u> behavior of matter

Soda can from fridge 36°F

1) Soda can in thermal contact with table (so heat can be exchanged)
2) Soda can absorbed heat

after 6 hrs they come to have the same temperature

<u>Conclusion</u>: two objects in thermal contact can exchange heat to arrive at a thermodynamic equilibrium (TD) or same temperature.

→ Temperature can be measured using thermometers (mercury; electronic; gas at constant volume, etc.)

Body temp ~100°F ← Fahrenheit °F   → $T_F = T_C \frac{9}{5} + 32$

Water boiling ← Celsius °C   → $T_C = \frac{5}{9}(T_F - 32)$
~100°C

absolute zero ← Kelvin °K   → $T_C = T_K - \underline{273.16\ °K}$
in universe is °K

triple point of water

Heat Transfer (or Exchange) Rate H: How fast heat is transferred

Soda in soda can

Faster heat transfer; larger contact area A b/w soda & table

Thick plate: slower heat transfer; heat need to travel longer (Δx)

Hot table: soda raises up to 70°F much faster: ΔT
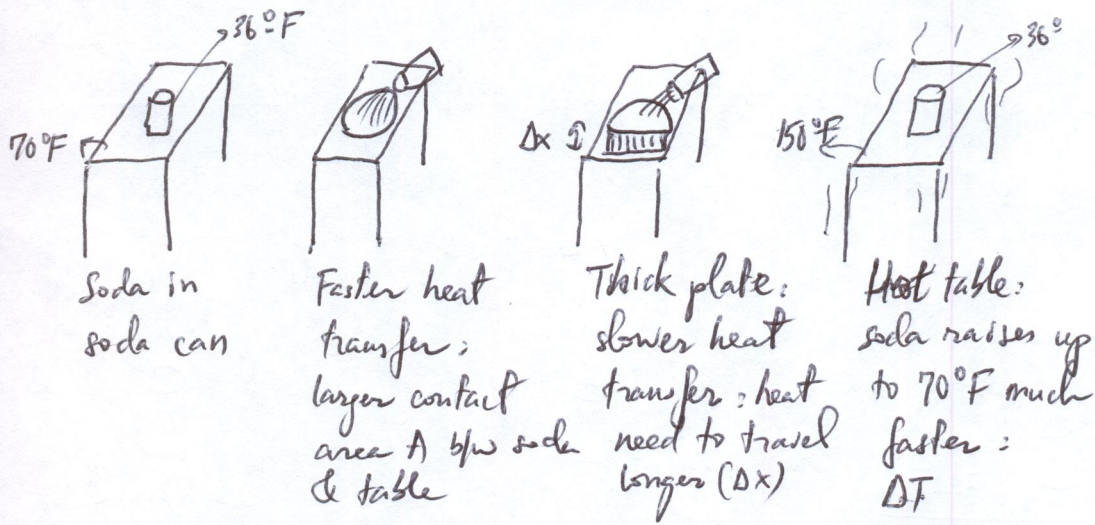
$$H = -k \frac{A}{\Delta x} \Delta T$$

H: Heat transfer rate b/w two objects

A: Contact area b/w the two objects.

ΔT: temperature difference b/w the objects

k: thermal constant, depends on materials (some are better thermal conductor → $\uparrow k$ → $\uparrow H$)

Negative sign = Heat "loss" rate

Dimensional analysis of H:

Basic dimensions: length (L), mass (M), time (T), temperature (T or temp)

Dimensions of energy:
$$[E] = \left[\tfrac{1}{2}\right] \cdot [m] \cdot [v]^2 = \frac{ML^2}{T^2}$$
$$\underset{1}{\underbrace{\quad}} \quad M \quad \left(\frac{L}{T}\right)^2$$

Dimensions of the thermal constant k:
$$[k] = \frac{[H] \cdot [\Delta x]}{[A] \cdot [\Delta T]} = \frac{\frac{[E]}{T} \cdot L}{L^2 \cdot Temp} = \frac{[E]}{T \cdot L \cdot Temp}$$

Power

$$[H] = \frac{[E]}{T}$$

Unit for thermal constant k = (not spring constant, nor wave number)

$$\begin{cases} SI: \dfrac{W}{m \cdot °K} \\[2mm] British: \dfrac{Btu \cdot in}{h \, ft^2 \, °F} \end{cases}$$

$\mathcal{R}$: R-factor $\qquad \mathcal{R} \equiv \dfrac{\Delta x}{k}$ $\qquad$ larger $\mathcal{R}$ better insulation

(smaller H)

$$H = -k \dfrac{A}{\Delta x} \Delta T = -\dfrac{A}{\underbrace{\left(\frac{\Delta x}{k}\right)}_{\mathcal{R}}} \Delta T = -\dfrac{A}{\mathcal{R}} \Delta T$$

$\mathcal{R} \rightarrow$ units $\begin{cases} SI: \dfrac{m^2\,{}^\circ K}{W} \\[2em] British: \dfrac{ft^2\,{}^\circ F}{\frac{Btu}{h}} \end{cases}$

Mechanisms of Heat Transfer:

Boiling water using a stove : heat travel : stove top $\rightarrow$ pan

$\rightarrow$ water $\rightarrow$ hand

pan

water $\rightarrow$ convection

Stove top

Hand

Radiation
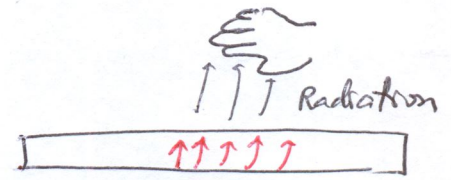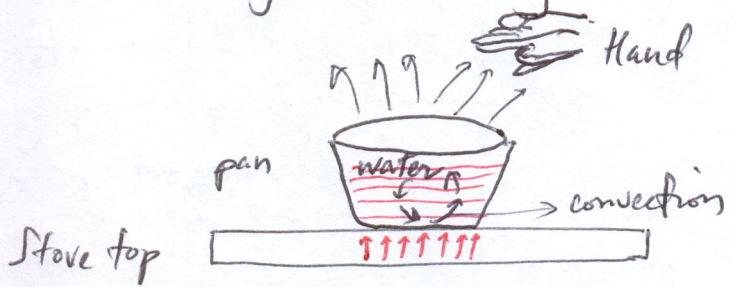
$\rightarrow$ Pan = metal wall & bottom

1) Stove top to pan : electrons in metal bottom of pan ~~moves~~
receive heat, move faster, hit other
electrons $\rightarrow$ <u>Heat transfer by Conduction</u>

2) Water molecules bottom to top : $\rightarrow$ <u>Heat transfer by convection</u>

3) Hand receives heat : $\rightarrow$ <u>Heat transfer by radiation</u>

Convection : Why hot molecules tend to rise?

(Baseboard heat @ floor level)

    hot ↔ more KE or more active → more separation b/w

  molecules → lower density → pushed up by bouyancy

    cold → low KE or less active → less separation b/w

  molecules → higher density → sinks down by gravity

(AC outlet @ ceiling level)

Radiation :   Stefan - Boltzman Law :

    Heat loss rate by radiation of an object of
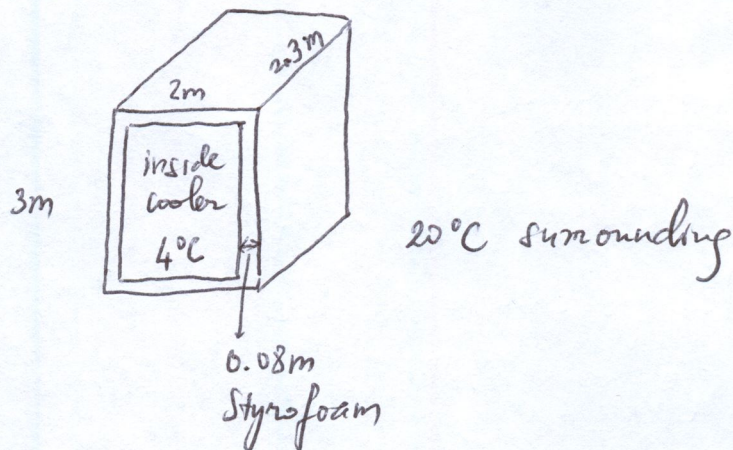
    area A & temperature T is given as

$$\underline{P} = e \; \sigma \, A T^4$$

    emissivity
    constant
    (dimension = 1
    or no unit)

Stefan - Boltzman constant

$$\sigma = 5.67 \cdot 10^{-8} \; \frac{W}{m^2 \, {}^{\circ}K^4}$$

16.57/   Rate of heat removal from cooler or H?

3m   2m   2.3 m   inside cooler 4°C   20°C surrounding

0.08m
Styrofoam

If there is no refrigeration unit, heat gets transferred into cooler @ rate $H = k \dfrac{A}{\delta x} \Delta T$

$$\begin{cases} k = 0.029 \dfrac{W}{m \, °K} \ (\text{Styrofoam thermal conductivity Table 16.2}) \\[2mm] A = 3 \times 2 \times 2 + 3 \times 2.3 \times 2 + 2 \times 2.3 \times 2 \\ \quad = 35 \, m^2 \\[2mm] \delta x = 0.08 \, m \\[2mm] \Delta T = \begin{cases} 20°C - 4°C = 16°C \\[2mm] (20°C + 273.16) - (4°C + 273.16) = 16°K \end{cases} \end{cases}$$

(Temperature differences in °C & °K are the same!)

To keep cooler interior @ constant 4°C → refrigeration unit to remove heat @ exact same rate H

$$H = \underset{\text{removal}}{-} k \frac{A}{\delta x} \Delta T = -\, 0.029 \cdot \frac{35}{0.08} \, 16 = -203 \, W$$

Heat balance & Specific Heat:

1) Objects 1, 2 @ T_1 & T_2 respectively ($T_1 \neq T_2$), when they are placed in thermal contact, after sufficient time they will arrive @ TD equilibrium or same final temperature T_f (which depends on m_1, m_2, c_1, c_2, besides T_1, T_2)

c : specific heat (specific to the object's material)

$$\hookrightarrow \begin{cases} c_{Cu} = 386 \dfrac{J}{kg\,°K} & \text{(to increase temp. of 1 kg Cu by} \\ & \text{1°K, we need to supply 386 J)} \\ c_{H_2O} = 4184 \dfrac{J}{kg\,°K} \end{cases}$$

2) In general if $\boxed{T_1 > T_2}$ $\Rightarrow$ $T_1 > T_f > T_2$ $\Rightarrow$

Heat loss by ① : ΔQ_1 (negative)

Heat gain by ② : ΔQ_2 (positive)

System of objects ① & ② is isolated (no thermal contact with outside world) $\Rightarrow$ $\boxed{\Delta Q_1 + \Delta Q_2 = 0}$

(From the outside world, these two objects exchange heat among themselves but together they didn't change)

3) $\Delta Q = m c \Delta T$ (SI unit for specific heat $c : \dfrac{J}{kg\,°K}$)

4) Heat exchange equation: $\underbrace{m_1 c_1 \left(T_f - T_1 \right)}_{\substack{<0 \\ \text{Negative (loss)}}} + \underbrace{m_2 c_2 \left(T_f - T_2 \right)}_{\substack{>0 \\ \text{Positive (gain)}}} = 0$

Ch 17 Thermal Behavior of Matter

Matter can be found in 3 states

|	Density	
gas	low	almost free molecules
liquid		interactions b/w molecules
solid	high	molecules are not free to move around (in metal: electrons are free to move around)

Gas → __Ideal Gas__ : assume that molecules are completely free (we ignore any interaction b/w them)
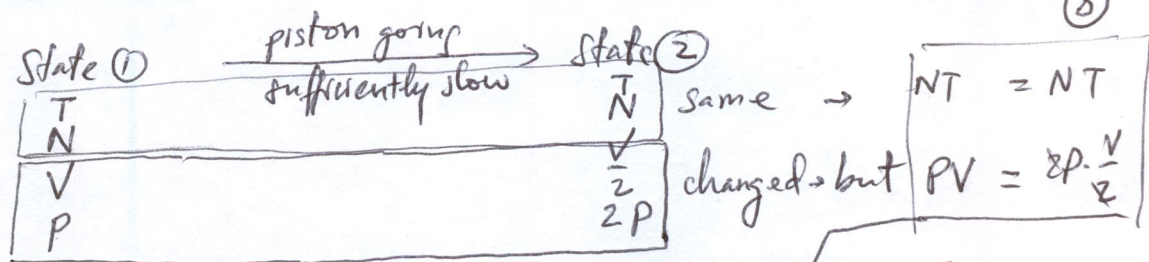
__Equation of State for an Ideal Gas__ :

- T : temperature of gas.
- N : # molecules of gas
- V : volume of gas
- P : pressure of gas (transfer of linear momentum to cylinder)
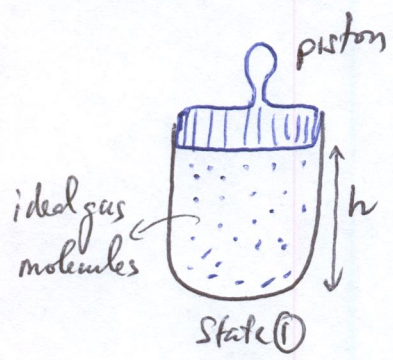
If we push the piston, it will go down since gas molecules can come closer together (gas is compressible)

piston

ideal gas molecules

h

State ①

$\frac{h}{2}$
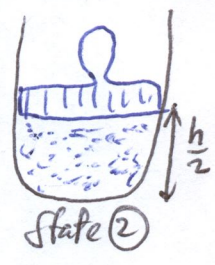
State ②

Notes:
1) Piston fits cylinder : no leak, # of gas molecules in state ① & state ② is N
2) Since height of cylinder is reduced by half.
State ① : V ; state ② : $\frac{V}{2}$
3) There are twice as many gas molecules transferring their linear momentum to the lower half part of cylinder in state ② compared to that in state ①
State ① : P ; state ② : $2P$
4) When pumping the bike wheel if we do it sufficiently slow the pump does not heat up. since molecules have time to adjust then don't get hit by the piston, their KE doesn't increase → their temp doesn't increase → Piston going down sufficiently slow → T is constant (isothermal)

State ① $\xrightarrow{\text{piston going sufficiently slow}}$ State ②

State ①	State ②	
T	T	Same →
N	N	
V	$\frac{V}{2}$	changed → but
P	2P	

$$NT = NT$$
$$PV = 2P \cdot \frac{V}{2}$$

Equation of State for an Ideal gas : $\boxed{PV = kNT}$

k = Boltzman constant

$$= 1.38 \times 10^{-23} \frac{J}{°K}$$

Note: PV unit in SI is Joule (J)

$$PV = kNT$$

$$N = n N_A \quad ; \quad \begin{cases} \text{Avogadro \# } N_A = 6.022 \cdot 10^{23} \\ n : \text{\# of moles} \end{cases}$$

$$PV = n \underbrace{k N_A}_{in} T$$

$$R = 8.314 \frac{J}{°K \, mol}$$

$$\boxed{PV = nRT} \quad \text{Eq. of State for an Ideal Gas}$$

Real Gas : Eq. of State or van der Waals Law

$$\boxed{\left(P + \frac{n^2 a}{V^2}\right)\left(V - nb\right) = nRT, \qquad a \,\&\, b \text{ are constants}}$$

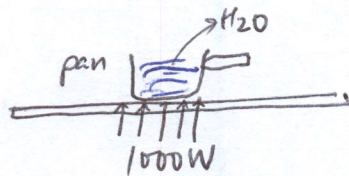16.49 | What is mass m when it is quicker to heat water using stovetop than the microwave ?

→ Heat Capacity $\boxed{C \equiv m \cdot c}$ $\qquad \rightarrow \Delta Q = mc\Delta T = C\Delta T$

$\qquad\qquad\qquad\qquad\qquad \underset{\text{mass specific heat}}{\downarrow}$

Stovetop

→ Need a pan $C = 1400 \frac{J}{\degree K}$
(which gets heat up in the process)

→ Heat supply per unit time :

$\boxed{\dfrac{\Delta Q}{\Delta t_s} = 1000 W}$

pan →H_2O

1000W

Heat supply = Heat absorbed by pan
$\qquad\qquad\qquad$ + Heat absorbed by H_2O

$\Delta t_s = \dfrac{\Delta Q}{1000}$

$\qquad = \dfrac{C_{pan} \cdot \Delta T + m_{H_2O} C_{H_2O} \cdot \Delta T}{1000}$

$\boxed{\text{Stovetop quicker :}} \qquad \Delta t_s < \Delta t_m$

$\dfrac{(C_{pan} + m_{H_2O} C_{H_2O}) \Delta T}{1000} \lesseqgtr \dfrac{m_{H_2O} C_{H_2O} \cdot \Delta T}{625}$

$C_{pan} \qquad < \dfrac{1000}{625} m_{H_2O} C_{H_2O} - m_{H_2O} C_{H_2O}$

$\qquad C_{pan} < m_{H_2O} C_{H_2O} \left(\dfrac{1000}{625} - 1 \right)$

$\boxed{\dfrac{C_{pan}}{C_{H_2O} \left(\frac{1000}{625} - 1 \right)} < m_{H_2O}}$

Microwave

→ Can use paper cup with negligible heat Capacity $C \approx 0$

→ $\boxed{\dfrac{\Delta Q}{\Delta t_m} = 625 W}$

paper cup $C \approx 0$ $\quad H_2O$

625W

Heat supply = Heat absorbed by H_2O

$\Delta t_m = \dfrac{\Delta Q}{625}$

$\qquad = \dfrac{m_{H_2O} C_{H_2O} \cdot \Delta T}{625}$
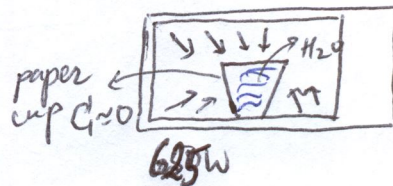
$\Rightarrow \dfrac{1400}{4184 \frac{375}{625}} = 0.558 kg < m_{H_2O}$

Thermal Behavior of Matter (cont.)

When heat is supplied to matter $\boxed{\Delta Q}$

1) Its temperature will change $\underline{\underline{\Delta T}}$
$$\Delta T = \frac{\Delta Q}{m \cdot c}$$

2) If sufficient heat is supplied →
Matter will undergo a <u>change</u> of <u>phase</u> $\Delta Q = mL$
↓ Latent heat

$$\text{Solid} \xrightarrow{L_f} \text{liquid} \xrightarrow{L_v} \text{gas}$$

↑ supply latent heat of fusion   ↑ supply latent heat of vaporization

L_s
supply latent heat of sublimation

3) If heat is supplied to a solid:
Expansion
- linear : $\alpha \equiv \dfrac{\frac{\Delta L}{L} \to \text{length}}{\Delta T \to \text{temp}}$
- volumic $\beta \equiv \dfrac{\frac{\Delta V}{V} \to \text{volume}}{\Delta T \to \text{temp.}}$

Change of Phase:

$T = 25°C$

$T\uparrow$
$\Delta Q = mc\Delta T$
Heat supply → $T\uparrow$
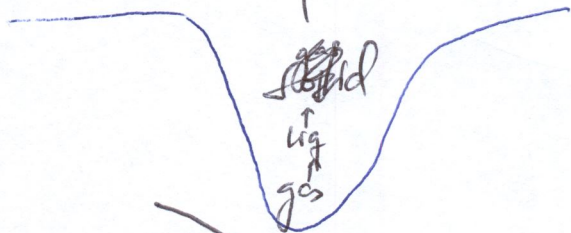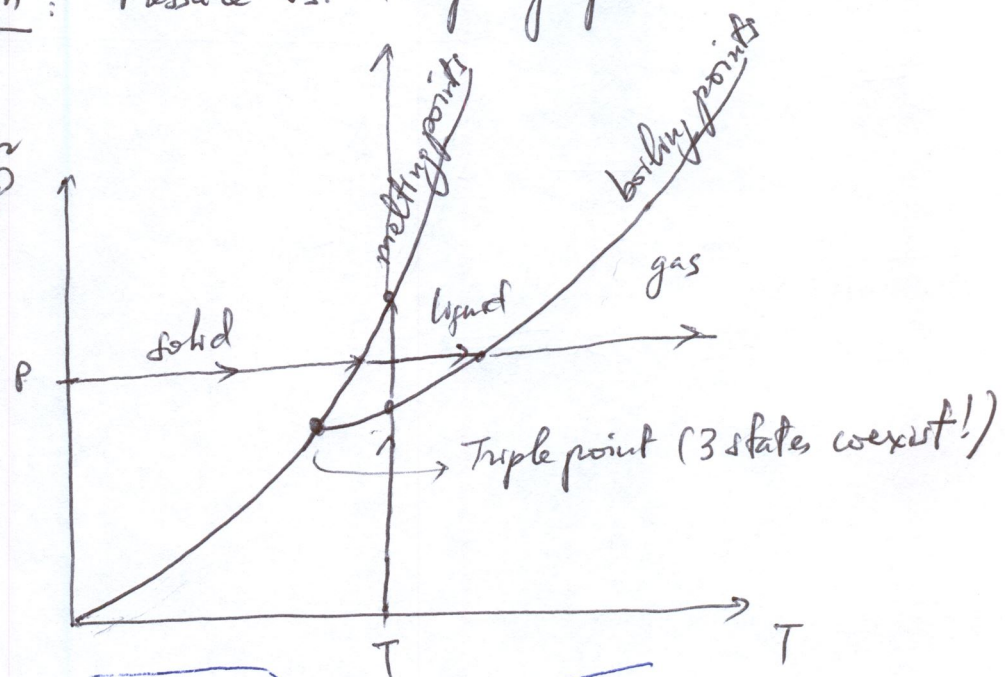
$T = 100°C$
Sufficient Heat is supplied:
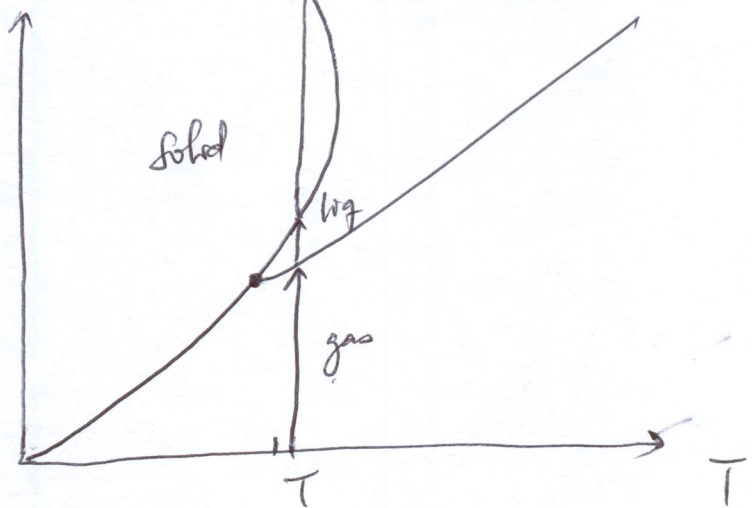H_2O starts to boil
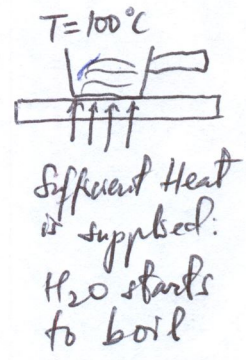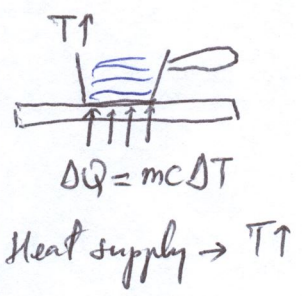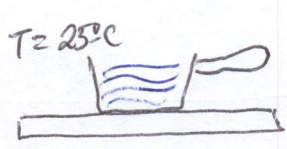
Temp is constant $= 100°C$
$T = 100°C$
if heat is supplied to H_2O @ $100°C$
→ no further increase in temperature!
Heat goes into latent heat of vaporization
$\Delta Q = mL_f$
Change of Phase
liquid → gas

Phase diagram : Pressure vs. Temp. graphs

Normal Matter

P

melting point   boiling point

solid   liquid   gas

P

→ Triple point (3 states coexist!)

T

T

gas
solid
liq
gas

Water: P

solid

liq

gas

T   T

H_2O: gas → liquid → [solid → liquid]

water is special
(we can melt ice by
pressing on it w/o
supplying heat).

16.76/

$$\Delta Q = mc \, \Delta T \qquad (16.3) \qquad \text{Heat transfer}$$

$$H = -kA \, \frac{\Delta T}{\Delta x} \qquad (16.5) \qquad \text{Heat transfer } \underline{\text{rate}}$$
$$\text{per unit time}$$

$$\frac{(16.3)}{\Delta t}: \quad \frac{\Delta Q}{\Delta t} = mc \, \frac{\Delta T}{\Delta t} = -kA \, \frac{\Delta T}{\Delta x}$$

Rate of temperature change $\quad \dfrac{\Delta T}{\Delta t} = -\dfrac{kA}{mc} \dfrac{\Delta T}{\Delta x} \quad$ or $\quad \boxed{\dfrac{\Delta T}{\Delta t} \propto \Delta T}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ "Newton's law of Cooling"

(proportional constant is $\dfrac{kA}{mc \, \Delta x} = \dfrac{1}{RC}$)

Data: $\quad \left. \begin{array}{l} T_H = 20°C \\ T_0 = -15°C \end{array} \right\} \Delta T = 35°C$ or $35°K$

$\boxed{\begin{array}{l} \dfrac{\Delta T}{\Delta t} = -\dfrac{1}{RC} \Delta T \\[2mm] \lim\limits_{\Delta t \to 0} \dfrac{\Delta T}{\Delta t} = -\dfrac{1}{RC} \lim\limits_{\Delta t \to 0} \Delta T \\[2mm] \dfrac{d(T - T_0)}{dt} = -\dfrac{1}{RC}(T - T_0) \end{array}}$

$$C_H = m_H c_H = 6.5 \times 10^6 \, \frac{J}{°K}$$

Thermal resistance $\quad R = 6.67 \times 10^{-3} \, \dfrac{°K}{W}$

$R = \dfrac{\Delta x}{kA}$ $\qquad$ Freezing point : $0°C$

If there is no heat supply, due to constant heat loss, house will reach $0°C$ after time t (?)

Rate of temperature change $\quad \dfrac{\Delta T}{\Delta t} = -\dfrac{1}{6.67 \times 10^{-3} \cdot 6.5 \times 10^6} \, (35) =$

$$\frac{d(\Delta T)}{dt} = -\frac{1}{RC} \Delta T \quad \rightarrow \quad \frac{d(\Delta T)}{\Delta T} = -\frac{dt}{RC} \quad \overset{\int}{\rightarrow} \quad \ln(\Delta T) = \frac{-t}{RC} + \text{const.}$$

$$e^{x+y} = e^x \cdot e^y \qquad \overset{\exp}{\rightarrow} \qquad \Delta T(t) = \boxed{\Delta T(0)} e^{-\frac{t}{RC}}$$

$$\Delta T(t) = \Delta T(0) \, e^{-\frac{t}{RC}}$$

When house reaches $0°C$: $\underbrace{[0 - (-15)]}_{15} = \underbrace{[20 - (-15)]}_{35} \, e^{-\frac{t}{6.67 \cdot 10^{-3} \cdot 6.5 \cdot 10^6}}$

Solve for t : $\quad \dfrac{15}{35} = e^{-\frac{t}{43.355 \cdot 10^3}} \quad \rightarrow \quad \ln\left(\dfrac{15}{35}\right) = -\dfrac{t}{43.355 \times 10^3} \rightarrow t = 36.73 \cdot 10^3 s$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 10.2 \, \text{hrs}$

16.67

Example 16.4
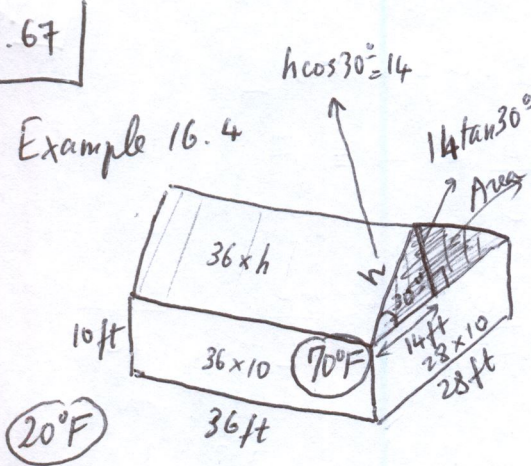
$h\cos 30° = 14$

$\tan 30 = \dfrac{x}{14}$

$x = 14\tan 30°$

Area $14 ft \times 14 \times \tan 30°$

$= \boxed{14 \times 14 \times \tan 30°} = 113$

Walls: $280 \times 2 + 360 \times 2 + 113 \times 2 = 1506 \ ft^2$

Roof Area $= 36 \times \dfrac{14}{\cos 30°} \times 2 = 1164 \ ft^2$

$h\cos 30° = 14 \ ft$

$h = \dfrac{14 \ ft}{\cos 30°}$

$R_{wall} = 0.17 + 11 + 0.65 + 0.55 = 12.37 \ \dfrac{ft^2 \ °F}{\frac{Btu}{h}}$

$R_{roof} = 0.17 + 30 + 0.65 + 0.55 = 31.37$

$H = -k\dfrac{A}{\Delta x}\Delta T = -\dfrac{A}{R}\Delta T$

$\begin{cases} H_{wall} = -\dfrac{1506}{12.37} \ 50 = -6073 \ \dfrac{Btu}{h} \\[4mm] H_{roof} = -\dfrac{1164}{31.37} \ 50 = -1853 \ \dfrac{Btu}{h} \end{cases}$

Total heat loss rate $6073 + 1853 = 7926 \ \dfrac{Btu}{h}$

$\begin{cases} \text{One month}: \quad 7926 \ \dfrac{Btu}{h} \cdot 24h \cdot 30 = 5.7 \cdot 10^6 \ Btu \\[4mm] \text{Gallons of oil}: \quad 5.7 \cdot 10^6 \ Btu \cdot \dfrac{1 \ Gallon}{10^5 \ Btu} = 57 \ Gallons \\[4mm] \text{Cost per month}: \quad 57 \ Gallons \cdot \dfrac{\$2.20}{Gallon} = \$126 \end{cases}$

10 windows $2.5 \times 5 \ ft^2 = 12.5 \ ft^2$ per window

$\begin{cases} \text{All 10 lose heat} \to R = 0.9 \quad (\text{single glazed!}) \\[2mm] \text{4 south facing : collecting solar energy @ rate of } 30 \ \dfrac{Btu}{h \cdot ft^2} \end{cases}$

Total heat loss rate: $\ \cancel{6073} \quad \cancel{125} \ +11026 + 1853 = -12879 \ \dfrac{Btu}{h}$

$H_{wall} = -\dfrac{1506 - 125}{12.37} \ 50 - \dfrac{125}{0.9} \ 50 + \dfrac{50 \times 30}{solar \ gain} = -11026 \ \dfrac{Btu}{h}$
with Window

Cost per month (with windows) $= 12879 \times 24 \times 30 \ \dfrac{\$2.20}{10^5} = \$204$

Solar gain worth $= 50 \times 30 \times 24 \times 30 \times \dfrac{\$2.20}{10^5} = \$23.76$

17.45

→ Fridge extract heat @ $95W = \frac{\Delta Q}{\Delta t}$

→ Water ($m = 0.75kg$) @ $0°C$ (freezing point)

For this water to freeze → extract the latent heat
of fusion $\Delta Q = m_w L_f = 0.75kg \cdot \frac{334kJ}{kg} = 250.5 kJ$

Table 17·1 : $L_f = 334 \frac{kJ}{kg}$ (Normal conditions)

→ $\Delta t = \frac{\Delta Q}{\frac{\Delta Q}{\Delta t}} = \frac{250500 J}{95 \frac{J}{s}} = 2636 s \cdot \frac{1 min}{60 s} = 43.9 min$

17.64

"Ice box"
Keeping fridge cold
using ice to absorb
heat.
Ice absorbs latent heat
of fusion to melt.

"Fridge"
Use electric motor to
extract heat

We need enough ice to absorb same amount of heat
that is extracted

$T_k = T_c + 273.16$
$\Delta T = 20°C - 0°C = 20°C$
$\Delta T = (20 + 273.16) - (0 + 273.16)$
$\quad = 20°K$
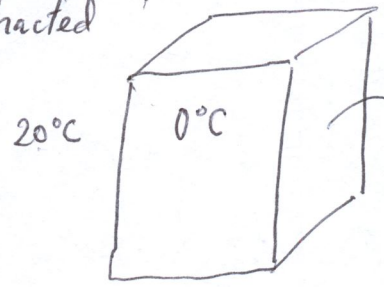
$20°C$ ☐ $0°C$ → $R = 0.12 \frac{°K}{W}$

Keep inside @ $0°C$ for 15 hrs

→ W/o power heat transfer in @ a rate $H = k \frac{A}{\Delta x} \Delta T$

$$H = \frac{20 °K}{0.12 \frac{°K}{W}} = 166.67 W \qquad = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{R}$$

→ In 15 hrs, total heat that will transfer in is

$$\Delta Q = 166.67 \frac{J}{s} \cdot 15 \cdot 3600 s = 9 \times 10^6 J$$

→ If we have enough ice that absorbs this heat then
no heat is left to raise the inside temp. of the fridge
$\Delta Q = m_{ice} L_f \rightarrow m_{ice} = \frac{\Delta Q}{L_f} = \frac{9 \times 10^6}{334 \times 10^3} kg = 26.94 kg$

Ch 18 Heat, Work, 1st Law of TD

$Q =$ heat absorbed by a system

$W =$ work done by same system

$\Delta U =$ change of internal energy of this system

Conservation of energy: $\qquad \Delta U = Q - W$
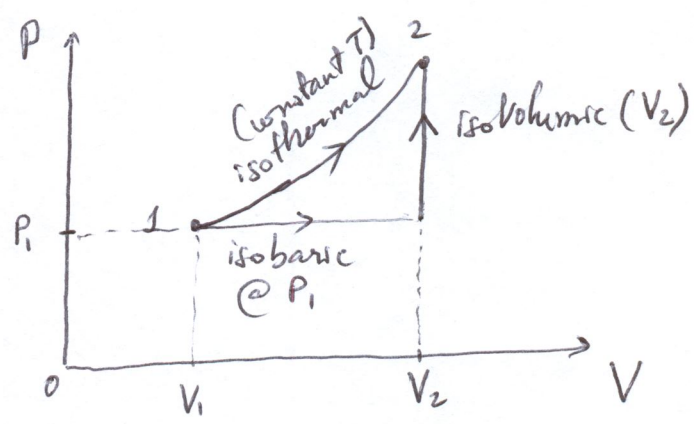
1) $\Delta U, T, P, V$ are <u>state</u> variables

Given a state, they are the same no matter how did the system arrive at that state. These state variables don't depend on a process

2) $Q \& W$ are <u>not</u> state variables. They depend on a particular process
$\begin{cases} \text{isobaric} & (P \text{ constant}) \\ \text{isovolumic} & (V \text{ constant}) \\ \text{isothermal} & (T \text{ constant}) \\ \text{adiabatic} & (Q = 0) \end{cases}$

→ $\boxed{\text{PV diagram:}}$

→ State is a point on the PV diagram

→ Work done by system:

$$W = \int P \, dV$$: geometrically, this integral is the area under the pressure curve in a PV diagram.

$W = F \cdot \Delta x = P \cdot \underbrace{A \cdot \Delta x}_{\Delta V}$

Ch 18 Heat, Work, 1st Law of TD

Q = heat absorbed by a system.

W = work done by same system. (In text: W = work done on system)

ΔU = change of internal energy of this system

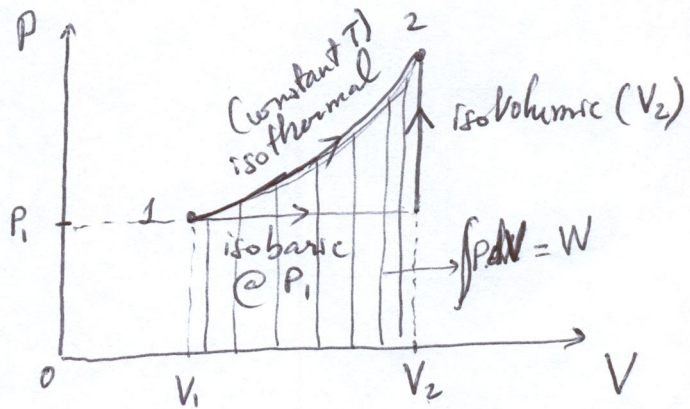Conservation of energy: $\Delta U = Q - W$

1) $\Delta U, T, P, V$ are <u>state variables</u>

Given a state, they are the same no matter how did the system arrive at that state. These state variables don't depend on a process

2) $Q \& W$ are <u>not</u> state variables. They depend on a particular process
$\begin{cases} \text{isobaric} & (P \text{ constant}) \\ \text{isovolumic} & (V \text{ constant}) \\ \text{isothermal} & (T \text{ constant}) \\ \text{adiabatic} & (Q = 0) \end{cases}$

→ PV diagram:
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→ Work done by system:
$$W = \int P\,dV \quad : \quad \text{geometrically, this integral is the area under the pressure curve in a PV diagram.}$$

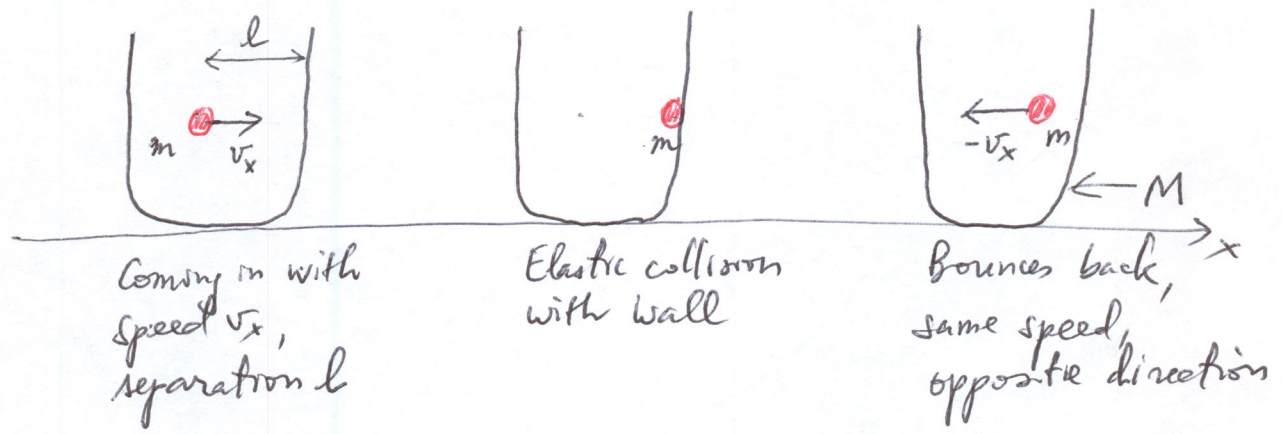$W = F \cdot \Delta x = P \cdot \underbrace{A \cdot \Delta x}_{\Delta V}$

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Microscopic to Macroscopic Connection in an Ideal Gas

→ Microscopic: looking at the pressure exerted by individual gas molecule

→ Macroscopic: adding up these pressures for N molecules
$(N_A = 6.028 \times 10^{23})$: $\quad P = \sum_{i=1}^{N} P_i$

Pressure by a single molecule comes from the transfer of momentum when it collides with the containing wall.

Coming in with speed v_x, separation l

Elastic collision with wall

Bounces back, same speed, opposite direction

→ What is the momentum transfer to wall?

Newton's 2nd Law: $\quad \vec{F}_{net} = \dfrac{d\vec{p}}{dt}$
(Most general version)

Net force on system of molecule + container in x-direction is zero

$\dfrac{d\vec{p}}{dt} = 0 \Rightarrow \vec{P}_i = \vec{P}_f$ $\begin{cases} i: \text{before collision} \\ f: \text{after collision} \end{cases}$

$$m v_x = -m v_x + \underbrace{MV_x}_{2mv_x}$$
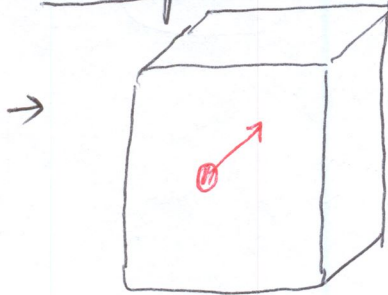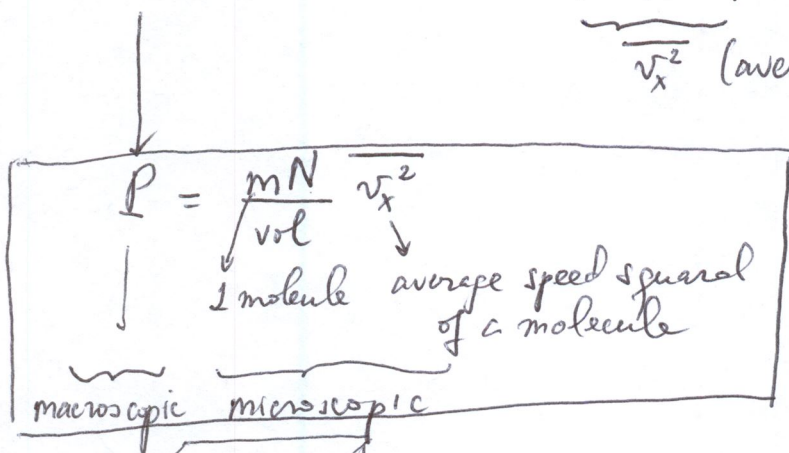
Momentum received by container after collision!

→ Relate this momentum transfer to pressure exerted by that molecule P_i

$$P_i = \frac{F_i}{A} = \frac{\frac{\Delta p_i}{\delta t}}{A} = \frac{\frac{2mv_{x_i}}{\frac{2\ell}{v_{x_i}}}}{A} = \frac{mv_{x_i}^2}{\underset{\text{Volume or vol}}{\boxed{A\ell}}}$$

→ Macroscopic pressure:

$$P = \sum_{i=1}^{N} P_i = \frac{mN}{vol}\left(\frac{\sum_{i=1}^{N} v_{x_i}^2}{N}\right)$$

$$\underbrace{\qquad\qquad}_{\overline{v_x^2}\ (\text{average of } v_x^2)}$$

$$\boxed{P = \frac{mN}{vol}\,\overline{v_x^2}}$$

$\underset{\text{macroscopic}}{\underbrace{\qquad}}$ $\underset{\text{microscopic}}{\underbrace{\text{1 molecule} \quad \text{average speed squared of a molecule}}}$

In general there are 3 directions for momentum transfer

$$\overline{v_x^2} = \frac{1}{3}\,\overline{v^2} \qquad (v \text{ total velocity in 3D})$$

$$P = \frac{1}{3}\frac{mN}{vol}\,\overline{v^2} = \frac{mN}{3V}\,\overline{v^2} \Rightarrow \boxed{PV = mN\frac{\overline{v^2}}{3}}$$

$$vol \equiv V$$

→ Comparing with the equation of state for an Ideal Gas $\boxed{PV = kNT}$

$$kT = m\frac{\overline{v^2}}{3} \qquad \xrightarrow{\times \frac{3}{2}} \qquad \boxed{\frac{3}{2}kT = \frac{1}{2}m\overline{v^2}}$$

$\underset{\text{macroscopic}}{\uparrow}$ $\underset{\text{microscopic}}{\uparrow}$

$$\frac{3}{2}kT \;=\; \frac{1}{2}m\overline{v^2}$$

$\underbrace{\qquad}_{\text{macroscopic}}$ $\underbrace{\qquad}_{\substack{\text{microscopic} \\ \;\rightarrow \text{ average KE of} \\ \text{one gas molecule in 3D}}}$

Note: 1) KE of gas molecules determines the overall gas temperature

2) If gas is confined in 1D : $\left(\overline{v_x^2} = \frac{1}{3}\overline{v^2} \Rightarrow \frac{1}{2}kT\right)$

$\downarrow$ KE of gas contributes $\frac{1}{2}kT$ per dimension or degree of freedom

3) Diatomic molecules :

w/o stick $\left\{ \begin{array}{l} \text{Atom ①} : 3 \text{ d.o.f.'s} \\ \text{Atom ②} : \underline{3 \text{ d.o.f.'s}} \\ \qquad\qquad\;\; 6 \text{ d.o.f's} \end{array} \right.$ w/ stick $\left\{ 5 \text{ d.o.f.'s} \right.$

$\downarrow$

$\rightarrow \frac{5}{2}kT$

Isothermal Process (constant T)

Work done by $\boxed{\text{ideal}}$ gas undergoing an isothermal process $\textcircled{1} \rightarrow \textcircled{2}$

$$W = \int_1^2 P\, dV = nRT \int_1^2 \frac{dV}{V} = nRT\left[\ln V\right]_1^2 = nRT\underbrace{\left(\ln V_2 - \ln V_1\right)}_{\ln\left(\frac{V_2}{V_1}\right)}$$

$$\begin{cases} \text{ideal gas} \\ \text{equation} \\ PV = nRT \\ T \text{ constant} \end{cases}$$

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$
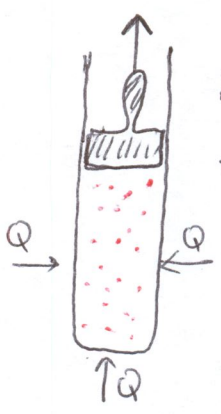
After pushing piston then let go with heat supply of $Q \rightarrow$ gas expand & does work on piston if process is isothermal:

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

ideal gas — isothermal expansion
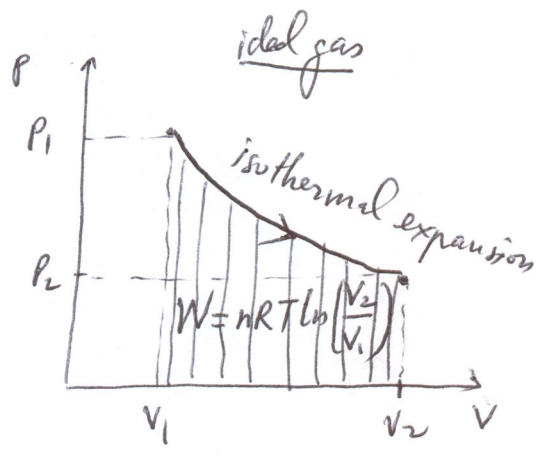
1) What is the change of internal energy of the ideal gas ΔU?

Ideal $\xrightarrow{a)}$ no interaction b/w molecules $\rightarrow$ only KE for internal energy

$\xrightarrow{b)}$ $\frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$

$\xrightarrow{c)}$ Isothermal: $\Delta T = 0 \Rightarrow \Delta KE = \Delta U = 0$

2) What is the heat absorbed Q?

1st Law of TD: $\Delta U = Q - W$ or $0 = Q - W \Rightarrow Q = W$

Ideal gas in isothermal process absorbs heat Q and returns equal amount of work.

Isovolumic Process (constant Volume)

$$W = \int_1^2 P dV = 0 \text{ (no area)}$$
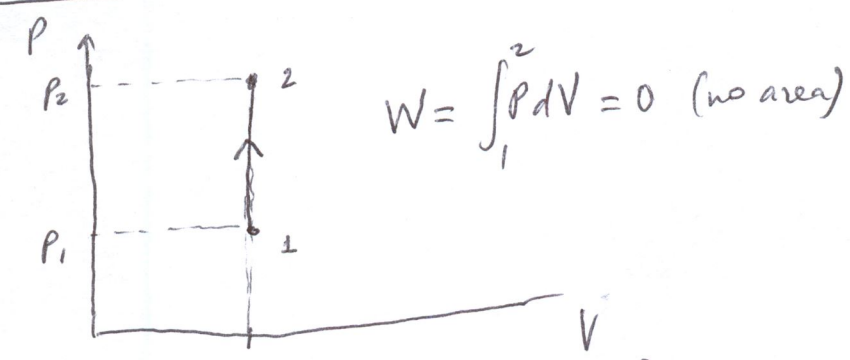
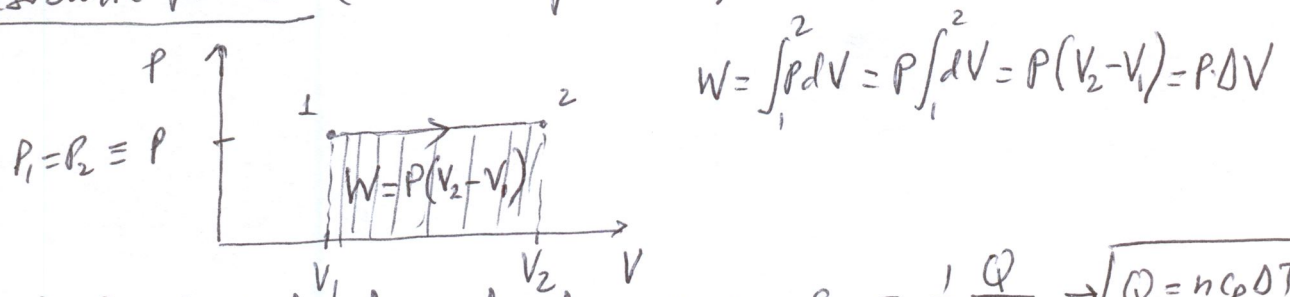1) 1st Law TD : $\Delta U = Q - W = Q$

Gas in a closed container

↑T if it absorbs Q
↓T if it ejects Q

2) Specific heat at constant volume

$$C_V = \frac{1}{n} \frac{Q}{\Delta T} = \frac{1}{n} \frac{\Delta U}{\Delta T} \rightarrow \boxed{\Delta U = n C_V \Delta T}$$

(depends on type of gas, monoatomic, diatomic, etc.)
KE contributes different amount of $\frac{1}{2} kT$)

Isobaric Process (constant pressure)

$$W = \int_1^2 P dV = P \int_1^2 dV = P(V_2 - V_1) = P \cdot \Delta V$$

$P_1 = P_2 \equiv P$

$$\boxed{W = P(V_2 - V_1)}$$

1) Specific heat at constant pressure $\quad C_p \equiv \frac{1}{n} \frac{Q}{\Delta T} \rightarrow \boxed{Q = n C_p \Delta T}$

2) Ideal gas equation : $\quad P \cdot \Delta V = n R \Delta T = W$
$\qquad \qquad \qquad \qquad \qquad \qquad \uparrow \text{Isobaric}$

3) 1st Law TD: $\qquad \Delta U = Q - W$

$$n C_V \Delta T = n C_p \Delta T - n R \Delta T$$

$$C_V = C_p - R \qquad \text{or} \qquad \boxed{C_p = C_V + R}$$

$$R = 8.314 \frac{J}{mol \,°K}$$

$C_p > C_V$ (a gas at constant P can absorb
more heat than one @ constant V)
since @ constant V can't use that
heat to do any work $\longrightarrow$

$$\boxed{Q = \Delta U + W \begin{cases} \text{const } P : \Delta U + W \\ \text{const } V : \Delta U \end{cases}}$$

Adiabatic Process $(Q=0)$ $\longleftrightarrow$ $\boxed{\Delta U = -W}$
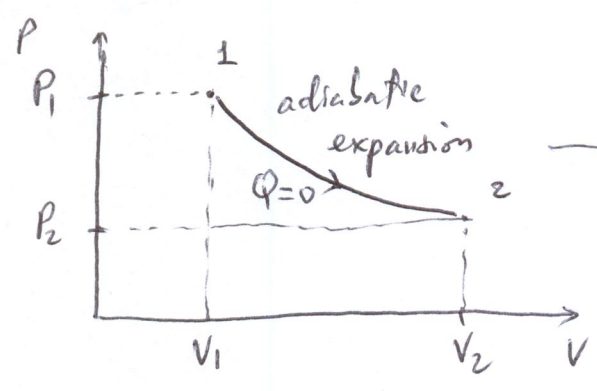
$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

γ: coefficient that depends on type of gas $= \frac{C_P}{C_V}$

(monoatomic or diatomic)

1) Adiabatic process $P \cdot V^{\gamma} = \text{constant}$

① → ② adiabatic expansion : $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

Ideal gas $PV = nRT$ & $P = nR\frac{T}{V} \to T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

2) Values for γ ; $\gamma \equiv \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$

$\gamma = \frac{C_V + R}{C_V} =$

Monoatomic gas : $\boxed{\gamma = \frac{\frac{3}{2}R + R}{\frac{3}{2}R} = \frac{\frac{3}{2}+1}{\frac{3}{2}} = \frac{5}{3} = 1.67}$ (d.o.f = 3)

Diatomic gas $\boxed{\gamma = \frac{\frac{5}{2}R + R}{\frac{5}{2}R} = \frac{\frac{5}{2}+1}{\frac{5}{2}} = \frac{7}{5} = 1.4}$

$C_V =$

Monoatomic gas $C_V = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{1}{n}\frac{\frac{3}{2}k\Delta T N}{\Delta T} = \frac{1}{\frac{N}{N_A}}\frac{3}{2}kN = \frac{3}{2}kN_A = \frac{3}{2}R$ (3 d.o.f)
$\underbrace{\phantom{\frac{3}{2}kN_A}}_{R}$

$n \equiv \frac{N}{N_A}$; $R \equiv k N_A$

Diatomic gas : $C_V = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{1}{n}\frac{\frac{5}{2}k\Delta T N}{\Delta T} = \frac{1}{\frac{N}{N_A}}\frac{5}{2}kN = \frac{5}{2}kN_A = \frac{5}{2}R$ (5 d.o.f)

18.58 Mixture of A_r & O_2 $\begin{cases} A_r: \text{ monoatomic } \gamma = 1.67 \\ O_2: \text{ diatomic } \gamma = 1.4 \end{cases}$

↳ Adiabatic expansion $\begin{cases} V_1 \longrightarrow V_2 = 2V_1 \\ P_1 \longrightarrow P_2 = \frac{P_1}{3} \end{cases}$

Fraction of Argon f in this mixture ? → will affect overall γ of mixture : via its contribution to the total internal energy of mixture.

Mixture: $C_v = f\frac{3}{2}R + (1-f)\frac{5}{2}R = R\left(\frac{3}{2}f + \frac{5}{2} - \frac{5}{2}f\right)$

$\underbrace{\phantom{f\frac{3}{2}R}}_{A_r} \quad \underbrace{\phantom{(1-f)\frac{5}{2}R}}_{O_2} \qquad = R\left(\frac{5}{2} - f\right)$

$$\boxed{\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{R\left(\frac{5}{2}-f\right) + R}{R\left(\frac{5}{2}-f\right)} = \frac{\frac{7}{2}-f}{\frac{5}{2}-f}}$$

Calculate γ from adiabatic expansion:

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$$

$$\ln\left(\frac{P_1}{P_2}\right) = \gamma \ln\left(\frac{V_2}{V_1}\right)$$

$$\rightarrow \gamma = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln(3)}{\ln(2)} = 1.58$$

$$\rightarrow 1.58 = \frac{3.5 - f}{2.5 - f} \rightarrow (2.5 - f)1.58 = 3.5 - f$$

$$2.5 \cdot 1.58 = 3.5 + f(1.58 - 1)$$

$$f = \frac{2.5 \cdot 1.58 - 3.5}{0.58} = 0.78$$
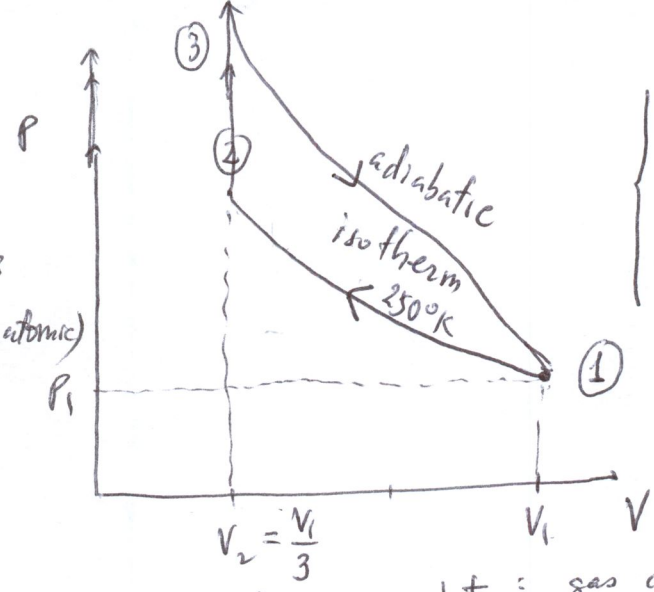
$\rightarrow \boxed{\begin{array}{l} A_r \ 78\% \\ O_2 \ 22\% \end{array}}$ $\underbrace{1.4}_{\text{diatomic}} < \gamma_{\text{mixture}} = 1.58 < \underbrace{1.67}_{\text{monoatomic}}$

closer → $f_{A_r} > f_{O_2}$

18.48

$\boxed{V_1 = 25L}$ ideal gas

$\gamma = 1.67$ (monoatomic)

$T = 250°K$

$\boxed{P_1 = 50 kPa}$

$1 \to 2$ isothermal compression

$2 \to 3$ isovolumic

$3 \to 1$ adiabatic expansion

$W = ?$ work done by gas $\begin{cases} + : \text{ gas did work} \\ - : \text{ gas received work or work done on gas} \end{cases}$

$$W = \underset{\substack{\text{isothermal} \\ \text{compression} \\ (-)}}{W_{12}} + \underset{0}{W_{23}} + \underset{\substack{\text{adiabatic} \\ \text{expansion} \\ (+)}}{W_{31}}$$

$$= \underbrace{\frac{nRT}{P_1 V_1} \ln\left(\frac{V_2}{V_1}\right)}_{\text{we have all}} + \underbrace{\frac{P_3 V_3 - P_1 V_1}{\gamma - 1}}_{\substack{\text{(initial = 3} \\ \text{final = 1)} \\ \text{we need } P_3}}$$

Use adiabatic process ③ → ① to find P_3

$$P_1 V_1^{\gamma} = P_3 V_3^{\gamma} \rightarrow P_3 = P_1 \left(\frac{V_1}{V_3}\right)^{\gamma} = P_1 \, 3^{\gamma}$$

$$\left(V_3 = V_2 = \frac{V_1}{3}\right)$$

$$\Rightarrow W_{31} = \frac{P_3 V_3 - P_1 V_1}{\gamma - 1} = \frac{3^{\gamma} P_1 \frac{1}{3} V_1 - P_1 V_1}{\gamma - 1} = \frac{(3^{\gamma - 1} - 1) P_1 V_1}{\gamma - 1}$$

$$W = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) + P_1 V_1 \frac{3^{\gamma - 1} - 1}{\gamma - 1} = P_1 V_1 \left[\ln\left(\frac{1}{3}\right) + \frac{3^{\gamma - 1} - 1}{\gamma - 1}\right]$$

$$= 50 \cdot 10^3 \cdot 25 \cdot 10^{-3} \left[\ln\left(\frac{1}{3}\right) + \frac{3^{0.67} - 1}{0.67}\right] J = 656 J \begin{array}{l} \text{(Gas did} \\ \text{work, delivered} \\ 656 J \text{)} \end{array}$$

$\boxed{1 m^3 = 10^3 L} \rightarrow \dfrac{10^3 L}{1 m^3}$ or $\dfrac{1 m^3}{10^3 L} \rightarrow 25L \cdot \dfrac{1 m^3}{10^3 L} = 25 \cdot 10^{-3} m^3$